LEVEL II

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A TWO-DIMENSIONAL COMPUTER MODEL FOR THE STEADY-STATE OPERATION OF MOSFETs

prepared by

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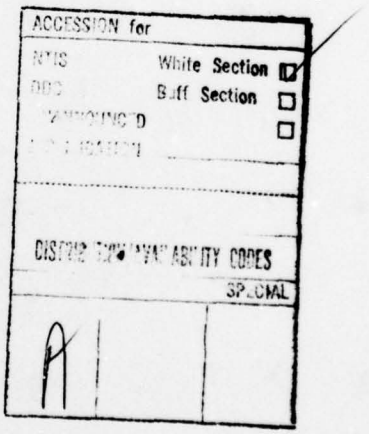

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This report describes in detail a two-dimensional computer model for the steady-state operation of an MOSFET, and is intended to serve as a USER'S MANUAL for that model. Detailed flow charts are included.

The model is shown to produce results in good agreement with those produced by a prototype model which is the proprietary property of IBM. It is also shown to agree well with experimental data obtained from a p-channel device whose parameters have been well characterized by means of test sites.

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FOREWORD

Custom integrated semiconductor circuits are often needed for military applications that involve a vast number of different, highly specialized, electronic systems, although seldom are these systems called for in large quantity. This combination produces an economically prohibitive situation. The large initial cost for designing integrated circuits, in conjunction with a small production requirement, creates an unreasonably large cost per IC chip. The source of this difficulty lies in empirical, and costly, engineering techniques used during both IC design, and production start-up. Empirical techniques are used during IC design to solve problems for which there is inadequate basic understanding. Similar empirical techniques are also used during production start-up to overcome problems that presently cannot be solved by design engineers.

An important source of this problem lies in the lack of adequate models for the design and development of IC structures. This effort is part of the DARPA funded research, pursued to ARPA Order No. 2985, and is intended to replace the empirical and costly conventional engineering approach to IC development for military systems with a computed aided cost effective method. In particular, this report describes, in detail, a two-dimensional computer model for the steady-state operation of an MOSFET and is intended to serve as a USER's MANUAL for that model.

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CHAPTER I

A Two-Dimensional Model for MOSFET Operation

1.0 Introduction

This report is an addendum to the Final Research and Development Technical Report under this contract on Computer Aided Engineering of Semiconductor Integrated Circuits, dated August 31, 1977. The present report presents further details concerning a two-dimensional mathematical model for an MOSFET, and is intended to serve as a USER's MANUAL for the computer program which has been developed to implement that model.

The model described herein is patterned from the earlier work of M.S. Mock who developed a two-dimensional MOSFET model for IBM [1-3]. This useful analytic tool, which has produced results in good agreement with experiment [4], was provided to us by IBM with the understanding that it would be regarded as their proprietary property. That computer program was utilized extensively in the development of a one-dimensional MOSFET model described in detail in Chapter I of the Final Report named above.

In order to make such a two-dimensional analytic tool available to government laboratories, and to the semiconductor industry, the development of the model reported herein was undertaken with the intention of disseminating the resultant computer program to those desiring it, accompanied by this USER's MANUAL.

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[1] M.S. Mock, Solid State Electronics, 16, 601 (1973).

[2] D.P. Kennedy, Mathematical Simulation of the Effects of Ionizing Radiation on Semiconductors, Scientific Report No. 1, AFCRL-71-0272 (1971).

[3] D.P. Kennedy, Mathematical Simulation of the Effects of Ionizing Radiation on Semiconductors, Final Report, AFCRL-72-0257 (1972).

[4] D.P. Kennedy, Private Communication.

It should be emphasized that the proprietary nature of the IBM program has been scrupulously observed during this development. Some of the algorithms implemented in our model are due to Mock, but only those which he has described in the literature [1-3]. No attempt has been made to copy the IBM program. We have, of course, made liberal use of that program as a means of providing numerical checks of the performance of our model. The availability of that program for this purpose is gratefully acknowledged.

2.0 Description of the Model

The computer model treats an n-channel enhancement mode MOSFET in which the conducting channel is formed by a gate-induced inversion layer just below the interface between a uniformly doped p-type substrate and the SiO_2 layer (hereinafter named the oxide) upon which the gate metallization is deposited. The inherent thinness of the inversion layer justifies the assumptions made that the net electron-hole recombination rate and the hole current are negligible in the steady state operation of this device. Details are given in the following subsections.

2.01 Basic Geometry

Figure 1(a) depicts such an n-channel device in which the source and drain consist of n^+ islands diffused into the p-substrate. The thinness of the gate-induced inversion layer of mobile electrons formed just below the oxide-silicon interface makes possible the adoption of the simplified rectangular geometry of Figure 1(b), wherein the vertical metallurgical p-n junctions (assumed to be abrupt transitions between p and n^+ regions) are located at the left and right borders of the rectangular region treated. Large gradients in electric potential and, consequently, in the electron and hole densities are expected in the immediate vicinity of those junctions, requiring a rather dense array of the geometric points at which those variables are calculated with the finite-difference equa-

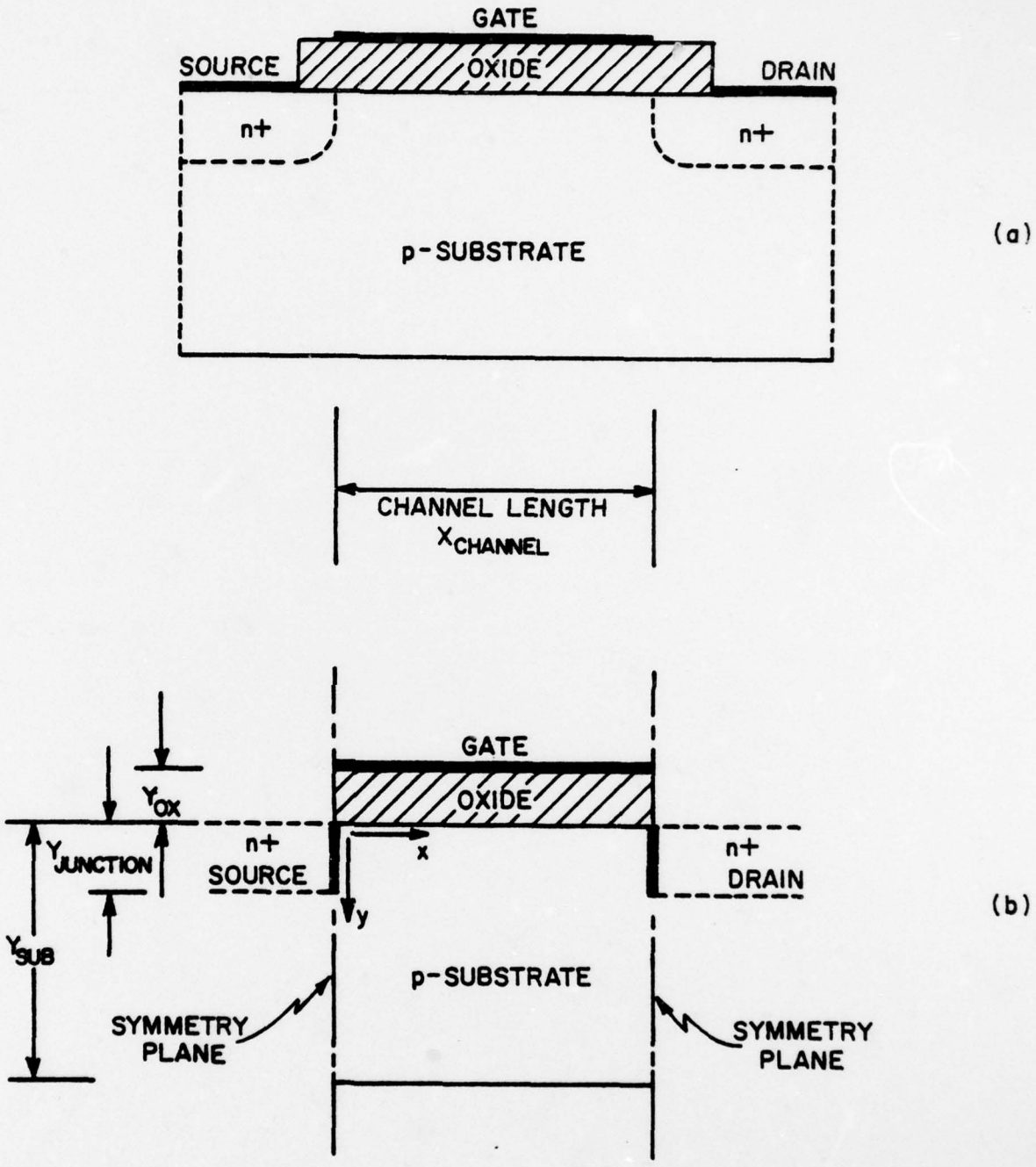

Figure 1 (a) Cross-section of an n-channel MOSFET.

(b) Rectangular region modeled. The abrupt metallurgical junctions between source-substrate and drain-substrate are regarded as planar and at the left and right borders of the region modeled.

tions employed. The same is true in the region just below the oxide-silicon interface where the inversion layer forms. On the other hand, the density of such points in regions remote from those surfaces can be relatively sparse. By placing the source and drain junctions at the borders of the rectangular region treated, such a graded lattice of points is readily defined. Figure 2 illustrates the graded lattice employed by the model with a 41 (horizontal) by 25 (vertical) array overlayed on the rectangle. The prescription adopted for generating this graded lattice is precisely that suggested by Mock [1].

2.02 Dimensionless Variables

For computational purposes, the adoption of dimensionless (normalized) variables at the outset results in two benefits: (1) unnecessary multiplications by redundant constants such as ε_o, q, kT/q, n_i, etc. can be precluded, and (2) numerical scaling causing most variables to lie within reasonable numerical ranges can be achieved.

Understanding that such dimensionless variables will be utilized in all of which follows (with few exceptions which will be clearly flagged), we shall designate physical variables (i.e., those which possess dimensions) in this subsection by means of primed symbols, using corresponding unprimed symbols for their dimensionless counterparts. This convention provides the advantage of utilizing conventional symbols such as $\bar{J}$ for current density, n and p for electron and hole densities, ψ for electric potential, etc., even though those variables are normalized. In this vein, the dimensionless variables employed in our model are defined in Table I.

2.03 Basic Equations

Having adopted the assumptions of negligible hole current and electron-hole recombination, the dimensionless equations describing the physical mechanisms within this semiconductor device are:

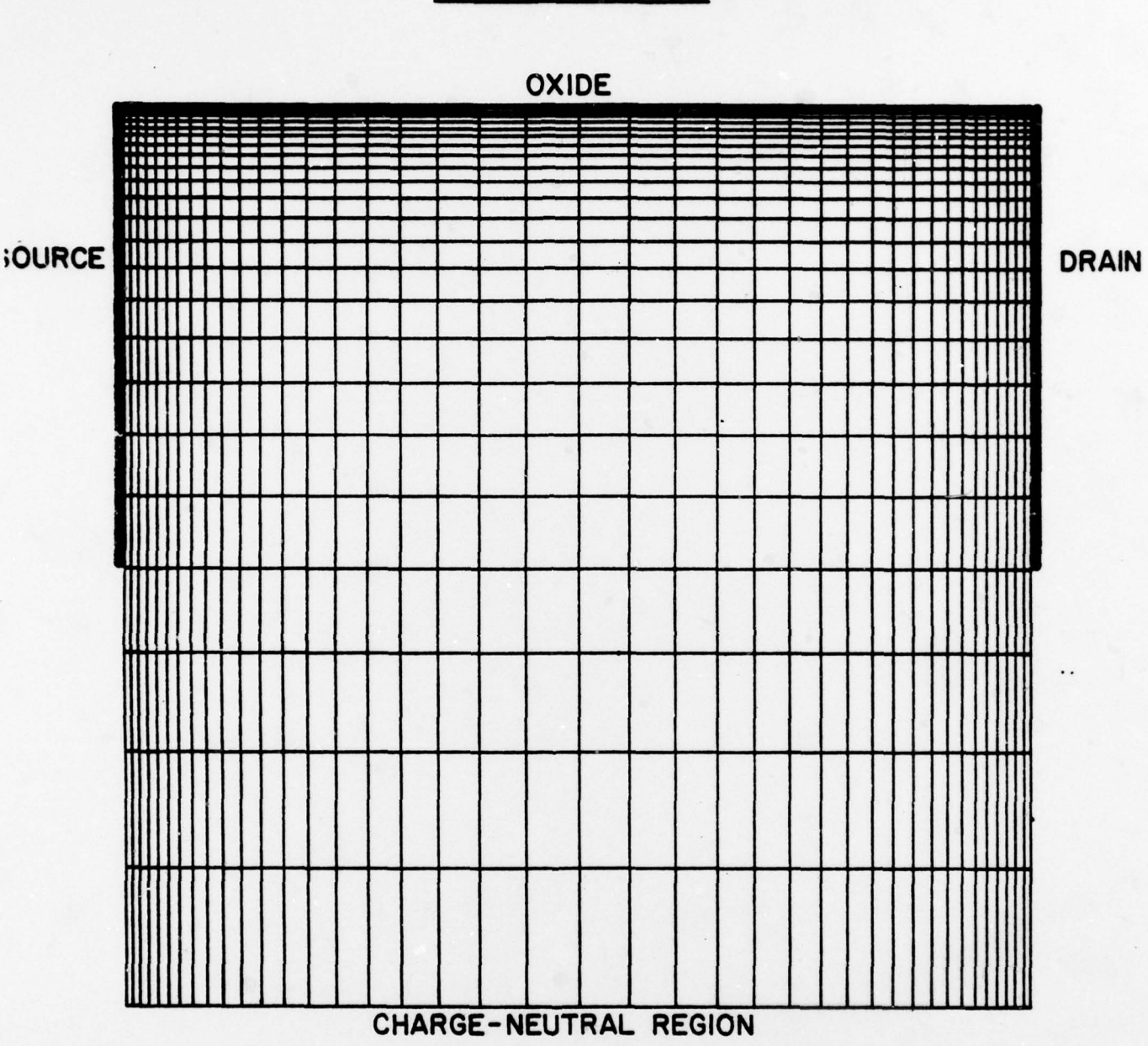

Figure 2 Graded lattice of points used in the finite-difference equations dealt with in the computer model.

TABLE I--DEFINITION OF DIMENSIONLESS VARIABLES
(κ_s is the relative permittivity of silicon)

Define:

$L_D' = \sqrt{\frac{\kappa_s \varepsilon_o' (k'T'/q')}{q' n_i'}}$	,	Intrinsic Debye length of silicon
μ_n'	,	Low field electron mobility

Then Let:

$\psi' = (k'T'/q')\psi$ $\phi_n' = (k'T'/q')\phi_n$ $\phi_p' = (k'T'/q')\phi_p$	,	Electric potential, and electron and hole quasi-Fermi potentials
$N_D' = n_i' N_D$ $N_A' = n_i' N_A$ $n' = n_i' n$ $p' = n_i' p$	,	Donor, acceptor, electron, and hole number densities
$x' = L_D' x$ $y' = L_D' y$	,	Cartesian coordinates
$\bar{J}' = \left[\frac{(k'T'/q')\mu_n' n_i' q'}{L_D'} \right] \bar{J}$	,	Electron current density
$Q_{ss}' = \left[\frac{(k'T'/q')\kappa_s \varepsilon_o'}{L_D'} \right] Q_{ss}$	,	Surface charge density
$\mu' = \mu_n' \mu$	,	Electron mobility

$$\nabla^2\psi = N_A + n - p \ , \tag{1}$$

$$n = \exp(\psi - \phi_n) \ , \tag{2}$$

$$p = \exp(\phi_p - \psi) \ , \tag{3}$$

$$\bar{J} = -\mu n \nabla\phi_n \ , \tag{4}$$

$$0 = \nabla\phi_p \ , \tag{5}$$

$$\nabla\cdot\bar{J} = 0 \ . \tag{6}$$

Equation (1) is Poisson's equation for the electric potential ψ, eqs. (2) and (3) express the use of Boltzmann statistics for the electron and hole number densities n and p (i.e., nondegenerate doping of the p-substrate is implied), eq. (4) relates the electron current density vector $\bar{J}$ to the electron quasi-Fermi potential ϕ_n and number density n in the usual manner [5], eq. (5) enforces zero hole current density, while eq. (6) assures conservation of charge under the assumed condition of zero net recombination of electron-hole pairs.

The above equations apply only in the rectangular p-substrate region. The conditions in the oxide are described by:

$$\nabla^2\psi = 0 \ , \tag{7}$$

and $n = p = \bar{J} \equiv 0$, with appropriate boundary conditions to match the tangential and normal electric field at the oxide-silicon interface.

The divergence-free flow of electron current expressed by (6) can be enforced by introducing a stream function $\theta(x,y)$, as did Mock, such that the x and y components of $\bar{J}$ are given by:

$$J_x = J_o \frac{\partial\theta}{\partial y} \ , \tag{8}$$

$$J_y = -J_o \frac{\partial\theta}{\partial x} \ , \tag{9}$$

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[5] S.M. Sze, Physics of Semiconductor Devices, p. 96, Wiley, N.Y. (1969).

thereby assuring that $\nabla \cdot \bar{J} = 0$. Here, J_o is a scalar constant whose determination will be discussed later. We next seek the differential equation which must be satisfied by this stream function.

Solving (2) for the electron quasi-Fermi potential ϕ_n, and substituting the result into (4), we obtain:

$$\bar{J} = \mu e^{\psi} \nabla (n e^{-\psi}) \ . \tag{10}$$

Identifying the x and y components of (10) with the corresponding expressions of (8) and (9), we have:

$$J_o \frac{\partial \theta}{\partial y} = \mu e^{\psi} \frac{\partial}{\partial x} (n e^{-\psi}) \ , \tag{11}$$

$$J_o \frac{\partial \theta}{\partial x} = -\mu e^{\psi} \frac{\partial}{\partial y} (n e^{-\psi}) \ . \tag{12}$$

Divide both sides of these intermediate equations by μe^{ψ}, then differentiate the first partially with respect to y, the second partially with respect to x, and add the two, obtaining:

$$\frac{\partial}{\partial x} \left(\mu^{-1} e^{-\psi} \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu^{-1} e^{-\psi} \frac{\partial \theta}{\partial y} \right) = 0 \ . \tag{13}$$

Eq. (13) is a second order differential equation in θ, given $\mu(x,y)$ and $\psi(x,y)$. Given its solution, the electron current density vector $\bar{J}$ is then known, from (8) and (9).

The manner in which the current density scale factor J_o is determined is best deferred until after a discussion of the boundary conditions which must be satisfied by the solutions of these basic equations.

2.04 Boundary Conditions

We regard the left and right borders of the rectangular region dealt with by the model as symmetry planes such that ψ, θ, n, p, and J_y are even functions of x, while J_x is an odd function. Image theory then requires that these functions all be periodic in x, the periodicity being twice the channel

length $X_{channel}$ of Figure 1(b). A consequence of this implied periodicity is that the source and drain islands shown dotted in Figure 1(a) are "squeezed out of the picture," being replaced by infinitesimally thin source and drain "contacts" which serve as sources or sinks for current and electric flux lines. The assignment of boundary conditions on ψ and n to these "contacts" bears careful scrutiny.

2.04.1 Boundary Conditions on ψ and n--We select the charge-neutral source island as the zero reference for electric potential, $\psi = 0$ (even though that region has been "squeezed out of the picture" in the model), and assume that the contact potentials of all metal-semiconductor and metal-oxide contacts are identical and, therefore, can be ignored.* The p-substrate, in the absence of any external bias voltages, assumes the normalized "built-in" potential:

$$\psi_{EQ} = -\ell n(N_D N_A) \ , \qquad (14)$$

which is also assumed by the gate. If the substrate and gate are then biased positively** with respect to the source at potentials V_{SUB} and V_G, respectively, the boundary conditions stipulated along the two horizontal borders in Figure 3(a) are obtained.

Consider next the source and drain "contacts" which, as stated above, are assumed to coincide with the metallurgical junction planes. Letting U be the normalized potential with zero reference redefined (for the present discussion) to be that of the charge-neutral p-region, Kennedy [6] shows rigorously that the potential U_J resulting in the plane of the metallurgical junction defining an abrupt p-n junction is:

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*Differences in contact potential of the gate and substrate contacts and of the oxide-silicon interface can be modeled by assigning appropriate numerical values to the surface charge term Q_{SS}, for which our model does account.

**Later is this development, we tacitly assume that V_{SUB} will always be a negative numeric.

[6] D.P. Kennedy, IEEE Trans. Electron Devices, ED-22, 988 (1975).

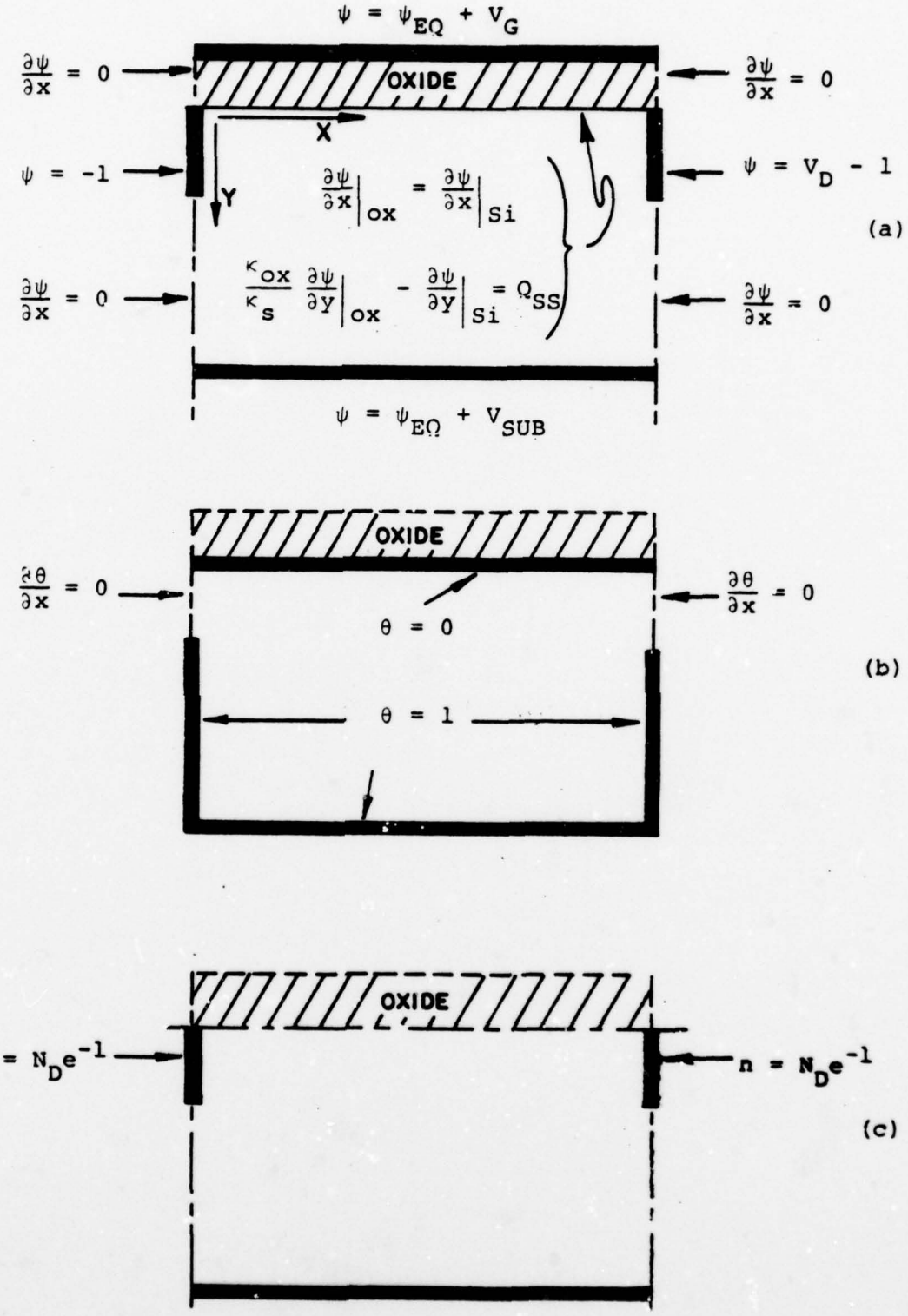

Figure 3 Boundary conditions for: (a) $\psi(x,y)$, (b) $\theta(x,y)$, and (c) $n(x,y)$.

$$U_J = U_T\left[\frac{N_D}{N_A+N_D}\right] + \left[\frac{N_A-N_D}{N_A+N_D}\right]\left[1 - \exp(-U_T)\right], \qquad (15)$$

where U_T is the total potential appearing across the p-n junction:

$$U_T = \ell n(N_D N_A) + U_{applied} . \qquad (16)$$

(Note that positive $U_{applied}$ in (16) results in a reverse-biased junction, consistent with Kennedy's convention.)

Consider some typical numerical values in (15) and (16). With $n_i' = 1.5\text{x}10^{10}$ cm^{-3}, $N_D' = 10^{19}$ cm^{-3}, $N_A' = 10^{16}$ cm^{-3} (the primes flag unnormalized constants), $\ell n(N_D N_A)$ assumes the numerical value 33.7, and the exponential term in (15) is entirely negligible except for negative values of $U_{applied}$ (i.e., forward bias) approaching that numerical magnitude. Furthermore, since $N_D >> N_A$, (15) simplifies tremendously, becoming:

$$U_J \simeq U_T - 1 . \qquad (17)$$

Since the drain junction is always operated reverse-biased, with $V_D > 0$, (17) is valid at the drain "contact."

Consider the source "contact," which does become forward-biased due to band bending near the oxide-silicon interface as the applied gate potential V_G is made positive with respect to the source. At the onset of strong inversion, the total normalized potential drop across the n^+p source junction right at the oxide-silicon interface will be reduced to $U_T = \ell n(N_D/N_A)$, which assumes the value $U_T = 6.9$, using the same numerical values for N_D' and N_A' assumed above. Again, the exponential term in (15) is negligible. We conclude that (17) is still a valid approximation for the source contact, at least for N_D/N_A ratios of 10^3 or higher. The approximation begins to become questionable only when $N_D/N_A < 10^2$, but at the source "contact" only.

Reverting to our convention of selecting the charge-neutral source island for zero reference potential, we conclude from (17) that the appropriate boundary conditions on ψ at the source and drain "contacts" are $\psi = -1$, and $\psi = V_D - 1$, respectively, as shown in Figure 3(a).

The remaining boundary conditions on ψ shown in that figure assure the symmetry assumed at the left and right borders of the region, and assure proper matching of the solutions of (1) and (7) at the oxide-silicon interface. Note that the condition imposed upon the normal derivatives of ψ at that interface allows for the inclusion of surface charge Q_{SS}, a feature not included by Mock. (The condition stated in the figure for the normal derivatives assumes an unfamiliar form because of the normalized variables employed.)

The numerical method later to be described for determining the electron number density $n(x,y)$ requires that n be specified along the borders of the rectangular region only at the source and drain "contacts." The fact that those "contacts" have potentials one normalized unit lower than those of the corresponding charge-neutral source and drain islands (where $n = N_D$) leads to the boundary conditions on n depicted in Figure 3(c).

2.04.2 Boundary Conditions on θ--Referring to (8) and (9), we note that θ is defined only in terms of its spatial derivatives. This means that its magnitude can be arbitrarily bounded. A convenient choice for those bounds is $0 \leq \theta \leq 1$, with θ specified to be zero along the oxide-silicon interface as in Figure 3(b). Tacitly assuming that the substrate (back-gate) will always be biased negatively with respect to the source ($V_{SUB} \leq 0$), so that no back-gate current flows, the appropriate boundary conditions on θ along the remaining borders in the figure are those shown. The stipulation $\frac{\partial \theta}{\partial x} = 0$ along the source and drain "contacts" assures zero tangential current at those "contacts," and is consistent with the implied symmetry conditions discussed earlier.

Bounding θ between zero and unity has two useful consequences, both stemming from eq. (8): 1) the numerical value of θ at any point (x,y) is the decimal fraction of the total drain current which crosses the plane x through the layer bounded by the oxide-silicon interface and the plane y and, 2) the constant J_o appearing in (8) and (9) represents, physically, the total drain current per unit width of the inversion channel.

2.05 Determination of the Drain Current Factor J_o:

The drain current per unit width of channel, J_o, is obtained as follows. Eq. (11) gives:

$$\frac{\partial}{\partial x}\,(ne^{-\psi}) = \mu^{-1}e^{-\psi}J_o\,\frac{\partial\theta}{\partial y}\,. \qquad (18)$$

Integrating with respect to x along a horizontal line (y = constant) which intersects the source and drain "contacts," one obtains:

$$N_D\left[e^{-V_D} - 1\right] = J_o\int_0^{X_{channel}} \mu^{-1}e^{-\psi}\,\frac{\partial\theta}{\partial y}\,dx\,. \qquad (19)$$

Solving for J_o:

$$J_o = \frac{-N_D[1-e^{-V_D}]}{\int_0^{X_{channel}} \mu^{-1}e^{-\psi}\,\frac{\partial\theta}{\partial y}\,dx}\,. \qquad (20)$$

This is the same result used by Mock for calculating J_o, except for the factor N_D appearing in (20), which is absent in Mock's result ([1], eq. (10)). The reason for this difference is seated in the fact that Mock chose to set the electron quasi-Fermi Potential ϕ_n equal to zero in the charge-neutral source region, which makes $\psi = \ell n(N_D)$ in that region, whereas we have chosen to set $\psi = 0$ there. Note that the sign

of J_o changes if one were to perform the integration from $x = 0$ to the "image" drain contact at $x = -X_{channel}$. Thus, J_o is an odd function of x, as it must be in order to satisfy the symmetry conditions on J_x and J_y described in Section 2.04.

3.0 Finite-Difference Algorithms

Figure 4 defines the indices (I,J), and their bounds, used to identify each lattice point involved in implementing finite-difference algorithms for determining ψ, θ, and n. (The figure does not attempt to convey the graded nature of the lattice actually used.) As shown, two "interlaced" lattices are employed. The first, depicted by circles, represents the set of points at which the electrostatic potential ψ and the corresponding electron and hole densities n and p are numerically determined while the second, depicted by squares, represents the set of points at which the stream function θ is evaluated. This choice, although complicating the satisfaction of the boundary conditions on θ, allows the use of simple differencing between adjacent lattice points in evaluating $\partial\theta/\partial x$ and $\partial\theta/\partial y$.* Mock also employed such an interlaced pair of lattices.

Note that the oxide-silicon interface coincides with the upper border, J = JMAX, of the θ lattice. Although not shown in Figure 4, the ψ lattice extends vertically an additional four rows (uniformly spaced) beyond the row J = JMAX + 1 shown, to model the electrostatic potential

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*In an earlier version of the model, the author chose to deal with a single lattice of points, rather than the interlaced lattices presently employed. In the development of the computer code for that choice, it was soon discovered that the nonuniform point spacings of the lattice precluded the use of simple differencing for evaluating derivatives. It was found necessary to fit parabolas through sets of three adjacent points in a row or column in order to evaluate the partial derivatives needed. While this was easily accomplished with a single subroutine for that purpose, nonetheless each such operation required 7 multiplications and 7 additions each time the subroutine was called. This imposed an unacceptable cost in computation time, and the approach was abandoned.

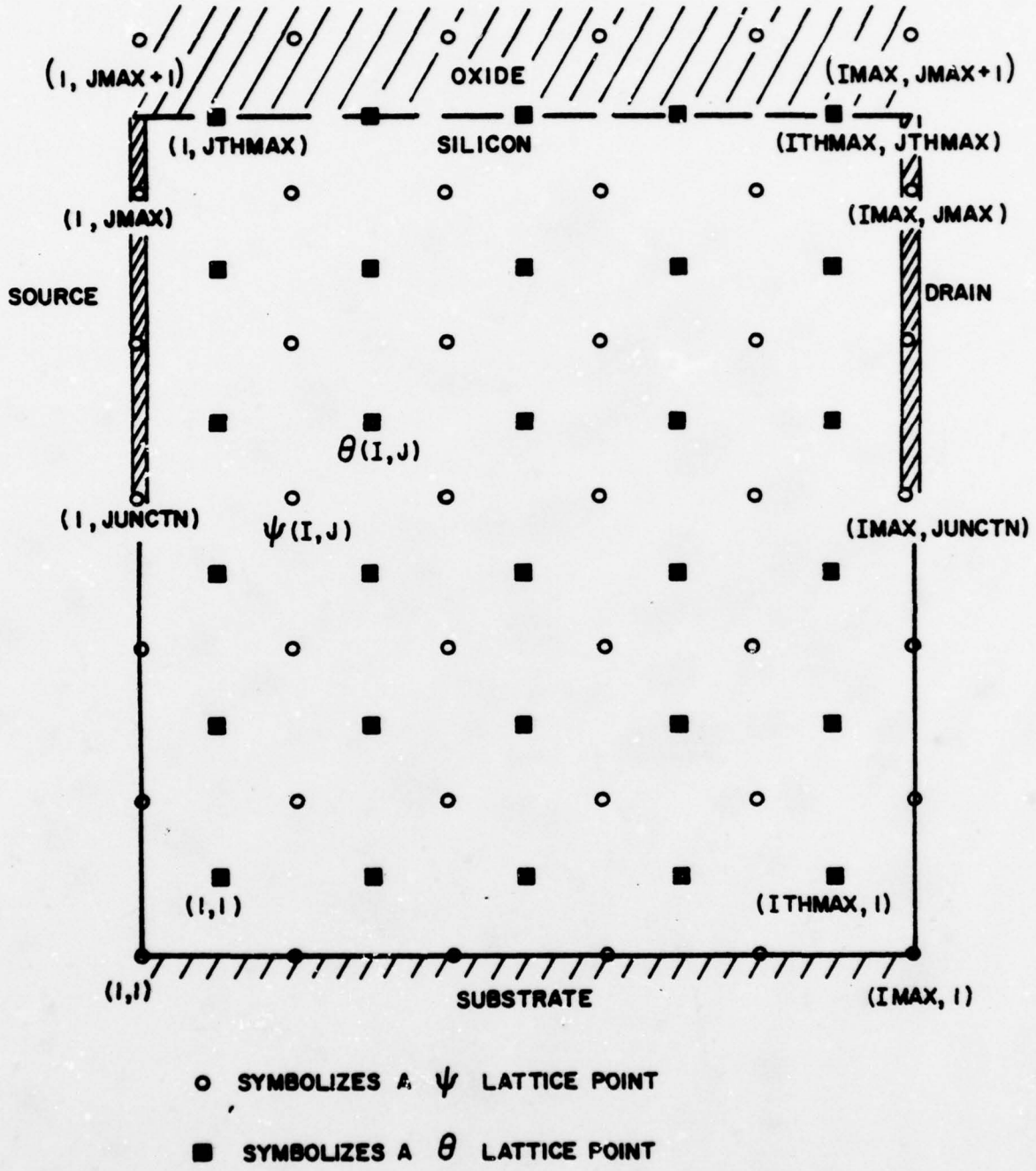

Figure 4 Layout of the interlaced ψ and θ lattices used in the finite-difference equations.

within the oxide region. Note, also, the convention adopted for the indices of the ψ lattice with respect to those of the θ lattice. (Given the ordered pair (I,J), the point of the θ lattice so identified is "northeast" of the corresponding point of the ψ lattice.)

3.01 Finite-Difference Algorithms for ψ

Consider the five point "star" of points in the ψ lattice shown in Figure 5. The dotted rectangle shown in that figure, having dimensions $\tilde{\Delta}x(I)$ by $\tilde{\Delta}y(J)$, is bounded at its four corners by the corresponding θ lattice points "NE, NW, SE and SW" of the central point P. (We shall adopt "points of the compass" notation, using subscripts in the difference equations to avoid repeated use of the index pairs (I,J), (I+1,J), etc., which are used in their FORTRAN counterparts in the computer program.) The lengths $\tilde{\Delta}x(I)$, $\tilde{\Delta}y(J)$, abbreviated $\tilde{\Delta x}_P$ and $\tilde{\Delta y}_P$ in the difference equations, are defined to be:

$$\left.\begin{aligned} \tilde{\Delta x}_P &= (\Delta x_E + \Delta x_W)/2 \ , \\ \tilde{\Delta y}_P &= (\Delta y_N + \Delta y_S)/2 \ . \end{aligned}\right\} \qquad (21)$$

This locates the θ lattice points always midway betweeen their neighboring ψ lattice points.

Applying Gauss's law to the rectangular region surrounding the point (I,J) in Figure 5, one obtains:

$$B_P\psi_S + D_P\psi_W + E_P\psi_P + F_P\psi_E + H_P\psi_N = Q_P \ , \qquad (22)$$

where:

$$\left.\begin{aligned} B_P &= \tilde{\Delta x}_P/\Delta y_S \ , \\ D_P &= \tilde{\Delta y}_P/\Delta x_W \ , \\ E_P &= -(B_P+D_P+F_P+H_P) \ , \\ F_P &= \tilde{\Delta y}_P/\Delta x_E \ , \\ H_P &= \tilde{\Delta x}_P/\Delta y_N \ , \\ Q_P &= \int\int_{BOX} (N_A+n-p)\,dxdy \ . \end{aligned}\right\} \qquad (23)$$

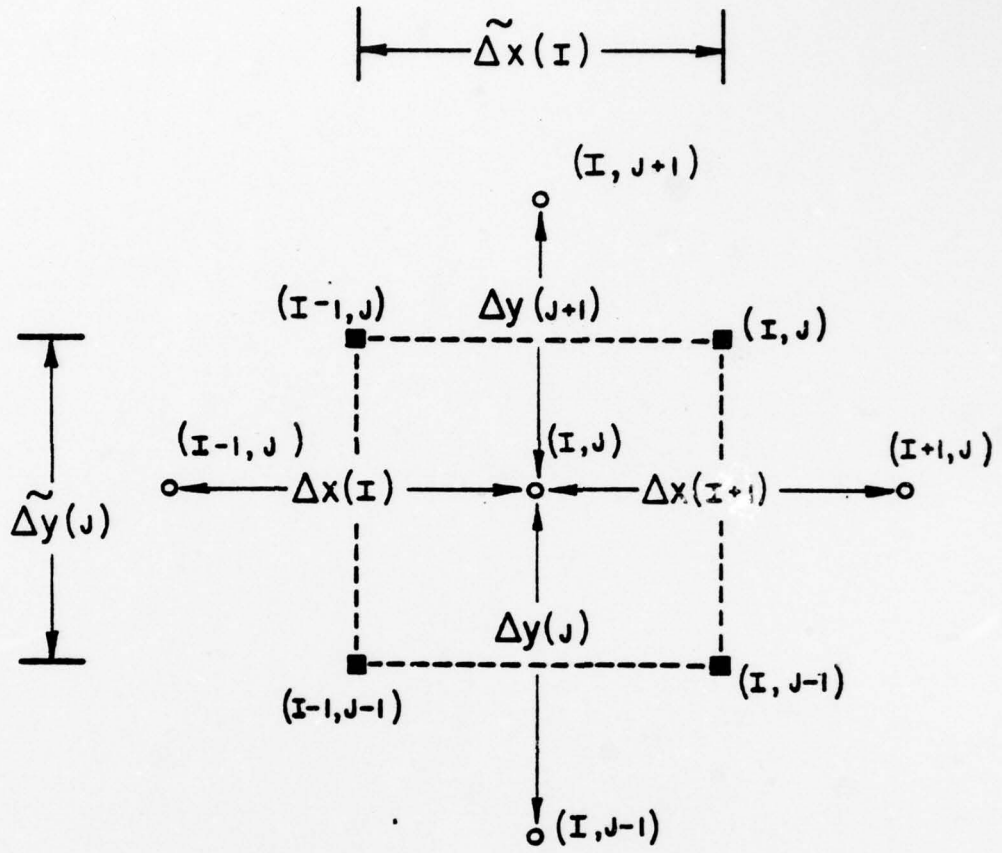

Figure 5 Five-point "star" of ψ-lattice points used in the finite-difference equation for ψ.

With a ψ lattice whose dimensions are I_{MAX} by (JMAX+5), the set of equations represented by (22) can be expressed in the matrix form:

$$[L] \cdot [\psi] = [Q] , \qquad (24)$$

where $[\psi]$ and $[Q]$ are rectangular matrices having the same dimensions as those of the ψ lattice, while [L] is a square matrix of dimensions (IMAX)*(JMAX+5) by (IMAX)*(JMAX+5). This matrix is sparse, having only five nonzero symmetrically positioned diagonal elements. Stone's iterative matrix factorization method [7] is employed in our model to solve (24) for the array ψ, given the array Q.

3.01.1 Implementation of the Boundary Conditions on ψ

The boundary conditions on ψ depicted in Figure 3(a) are enforced by appropriate modifications of the factors B_p, D_p, E_p, F_p, H_p, and Q_p in (22) at those ψ lattice points coinciding with the borders of the rectangular region modeled. These are next discussed.

3.01.1(a) Border Segments where ψ Is Specified--For ψ lattice points at which ψ is fixed, the coefficients B_p, D_p, F_p, and H_p are set to zero, while E_p is set to unity. Then the term Q_p in (22) is set equal to the known value of ψ at the point in question. In accordance with (22), this forces ψ_p to assume that known value at such border points.

3.01.1(b) Border Segments along Symmetry Planes--The boundary condition $\partial\psi/\partial x = 0$ is enforced at the symmetry planes in the following manner. At the left border, the coefficient D_p in (22) is set to zero, while F_p is doubled. The converse is applied at the right border. This assures the required symmetry of ψ at those border segments.

3.01.1(c) The Oxide-Silicon Interface--Since the ψ lattice has no points coinciding with the interface, the enforcement

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[7] H.L. Stone, SIAM Jour. Numer. Anal., 5, 530 (1968).

of the proper boundary conditions on ψ there is somewhat more involved. We develop special forms of the finite-difference equation for ψ, which apply for the two rows of the ψ lattice, viz. J = JMAX and J = (JMAX+1), which bracket that interface. Figure 6 shows one set of points in those rows of the ψ lattice, and defines spacings $\Delta\eta_{ox}$ and $\Delta\eta_{si}$ which locate the interface. Through an appropriate choice of the ratio $(\Delta\eta_{ox}/\Delta\eta_{si})$, we develop an expression for the potential ψ_{surf} at the interface, in terms of the known potentials ψ_{ox} and ψ_{si} of the ψ lattice points north and south of it, such that the required discontinuity in $\partial\psi/\partial y$ is automatically satisfied. Then, an application of Gauss's law to the two rectangular regions shown crosshatched in the figure leads to the required special forms of the finite-difference equation in ψ for the two rows in question.

The boundary condition on $\partial\psi/dy$ at the interface is:

$$\left(\frac{\partial\psi}{\partial y}\right)_{si} = \left(\frac{\kappa_{ox}}{\kappa_{si}}\right)\left(\frac{\partial\psi}{\partial y}\right)_{ox} - Q_{SS} \ , \qquad (25)$$

which comes from applying Gauss's law to an inifinitesimally thin gaussian "pillbox" bracketing the interface, with the result expressed in terms of the normalized variables employed in our model. Here, κ_{ox} and κ_{si} are the relative permittivity of the oxide and silicon regions, respectively, while Q_{SS} is the surface charge density (if any) residing at the interface. In addition to this condition, we have:

$$\psi_{si} = \psi_{ox} + \left(\frac{\partial\psi}{\partial y}\right)_{ox} \Delta\eta_{ox} + \left(\frac{\partial\psi}{\partial y}\right)_{si} \Delta\eta_{si} \ , \qquad (26)$$

$$\left(\frac{\partial\psi}{\partial y}\right)_{ox} = \left[\frac{\psi_{surf} - \psi_{ox}}{\Delta\eta_{ox}}\right] \ , \qquad (27)$$

$$\left(\frac{\partial\psi}{\partial y}\right)_{si} = \left[\frac{\psi_{si} - \psi_{surf}}{\Delta\eta_{si}}\right] \ , \qquad (28)$$

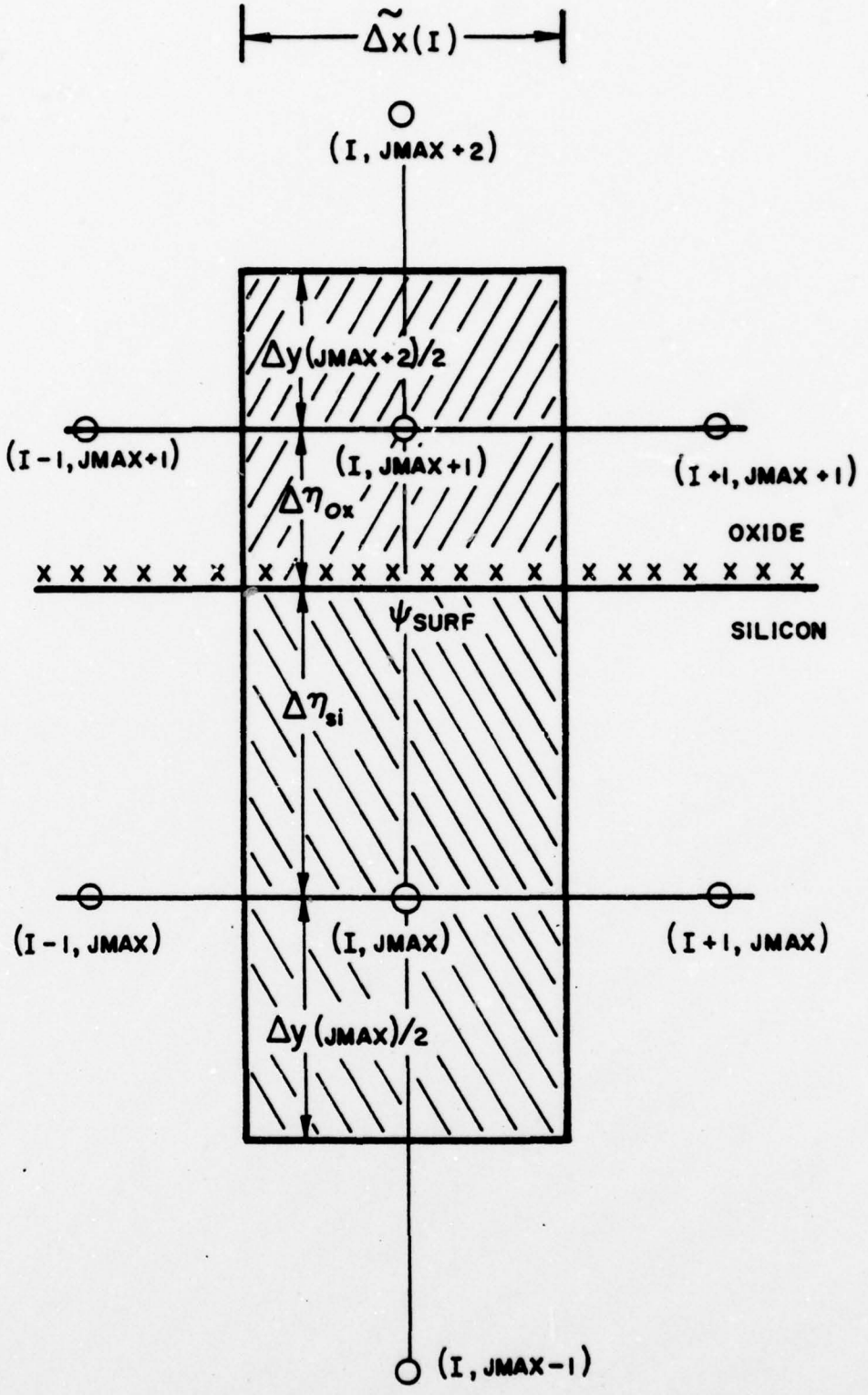

Figure 6 Boundary conditions on ψ at the oxide-silicon interface are enforced through an appropriate choice for the location of the interface, leading to special finite-difference equations applicable for the two ψ-lattice rows bracketing the interface.

with positive y defined to be vertically downward from the interface. These four equations relate five unknowns--$\Delta\eta_{ox}$, $\Delta\eta_{si}$, ψ_{surf}, $(\partial\psi/\partial y)_{ox}$, $(\partial\psi/\partial y)_{si}$--which means that the ratio $(\Delta\eta_{ox}/\Delta\eta_{si})$ can be specified at our convenience.

To find ψ_{surf} (and discover a convenient choice for $\Delta\eta_{ox}/\Delta\eta_{si}$), substitute (25) for $(\partial\psi/\partial y)_{si}$ and (27) for $(\partial\psi/\partial y)_{ox}$ into (26), obtaining:

$$\psi_{surf} = \left\{ \frac{\psi_{si} + \left(\frac{\kappa_{ox}}{\kappa_{si}}\right)\left(\frac{\Delta\eta_{si}}{\Delta\eta_{ox}}\right)\psi_{ox} + Q_{SS}\Delta\eta_{si}}{1 + \left(\frac{\kappa_{ox}}{\kappa_{si}}\right)\left(\frac{\Delta\eta_{si}}{\Delta\eta_{ox}}\right)} \right\} . \quad (29)$$

Choosing:

$$\Delta\eta_{ox} = \left(\frac{\kappa_{ox}}{\kappa_{si}}\right)\Delta\eta_{si} , \quad (30)$$

we obtain the simple relationship:

$$\psi_{surf} = \left[\frac{\psi_{ox} + \psi_{si}}{2}\right] + Q_{SS}\,\frac{\Delta\eta_{si}}{2} . \quad (31)$$

Furthermore, since $(\Delta\eta_{ox} + \Delta\eta_{si}) = \Delta y(JMAX+1)$, we find:

$$\Delta\eta_{si} = \Delta y(JMAX+1)/[1 + (\kappa_{ox}/\kappa_{si})] , \quad (32)$$

$$\Delta\eta_{ox} = \Delta y(JMAX+1)\left[\frac{(\kappa_{ox}/\kappa_{si})}{1 + (\kappa_{ox}/\kappa_{si})}\right] . \quad (33)$$

The result of the above procedure is that the potential at the interface, ψ_{surf}, is now known in terms of the potential at its north and south ψ lattice neighbors, with the simple relationship expressed by (31) resulting from locating the interface as prescribed by (32) and (33). These three equations assure that the boundary condition on $(\partial\psi/\partial y)$ at the interface is automatically satisfied, as is the boundary condition on $(\frac{\partial\psi}{\partial x})$.

Now apply Gauss's law to the rectangular region shown in Figure 6 which encompasses the ψ lattice point (I,JMAX) just below the interface. Again, adopting "points of the compass" notation, the electrostatic flux entering the upper surface of that rectangle is $-\tilde{\Delta}x_p(\psi_p-\psi_{surf})/\Delta\eta_{si}$. Using (31) to relate ψ_{surf} to ψ_N and ψ_p, the resulting finite-difference equation still assumes the form given by (22). However, the terms H_p and Q_p appearing in (23) (and only those) are now modified becoming:

$$\left.\begin{aligned} H_p &= \tilde{\Delta}x_p/2\Delta\eta_{si} \ , \\ Q_p &= \iint_{BOX} (N_A+n-p)\,dxdy - Q_{SS}\tilde{\Delta}x_p/2 \ , \end{aligned}\right\} \qquad (34)$$

which apply for any ψ lattice point in the row J = JMAX.

An identical procedure, applied to the rectangular region encompassing the point (I,JMAX+1) in Figure 6 shows the terms B_p and Q_p to be:

$$\left.\begin{aligned} B_p &= \tilde{\Delta}x_p/2\Delta\eta_{ox} \ , \\ Q_p &= -\left(\frac{\kappa_{si}}{\kappa_{ox}}\right)Q_{SS}\tilde{x}_p/2 \ , \end{aligned}\right\} \qquad (35)$$

which apply for any Ψ lattice point in the row J = JMAX+1.

As a check of these results, it is satisfying to find that if there <u>were</u> no interface (i.e., if $\kappa_{ox} \equiv \kappa_{si}$, and $Q_{SS} \equiv 0$), then (32) and (33) result in $\Delta\eta_{si} = \Delta\eta_{ox} = \Delta y(JMAX+1)/2$, so that both (34) and (35) reduce to the corresponding expressions for B_p, H_p, and Q_p given in (23), which apply for all rows of the ψ lattice except the two which bracket the oxide-silicon interface.

Mock did not use the above procedure for dealing with the interface. Instead, he defined a transformation of coordinate variable $y \rightarrow z$ in the oxide region, chosen so that $(\partial\psi/\partial z)_{ox} = (\partial\psi/\partial y)_{si}$ at the interface, even though

$(\partial\psi/\partial y)$ is discontinuous there. (This is equivalent to defining a modified oxide thickness.) While this approach possesses the advantage of requiring no special forms of the finite-difference equations at the interface, it precludes the inclusion of surface charge Q_{SS} in the model. Since we wish to include such, we choose not to follow Mock's approach, and implement the above described procedure instead.

3.01.2 Numerical Algorithm for the Charge Term Q_P

The charge term Q_P in (23) represents the net bulk charge enclosed by the rectangular region surrounding each ψ-lattice point, the corners of that region coinciding with the points of the θ-lattice NE, NW, SE, and SW of the point P. A simple approximation for Q_P is obtained from (23) to be:

$$Q_P \approx (N_A + n - p)\tilde{\Delta}x_P\tilde{\Delta}y_P \ , \qquad (36)$$

which assumes that n and p can be regarded as constant throughout the rectangular region.

While more elaborate approximations for Q_P are possible (e.g., a two-dimensional implementation of Simpson's rule) they involve more complex programming logic to handle lattice points adjacent to the four borders of the lattice, and are more costly in terms of computation time. Such more sophisticated procedures were tried by the author , but were found to be unnecessary, and were subsequently abandoned.

3.02 Finite-Difference Algorithm for the Stream Function θ

The partial differential equation in θ, eq. (13), is repeated here for convenience of reference:

$$\frac{\partial}{\partial x}\left(\mu^{-1}e^{-\psi}\,\frac{\partial\theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu^{-1}e^{-\psi}\,\frac{\partial\theta}{\partial y}\right) = 0 \ . \qquad (13)$$

Our present version of the computer model does not incorporate provisions for field-dependent electron mobility. Therefore, μ^{-1} can be factored from (13), and ignored. If the field dependence of μ is to be retained, as a refinement to be later incorporated, several subroutines in the computer program will require revision.

Figure 7 focuses upon a point (I,J) of the θ lattice, and its nearest neighbors. Simple differencing gives the partial derivatives of θ at the four points shown as crosses in the figure:

$$\left.\begin{aligned}
\left(\frac{\partial\theta}{\partial x}\right)_E &= \left[\frac{\theta_E - \theta_P}{\tilde{\Delta}x_E}\right] , \\
\left(\frac{\partial\theta}{\partial x}\right)_W &= \left[\frac{\theta_P - \theta_W}{\tilde{\Delta}x_W}\right] , \\
\left(\frac{\partial\theta}{\partial y}\right)_N &= \left[\frac{\theta_P - \theta_N}{\tilde{\Delta}y_N}\right] , \\
\left(\frac{\partial\theta}{\partial y}\right)_S &= \left[\frac{\theta_S - \theta_P}{\tilde{\Delta}y_S}\right] .
\end{aligned}\right\} \qquad (37)$$

The numerical value of $e^{-\psi}$ at each of those four points is determined in the manner next to be described.

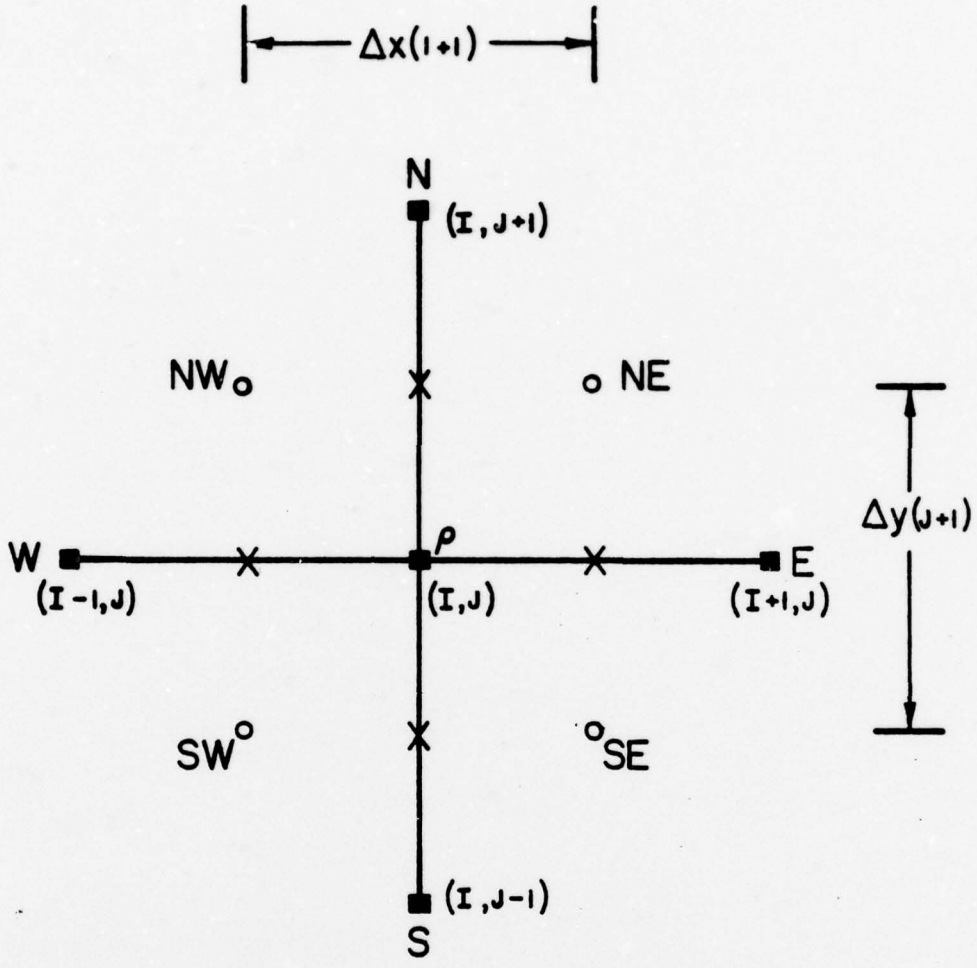

Figure 7 Five point "star" of θ-lattice points used in the finite-difference equation for θ.

Consider the four ψ lattice points located NE, SE, SW, and NW of the θ lattice point P. Assume a linear dependence of ψ along the borders of the rectangle defined by those four points, then evaluate an average value of $e^{-\psi}$ along each of those four borders, as did Mock [1], assigning the four averaged values to the respective points indicated by the crosses in the figure. (Mock describes this approach as a critical factor in achieving accuracy.) The result is:

$$\left.\begin{aligned} \langle e^{-\psi}\rangle_E &= \left[\frac{e^{-\psi_{NE}} - e^{-\psi_{SE}}}{\psi_{SE} - \psi_{NE}}\right], \\ \langle e^{-\psi}\rangle_W &= \left[\frac{e^{-\psi_{NW}} - e^{-\psi_{SW}}}{\psi_{SW} - \psi_{NW}}\right], \\ \langle e^{-\psi}\rangle_N &= \left[\frac{e^{-\psi_{NE}} - e^{-\psi_{NW}}}{\psi_{NW} - \psi_{NE}}\right], \\ \langle e^{-\psi}\rangle_S &= \left[\frac{e^{-\psi_{SE}} - e^{-\psi_{SW}}}{\psi_{SW} - \psi_{SE}}\right]. \end{aligned}\right\} \qquad (38)$$

With the $e^{-\psi}$ values thus determined at the four points midway between the N, E, S, and W points of the θ lattice, the finite-difference form of (13) becomes (assuming constant mobility μ):

$$\Delta y_P\left[\langle e^{-\psi}\rangle_E\left(\frac{\theta_E-\theta_P}{\tilde{\Delta}x_E}\right) - \langle e^{-\psi}\rangle_W\left(\frac{\theta_P-\theta_W}{\tilde{\Delta}x_W}\right)\right]$$

$$+ \Delta x_P\left|\langle e^{-\psi}\rangle_S\left(\frac{\theta_S-\theta_P}{\tilde{\Delta}y_S}\right) - \langle e^{-\psi}\rangle_N\left(\frac{\theta_P-\theta_N}{\tilde{\Delta}y_N}\right)\right| = 0, \qquad (39)$$

where Δx_P and Δy_P have been introduced as shorthand notation for $\Delta x(I+1)$, $\Delta y(J+1)$ in Figure 7. (See Figure 5 for the geometric definitions of the $\tilde{\Delta}x$ and $\tilde{\Delta}y$ factors in the above equation.)

As was the case with the finite-difference equation for ψ discussed earlier, the set of equations (39) can be represented in matrix form. The resulting sparse matrix contains only five nonzero symmetrically positioned diagonal elements. The matrix equation is solved for θ, given ψ, using the block-line iterative procedure described by Varga [8].

The finite-difference equation for θ expressed by (39) is deceptively simple. Its implementation produces nontrivial challenges because of the exponential dependence upon ψ evident in (38). To illustrate, consider a modest drain voltage of 10 volts. In our normalized variables, this translates to a numerical value on the order of 400. But the IBM 370/165 used in this development produces overflow and underflow with the operation $e^{\pm|x|}$ when $|x|$ exceeds approximately 175. Not only is this limit obviously violated in regions near the drain, in the example cited, but along the entire channel as well when the gate voltage exceeds about 4.5 volts.

The modus operandi developed for dealing with this problem is the following. One <u>never</u> deals with positive x values in the operation e^{x}. One <u>always</u> alters the equation being treated so that only negative exponents are encountered. Furthermore, one endeavors <u>always</u> to reformulate that equation so that x represents <u>differences</u> in the potential between adjacent ψ lattice points, rather than the magnitude of ψ itself. Finally, one always tests the magnitude of the negative exponent before attempting the operation of exponentiation, avoiding "underflow" messages by use of the approximation $e^{-x} \approx 0$ when x is found sufficiently large.

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[8] R.S. Varga, Matrix Iterative Analysis, p. 194, Prentice-Hall, New York (1962).

To implement these restrictions, the average $e^{-\psi}$ functions in (38) are first reexpressed by factoring the exponential term whose exponent is least negative. This leaves within the brackets an exponential term whose negative exponent represents differences in ψ between adjacent lattice points. Following this, the four terms in (38) are again factored by the exponential term outside of the brackets whose exponent is least negative. Since the right hand side of (39) is zero, the latter factor can be cancelled from the equation. Again, this results in exponential terms whose negative exponents represent differences in ψ between adjacent lattice points. The subprogram which implements this procedure selects one of 16 possible cases in the process of determining which exponents are least negative.

3.02.1 Implementation of the Boundary Conditions on θ

The finite-difference equation in θ, equation (39), can be reexpressed to read:

$$B_P\theta_S + D_P\theta_W + E_P\theta_P + F_P\theta_E + H_P\theta_N = 0 \ , \qquad (40)$$

which then takes on the same form as the finite-difference equation in ψ, equation (22). As was the case with ψ, the boundary conditions on θ shown in Figure 3(b) are established by means of appropriate modifications in the coefficients B_P, D_P, E_P, F_P, and H_P in (40).

3.02.1(a) Border Segments where θ=1--The fact that the θ lattice interlaces the ψ lattice (Figure 4) means that the boundary condition θ=1 cannot strictly be enforced at the borders where that condition applies (Figure 3). However, those borders are remote from the region under the oxide-silicon interface where "the action" occurs, so this is not considered to be a serious drawback. What is done is to enforce θ=1 along the column (or row) of θ lattice points

imagined to be present just outside of the rectangular region modeled. Thus, for example, at the left border in Figure 4 the coefficient D_p is calculated in the usual manner. Then the right hand side of (40) is set equal to the negative of D_p and the term involving θ_W in (40) is omitted, which is equivalent to enforcing θ_W to be unity. Similar procedures are applied at the right border and along the lower border of the substrate.

3.02.1(b) Border Segments where $\partial\theta/\partial x = 0$--Again, we imagine two columns of the θ lattice just outside of the rectangular region modeled in Figure 4. At the left border, therefore, we wish to enforce $\theta_W = \theta_P$. This is achieved by calculating D_p and E_p in the usual manner, then summing them to obtain a revised value for E_p. The coefficient D_p in (40) is then set to zero. A similar procedure is applied at the right border.

3.02.1(c) The Oxide-Silicon Interface where $\theta=0$--Here, the row J = JTHMAX of the θ lattice coincides with the interface. The condition $\theta=0$ is enforced along that row by setting B_p, D_p, F_p, and H_p to zero, while setting E_p to unity in (40).

3.03 Finite-Difference Algorithm for the Drain Current Factor J_o

The row of the ψ lattice just below the oxide-silicon interface is selected for the line integral contained in (20), to evaluate the drain current factor J_o. The trapezoidal rule is applied to evaluate the integral. Simple differencing of θ on the interlaced θ lattice provides $\partial\theta/\partial y$ at the centroid of each interval Δx. The value of $e^{-\psi}$ at that centroid is again specified to be an average obtained by assuming a linear variation of ψ between adjacent ψ lattice points along the row:

$$\langle e^{-\psi} \rangle = \left[\frac{e^{-\psi_E} - e^{-\psi_W}}{\psi_W - \psi_E} \right] . \qquad (41)$$

As in the case of evaluating θ discussed earlier, special procedures are employed to avoid overflow and underflow due to machine limitations.

3.04 Finite-Difference Algorithm for the Electron Number Density n

Equation (10) when separated into its x and y components, gives:

$$\frac{\partial}{\partial x}\left(ne^{-\psi}\right) = \frac{J_x}{\mu} e^{-\psi} , \tag{42}$$

$$\frac{\partial}{\partial y}\left(ne^{-\psi}\right) = \frac{J_y}{\mu} e^{-\psi} . \tag{43}$$

Expressing (42) in finite-difference form:

$$\left[\frac{n(I+1,J)e^{-\psi(I+1,J)} - n(I,J)e^{-\psi(I,J)}}{\Delta x(I+1)}\right] = \frac{J_x}{\mu} \langle e^{-\psi(I,J)} \rangle . \tag{44}$$

(Here we resort to the use of the indices (I,J) identifying ψ lattice points because the "points of the compass" notation employed earlier becomes awkward.) Note that an average value for $e^{-\psi}$ is assigned to the point midway between the points I+1 and I of the row J. That is:

$$\langle e^{-\psi(I,J)} \rangle = \left[\frac{e^{-\psi(I+1,J)} - e^{-\psi(I,J)}}{\psi(I,J) - \psi(I+1,J)}\right] . \tag{45}$$

Equations (44) and (45), when combined, can be reexpressed so as to involve only differences in ψ:

$$n(I+1,J) = n(I,J)e^{-[\psi(I,J)-\psi(I+1,J)]} + \frac{J_x}{\mu}\left[\frac{1 - e^{-[\psi(I,J)-\psi(I+1,J)]}}{\psi(I,J) - \psi(I+1,J)}\right] \Delta x(I+1) . \tag{46}$$

Alternatively:

$$n(I,J) = n(I+1,J)e^{-[\psi(I+1,J)-\psi(I,J)]} - \frac{J_x}{\mu}\left[\frac{1 - e^{-[\psi(I+1,J)-\psi(I,J)]}}{\psi(I+1,J) - \psi(I,J)}\right]\Delta x(I+1) \quad . \qquad (47)$$

Equations (46) and (47), together with the vertical counterpart of (46) derived in the same manner from (43), form the basis for the finite-difference algorithm for the electron density n, which proceeds as follows. Consider any row J $\geq$ JUNCTN (see Figure 4) which intersects the vertical source and drain "contacts" at the left and right borders where n is known. Starting at the left border, (46) is applied recursively to "march" along that row, toward the right border, until a potential minimum is encountered. When this occurs, then one switches to the right border and applies (47), recursively, to "march" to the left toward the last point at which n was previously calculated. In this manner, the electron member density is filled in row by row, for all rows of the ψ lattice which intersect the source and drain "contacts." Note that the exponentials encountered with this procedure always involve negative exponents.

The next step in the procedure then utilizes the vertical counterpart of (46), derived from (43). In this case, one starts at the row J = JUNCTN, where n is now known, and "marches" vertically downward to the substrate contact, column by column. Again, only negative exponents are encountered in any exponentiation operations required.

To avoid underflow, appropriate approximations are invoked in the subprogram which implements this procedure when the negative exponents encountered become sufficiently large. Similarly, appropriate approximations are applied when the denominators in (46) and (47), and the vertical counterpart of (46) approach zero, to avoid divide checks.

The "marching" algorithm described above is precisely the one suggested by Mock [1], who states that the procedure of switching directions at the point where ψ passes through a minimum suppresses the buildup of roundoff errors. In spite of this assertion, one intuitively feels uneasy about any numerical method which only makes use of information about the solution at one border without "pumping into" the solution some information about the boundary condition at the other border as well. The "marching" method fails to do so, and blithely assumes that, where the two solutions meet, continuity in n will not be violated.

The author felt sufficiently uneasy about this to try a different approach. Combining pairs of equations of the form of (46) by summing them, so as to couple the points I-1, I, and I+1 in a common row J, a tridiagonal matrix results from the set of equations relating n along that row. Given J_x and ψ as knowns, Gaussian elimination is then implemented to obtain n along such a row. This approach achieves the goal of "pumping into" the equations simultaneous information about the boundary conditions at both ends of the row in question. Interestingly, the two methods are found to give identical results (within the three significant figures printed out) in regions where the electron density is numerically important, i.e., in the inversion layer. Both approaches produce <u>negative</u> electron densities in regions where the numerical magnitude of n is negligible in comparison with the acceptor density N_A. (These nonphysical negative densities are believed due to roundoff errors.) Insofar as these negative densities are concerned, the "marching" method is found superior. With it, they occur with values of n an order of magnitude lower than observed with the more sophisticated tridiagonal matrix approach.

4.0 The Iterative Procedure

The basic equations defined in Section 2, and their finite-difference counterparts developed in Section 3, constitute a set of coupled, nonlinear equations for which a self-consistent solution is obtained by means of an iterative procedure. The one followed is basically the same as that adopted by Mock [1], with one important modification which will be clarified in Section 4.03.

The iterative procedure starts with initial "guesses" for $\psi(x,y)$, $\theta(x,y)$, $n(x,y)$ and $p(x,y)$ which will be discussed in Section 4.04. It then proceeds as follows. Given n and p, the charge Q_P associated with each ψ lattice point P is then known (Subsection 3.01.2). Equation (22) is then solved, using Stone's method, to provide a new estimate for ψ. A new estimate for θ is then generated, followed by new estimates for n and p consistent with the new ψ and θ distributions. The procedure is then repeated until convergence is achieved, as determined by convergence criteria specified as input data by the user. Two differing procedures--PICARD iteration, and GUMMEL's algorithm--are employed in the implementation of this iterative sequence. These will next be discussed.

4.01 Picard Iteration

The Picard iteration procedure utilizes only "past history" to predict the next iterate for ψ, and is employed to achieve "coarse" convergence. Consider the m^{th} iteration step to have just been completed, so that $\psi^{(m)}$, $\theta^{(m)}$, $n^{(m)}$, and $p^{(m)}$ have been determined. (Values for step (m-1) are also available, as needed, except when m=0, which corresponds to the initial "guess.") Given these, (22) is solved, yielding a new solution for the potential distribution, which we designate $v^{(m+1)}$. With it, the improved approximation $\psi^{(m+1)}$ for the correct potential $\psi(x,y)$ is obtained from:

$$\psi^{(m+1)} = \psi^{(m)} + \alpha_m[v^{(m+1)}-\psi^{(m)}] + \beta_m[\psi^{(m)}-\psi^{(m-1)}] \ , \qquad (48)$$

where α_m and β_m are underrelaxation parameters, adjusted for each iteration step m in the manner to be described next. With $\psi^{(m+1)}$ thus specified, the corresponding new estimates for $\theta^{(m+1)}$, $n^{(m+1)}$ are then determined.

As was done by Mock [1], we adopt the Chebyscheff sequence of relaxation parameters α_m and β_m discussed by Forsythe and Wasow [9] for estimating appropriate values of those parameters at each iteration step m and, as did Mock, we reinitiate the sequence whenever the new residual $|v^{(m+1)}-\psi^{(m)}|_{max}$ turns out to be greater than the residual found in step m. (Mock interprets this as an indicator that the nonlinearity of the problem is becoming evident.) This sequence of relaxation parameters is given by, [9]:

$$\alpha_m = \frac{4\cosh(m\omega)}{(b-a)\cosh([m+1]\omega)} , \tag{49}$$

$$\beta_m = \frac{\cosh([m-1]\omega)}{\cosh([m+1]\omega)} , \tag{50}$$

where

$$\omega = \cosh^{-1}\left[\frac{b+a}{b-a}\right] . \tag{51}$$

The parameters b and a in (51) are the upper and lower bounds, respectively, of the eigenvalues of the matrix equation (24). The initial values of this Chebyscheff sequence, as prescribed by Forsythe and Wasow, are determined by,

$$\alpha_o = 2/(b+a) , \tag{52}$$

$$\beta_o = 0 . \tag{53}$$

The implementation of this procedure requires an estimate for the upper and lower bounds b and a of the eigenvalues of the matrix equation (24). The

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[9] G.E. Forsythe and W.R. Wasow, Finite-Difference Equations for Partial Differential Equations, pp. 226-235. Wiley, New York (1960).

estimate for a is not critical, but that of b is. Too small an estimate for b leads to failure of the relaxation procedure to converge, while too large an estimate leads to extremely slow convergence. We have adopted the estimate for b suggested by Mock ([1], eq. (21))*:

$$b = \frac{Y_{sub}}{4X_{channel}} \iint (n+p)\,dxdy \ , \qquad (54)$$

the double integral being evaluated over the entire rectangular substrate region by application of the trapezoidal rule in two dimensions. The parameter b is evaluated using initial estimates for n and p to be described below.**

Mock [1] points out that since large values of b lead to slow rates of convergence, and since its magnitude is proportional to the total number of free carriers in the substrate region, a savings in computation time can be achieved by applying the Picard iteration process, initially, with a reduced donor density assigned to the source and drain, until coarse convergence criteria are satisfied. Having achieved this, he then assigns the correct donor density to those regions and switches to Gummel's iterative algorithm for the remainder of the computation, with tightened convergence criteria applied.

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*Mock's eq. (21) contains a factor ε_s which is missing in (54). This is because of differences in Mock's choice of normalized variables and the choice which we have adopted.

**At one time, the program included steps to reevaluate the parameter b each time the Chebyscheff sequence was reinitiated, on the basis that the estimates for n and p at that stage of the computation should be better than the estimates previously employed for that purpose. Computer experiments later showed that not only is this unnecessary but also that it usually caused the number of iterations required for convergence to increase, rather than decrease.

4.02 Gummel's Algorithm

Unlike the Picard iteration method just described, which uses only "past history," Gummel's algorithm [10] incorporates a new prediction for p and n at each iterative step. Because this prediction is based upon a linear approximation of the exponential functions appearing in the Boltzmann relations for those densities, it is efficient in terms of computation speed only when the estimates of ψ, p, and n are close to the "correct" values when it is initiated.

Let $\psi^{(m)}$, $n^{(m)}$ and $p^{(m)}$ be the values obtained at iteration step m. Let $\psi^{(m+1)} = \psi^{(m)} + \delta^{(m+1)}$, and assume that $\delta^{(m+1)} << \psi^{(m)}$. Then:

$$e^{\psi^{(m+1)}} \approx e^{\psi^{(m)}} [1+\delta^{(m+1)}] . \qquad (55)$$

Applying this approximation to (2) and (3), one has

$$n^{(m+1)} \approx n^{(m)} [1 + \delta^{(m+1)}] , \qquad (56)$$

$$p^{(m+1)} \approx p^{(m)} [1 - \delta^{(m+1)}] , \qquad (57)$$

which assumes that the electron quasi-Fermi potential ϕ_n does not change appreciably between iteration steps (m) and (m+1). Assuming these conditions to be met, Poisson's equation, (1), gives for $\psi^{(m+1)}$:

$$\nabla^2\psi^{(m+1)} - [n^{(m)}+p^{(m)}]\psi^{(m+1)} = [N_A+n^{(m)}-p^{(m)}] - [n^{(m)}+p^{(m)}]\psi^{(m)} , \qquad (58)$$

which is Gummel's algorithm.

The finite-difference equivalent of (58) can be cast into the same form as (22). Only the coefficient E_p in (23)

[10] H.K. Gummel, IEEE Trans. Electron Devices ED-11, 455 (1964).

is modified due to additional term $[n^{(m)} + p^{(m)}]\psi^{(m+1)}$ appearing on the left side of (58). Similarly, the charge term Q_p in (23) is modified due to the corresponding additional term on the right side of (58). The resulting matrix equation, corresponding to (24), is solved for $[\psi^{(m+1}]$, given $[Q^{(m)}]$, using Stone's iterative procedure.

4.03 A Modification of Mock's Iterative Procedure

As described above, Mock uses both the Picard iteration and Gummel's algorithm, the former being used to achieve coarse convergence at reduced donor density, the latter being used to achieve final tightened convergence at full donor density. Initially, the author adopted this same procedure. However, computational experiments with our computer model have revealed that this procedure does not seem to be necessary. The author has found that not only is it possible to apply Gummel's algorithm directly at the outset, with full donor density, but also that the rate of convergence can be substantially enhanced if one applies overrelaxation in so doing. This approach proceeds as follows.

Let $\psi^{(m)}$ be the potential corresponding to the m^{th} iterative step and let $v^{(m+1)}$ be the result of applying Gummel's algorithm at the $(m+1)^{th}$ iterative step. Then the new potential $\psi^{(m+1)}$ is obtained as:

$$\psi^{(m+1)} = \psi^{(m)} + \Omega(v^{(m+1)}-\psi^{(m)}) \quad , \qquad (59)$$

where Ω is an empirically determined relaxation parameter having a numerical value greater than unity.

That this procedure can result in a substantial reduction in computation time is illustrated in Figure 8, which plots the number of iterations required to achieve tight convergence ($\max|\psi^{(m+1}-\psi^{(m)}| \leq .02$) versus gate voltage, with drain voltage fixed, for a particular device. In this

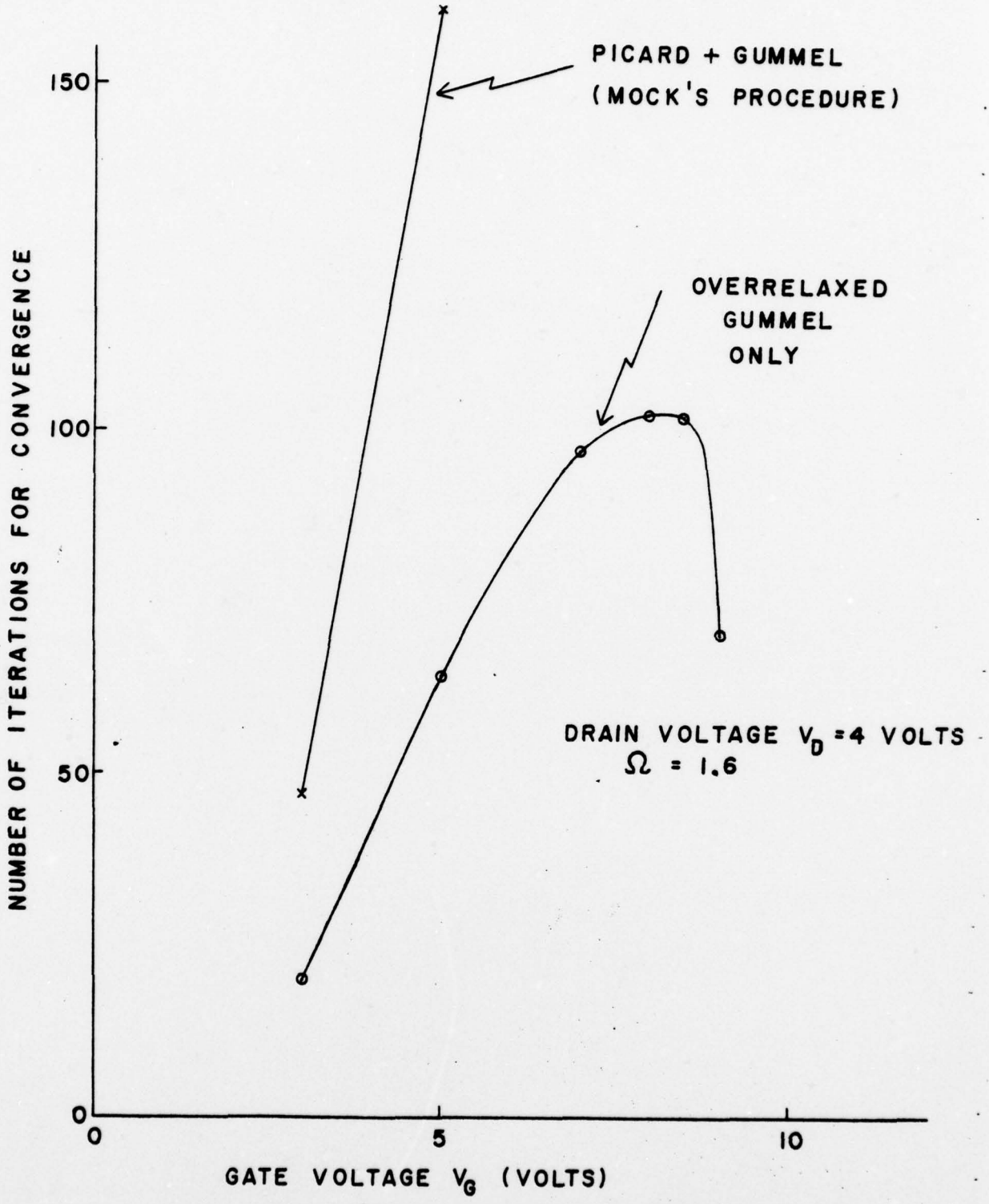

Figure 8 Number of iterations required for convergence of ψ to within .02 kT/q versus the gate voltage V_G, comparing Mock's iterative procedure with the overrelaxed Gummel procedure.

case, Ω was set to be 1.6, in the overrelaxation of Gummel's algorithm. In this experiment, four of the six points (V_G = 3,5,7,9) shown for the curve labeled "OVER-RELAXED GUMMEL ONLY" were generated in a single computer run requiring 190 seconds of CPU time (the remaining two points were generated in separate runs to fill in further detail). On the other hand, at V_G = 7 volts, the combined "PICARD + GUMMEL" procedure failed to converge at all within a 6-minute cut-off time that was specified for that particular run!

Figure 8 reveals that the number of iterations required for convergence is strongly dependent upon the degree of charge inversion in the conducting channel. Computational experiments have also shown that the effectiveness of over-relaxation is much more dramatic in the strong inversion case. This is illustrated in Figure 9, which plots the number of iterations required to achieve convergence with the overrelaxed Gummel procedure versus the overrelaxation parameter Ω. (The threshold voltage for this particular device is $\approx$2.5 volts.) The numerical values shown along the two curves give CPU time, in seconds, required with the Amdahl 470 V/6-II computer used for these experiments, and include compilation time in each instance.

The discovery that the Gummel algorithm alone can be used, and particularly the fact that overrelaxation can be applied with marked success with that algorithm, was made only during the final phases of this project. Consequently, the computer model has been revised so as to allow the user the flexibility of specifying the implementation of either Mock's iterative procedure, or the overrelaxed Gummel procedure alone, by means of a "flag" incorporated as part of the input data specified by him. Similarly, the value of the overrelaxation parameter Ω to be used is at the user's disposal, inasmuch as time did not permit an investigation of possible methods for automatically determining an optimum value (or possibly a sequence of values) for this parameter.

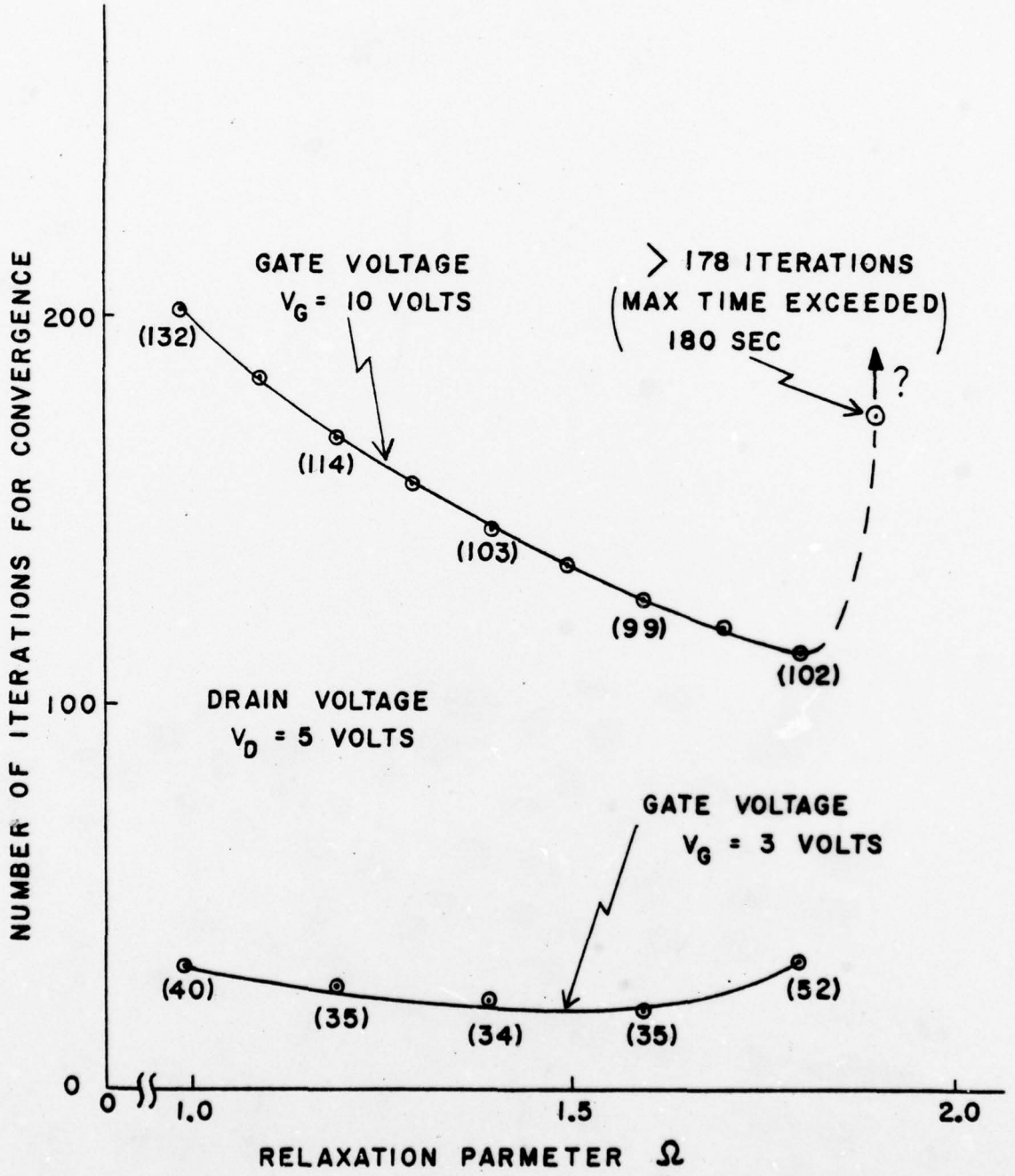

Figure 9 Number of iterations required for convergence of ψ to within .02 kT/q versus the overrelaxation parameter Ω used with Gummel's algorithm. The numerical values show CPU time, in seconds, required with an Amdahl 470 V/6-II computer.

4.04 Initial Estimates for $\psi(x,y)$, $\theta(x,y)$, $n(x,y)$, and $p(x,y)$

The initial estimates for ψ, θ, n, and p have been found to be a crucial factor in determining the number of iterations required to achieve convergence to a self-consistent solution. Early versions of our computer model were found to require from two to three times the number of iterations that were required with the IBM program developed by Mock. Much effort has been expended, in developing our model, to improve upon this early experience. These efforts have proven successful. Particularly, when only the Gummel algorithm is utilized, as described above, our model now requires from two to three times less iterations than does the IBM program, in most cases. This is attributed to the adequacy of the algorithms finally adopted for supplying the initial estimates. These will next be described.

4.04.1 Initial Estimate for $\psi(x,y)$

Figure 10(a) shows a sketch of the equipotential contours obtained from a two-dimensional, self-consistent solution of equations (1) through (7) for an MOSFET with a channel length of 5μm, operating in strong inversion in the saturation mode. The cross-hatched regions shown in the figure identify regions in which $\psi(x,y)$ can be regarded as essentially having one-dimensional dependence, $\psi(x,y) \approx \psi(x)$, where $\psi(x)$ is found, to good approximation, to exhibit the parabolic dependence on x predicted by the widely used depletion approximation of abrupt p-n junction theory. This parabolic dependence in the vicinity of the source and drain "contacts" is sketched in Figure 10(b) as the two dashed curves shown.

The dotted region in Figure 10(a) identifies the charge-neutral region of the substrate, where $\psi = \psi_{eq} + V_{SUB}$, and where $\nabla\psi=0$. The two depletion parabolas of Figure 10(b) merge

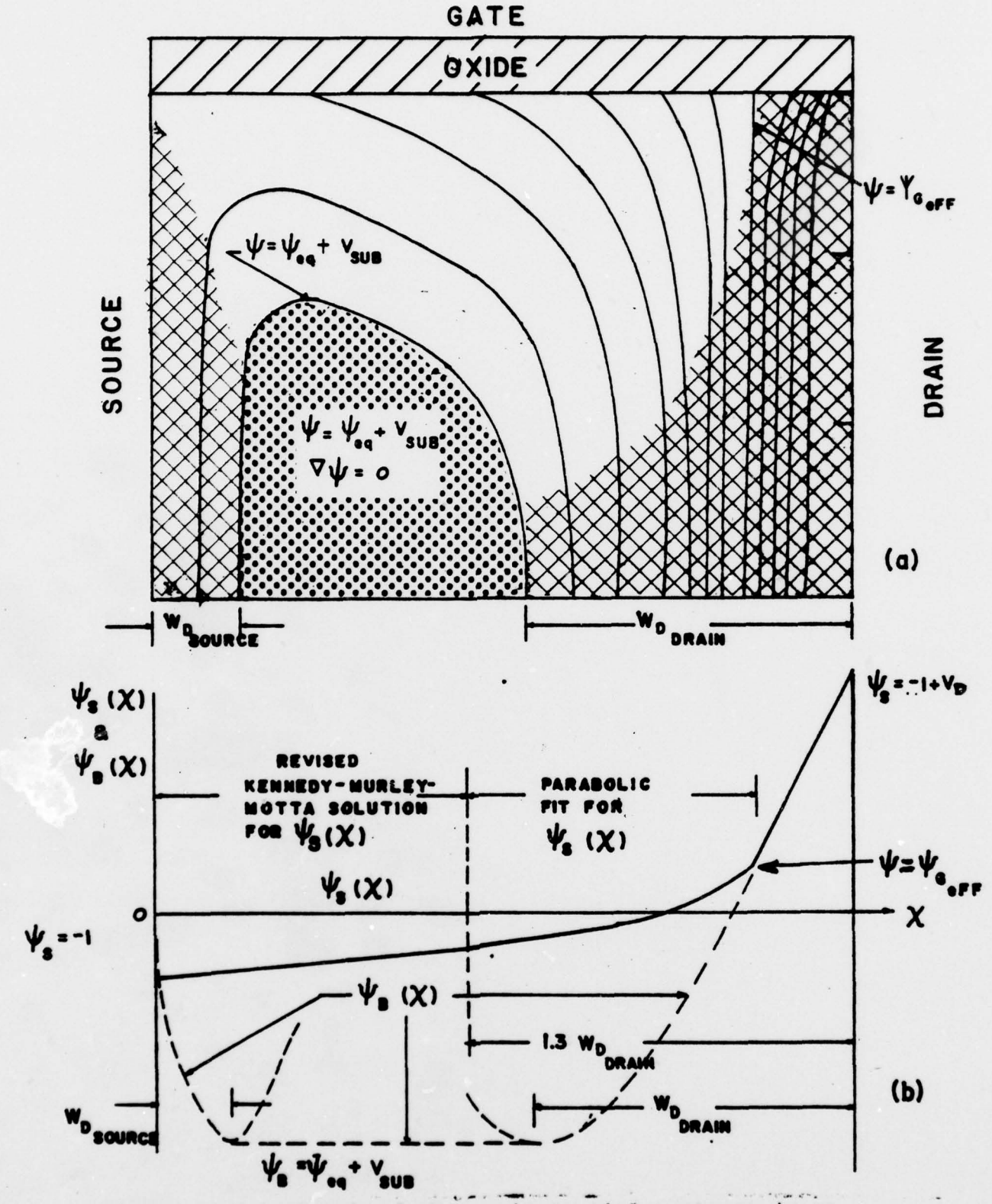

Figure 10 (a) Sketch of the equipotential contours in an MOSFET operating in saturation, providing insight into how to supply an initial estimate for $\psi(x,y)$. (b) Sketch of the potential $\psi_s(x)$ along the oxide-silicon interface, and $\psi_B(x)$ deep within the bulk material.

smoothly into the constant potential existent in that charge-neutral region. The composite dashed curve in Figure 10(b), for these three regions, serves to define the bulk potential $\psi_B(x)$, which is applicable deep within the substrate region, provided that $y < y_{JUNCTION}$ (see Figure 1(b)).

In contrast with the above regions, the unshaded region in Figure 10(a) depicts the region where $\psi(x,y)$ must be regarded as a two-dimensional function of the position coordinates x and y. At the oxide-silicon interface, y=0, it is effectively bounded on the left by the source itself, and on the right by the equipotential contour $\psi = \psi_{G_{eff}}$, which defines the point along the interface at which the component of electric field, E_y, normal to the interface vanishes. (In terms of the energy band diagrams commonly employed in MOSFET theory, this is the point along the channel at which the flatband condition occurs.) That contour is labelled in Figure 10(a) as $\psi = \psi_{G_{eff}}$, where:

$$\psi_{G_{eff}} = \psi_G + (k_{si}/k_{ox})y_{ox}Q_{ss} \quad , \tag{59}$$

with $\psi_G = \psi_{EQ} + V_G$, as defined in Figure 3(a). Equation (59) defines the surface potential at the interface ψ_s which leads to vertical electric field reversal. This condition occurs, of course, only in the saturation mode, when the drain potential ψ_D exceeds $\psi_{G_{eff}}$.

The approach that we have adopted for estimating the two-dimensional potential distribution $\psi(x,y)$ in the unshaded region of Figure 10(a) is the following. An estimate for the surface potential $\psi_s(x)$ is first obtained by using the one-dimensional theory for MOSFET operation developed by Kennedy and Murley [11], as modified by Motta [12]. In principle,

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[11] D.P. Kennedy and P.C. Murley, IBM Jour. Res. & Dev., 17, 1 (1973).

[12] R.F. Motta, Steady State Theory for the Metal-Oxide-Semiconductor Field Effect Transistor, Ph.D. Dissertation, University of Florida, 1976.

this solution is applicable up to the value of x at which $\psi_s(x) = \psi_{G_{eff}}$. It has been found empirically, however, that it is better to terminate the Kennedy-Murley-Motta solution for $\psi_s(x)$ at a value of x about 1.3 times the drain depletion width, $W_{D_{drain}}$, from the drain itself, and to extend $\psi_s(x)$ parabolically beyond that point of termination. Said parabolic extension is then matched in magnitude and slope to the Kennedy-Murley-Motta solution at that point, defining two of the three constants required to describe a parabola. The third constant depends upon whether or not the flatband condition is reached at some x between the point at which the parabolic fit is applied and the location of the drain, itself. If $\psi_D < \psi_{G_{eff}}$, the parabolic extension of $\psi_s(x)$ is forced to equal the drain potential ψ_D at $x = x_{channel}$ (see Figure 1). Otherwise, $\psi_s(x)$ is forced to equal $\psi_{G_{eff}}$ at the value of x at which the parabolic drain depletion potential equals $\psi_{G_{eff}}$. The latter case is the one illustrated in Figure 10(b).

Given an estimate for $\psi_s(x)$ and $\psi_B(x)$, as determined in the manner described above, one-dimensional solutions of Poisson's equation (with y the geometric variable) are then obtained along each plane x corresponding to an index I of the ψ lattice. These one-dimensional solutions are forced to: (a) match $\psi = \psi_B(x)$, with zero slope $d\psi/dy$ in the shaded regions of Figure 10(a), and (b) match the "known" solution $\psi = \psi_s(x)$ at the oxide-silicon interface. Furthermore, these one-dimensional solutions are forced to satisfy the boundary condition on $d\psi/dy$ at the oxide-silicon interface by adjusting the electron quasi-Fermi potential ϕ_n so that eq. (25) is satisfied. (The latter function is assumed to be independent of y over the range of y covered by these one-dimensional solutions--an approximation.)

In the above manner, "reasonable" quasi-two-dimensional first estimates for $\psi(x,y)$ are established throughout the silicon substrate. The estimate for $\psi(x,y)$ in the oxide region assumes a linear dependence on y which matches the

"known" surface potential $\psi_s(x)$ at the interface and the gate potential ψ_G at the gate electrode.

4.04.2 Initial Estimate for $\theta(x,y)$

The initial estimate of the stream function $\theta(x,y)$ is based upon the observation that θ takes on the value of unity at about the point at which the electron number density becomes intrinsic (i.e., n=1), throughout most of the substrate region. Furthermore, θ is found to be essentially a function only of the coordinate x, except in regions immediately adjacent to the source and drain "contacts." This being the case, the one-dimensional solution for $\psi(y)$, utilized as described above in the unshaded region of Figure 10(a), is then used at the vertical plane corresponding with the edge of the source depletion region to estimate the location y_1 at which n=1. A parabolic estimate for $\theta(y)$ is then established which takes on the value of unity, with zero slope, at $y=y_1$, and which takes on the value of zero at the oxide-silicon interface, y=0. That parabolic estimate for θ is then assigned along all vertical planes corresponding to the index I of the θ lattice, thereby establishing a one-dimensional initial estimate for θ.

4.04.3 Initial Estimates for $n(x,y)$ and $p(x,y)$

Given initial estimates for $\psi(x,y)$ and $\theta(x,y)$, the corresponding initial estimate for the electron density $n(x,y)$ is obtained using the "marching" method described in Section 3.04. The hole density $p(x,y)$ is obtained by applying the Boltzmann relationship, eq. (3), since the hole quasi-Fermi potential ϕ_p is assumed invariable (eq. (5)), and takes on the value which causes p to equal N_A in the charge-neutral substrate region.

CHAPTER II

Evaluation of the Computer Model's Performance

In this chapter, we show comparisons of the results obtained from our computer model with those obtained for identical cases from the IBM program devised by Mock. We also show comparisons of the predictions obtained from our model with experimental data obtained for a particular MOSFET structure whose processing parameters have been characterized from test sites fabricated as an integral part of its chip.

1.0 Comparisons with Mock's Program

Figures 11 through 34 show comparisons of the results obtained from our model with those obtained from the program developed for IBM by Mock. The figures are ordered in groups of three: (1) $\psi(x)$ versus x along the oxide-silicon interface, (2) $\psi(y)$ versus y along the vertical plane perpendicular to the interface midway between the source and the drain, and (3) n(y) versus y along the same vertical plane. The eight cases represented by these data represent extremes of operation as follows:

1) Short channel (3μm) ($N_D = 10^{19}$ cm^{-3}, $N_A = 2\text{x}10^{16}$ cm^{-3}):
   - (a) Weak inversion, triode mode
   - (b) Weak inversion, saturation mode
   - (c) Strong inversion, triode mode
   - (d) Strong inversion, saturation mode

2) Long channel (10μm) ($N_D = 10^{19}$ cm^{-3}, $N_A = 2\text{x}10^{15}$ cm^{-3}):
   - (a) Weak inversion, triode mode
   - (b) Weak inversion, saturation mode
   - (c) Strong inversion, triode mode
   - (d) Strong inversion, saturation mode

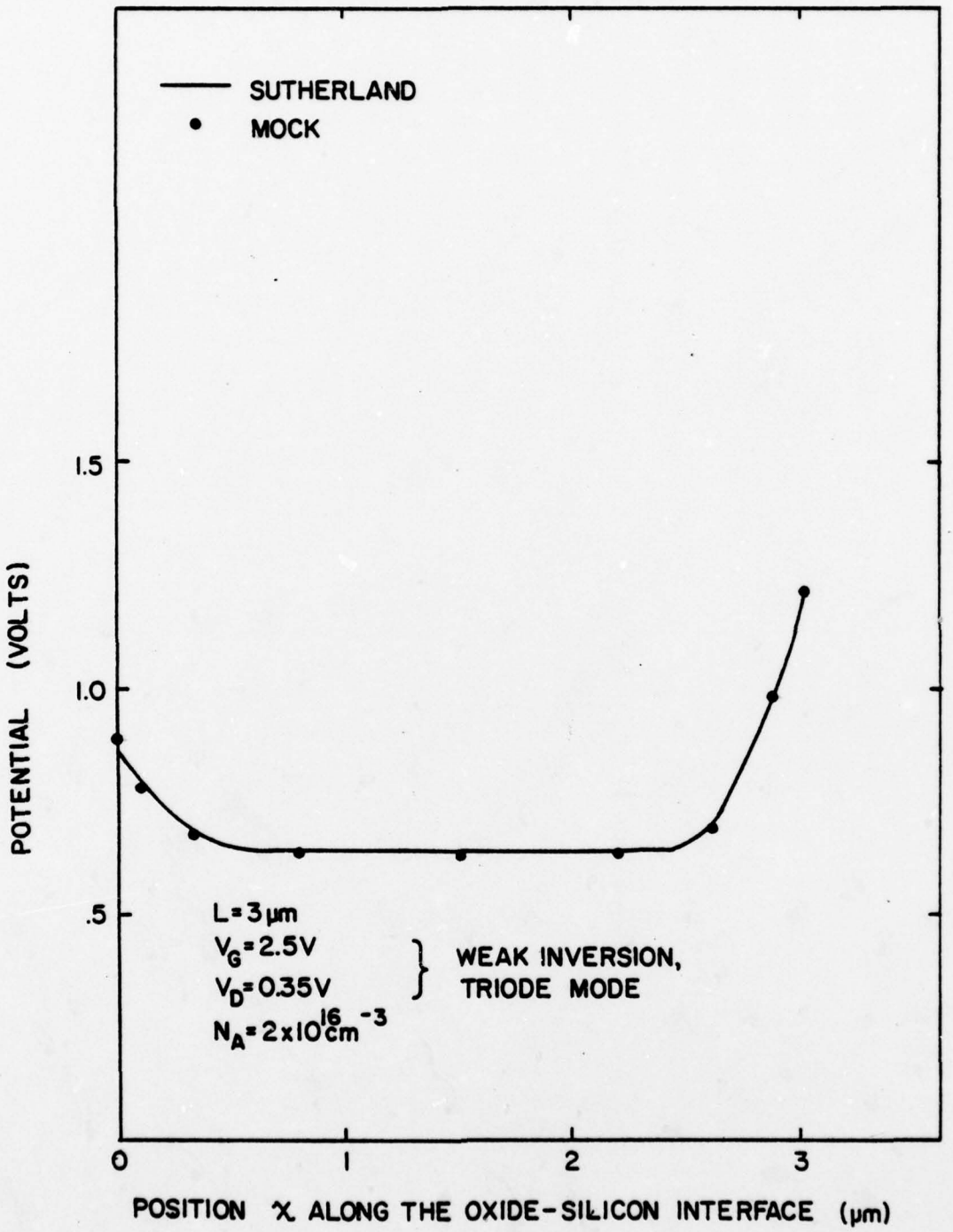


Figure 11 Comparison of the potential $\psi(x)$ along the oxide-silicon interface, generated by this model and by Mock's model.

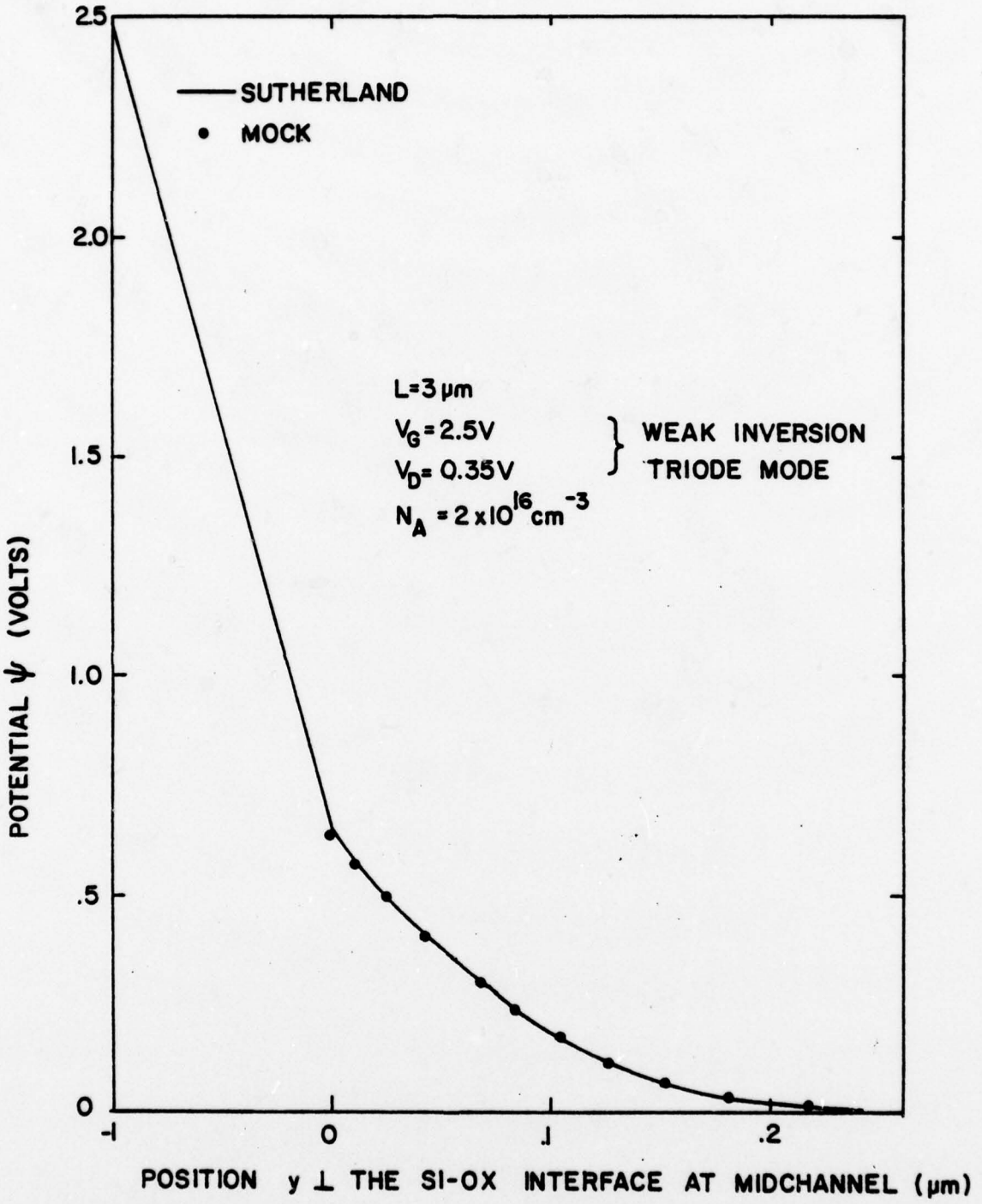


Figure 12 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

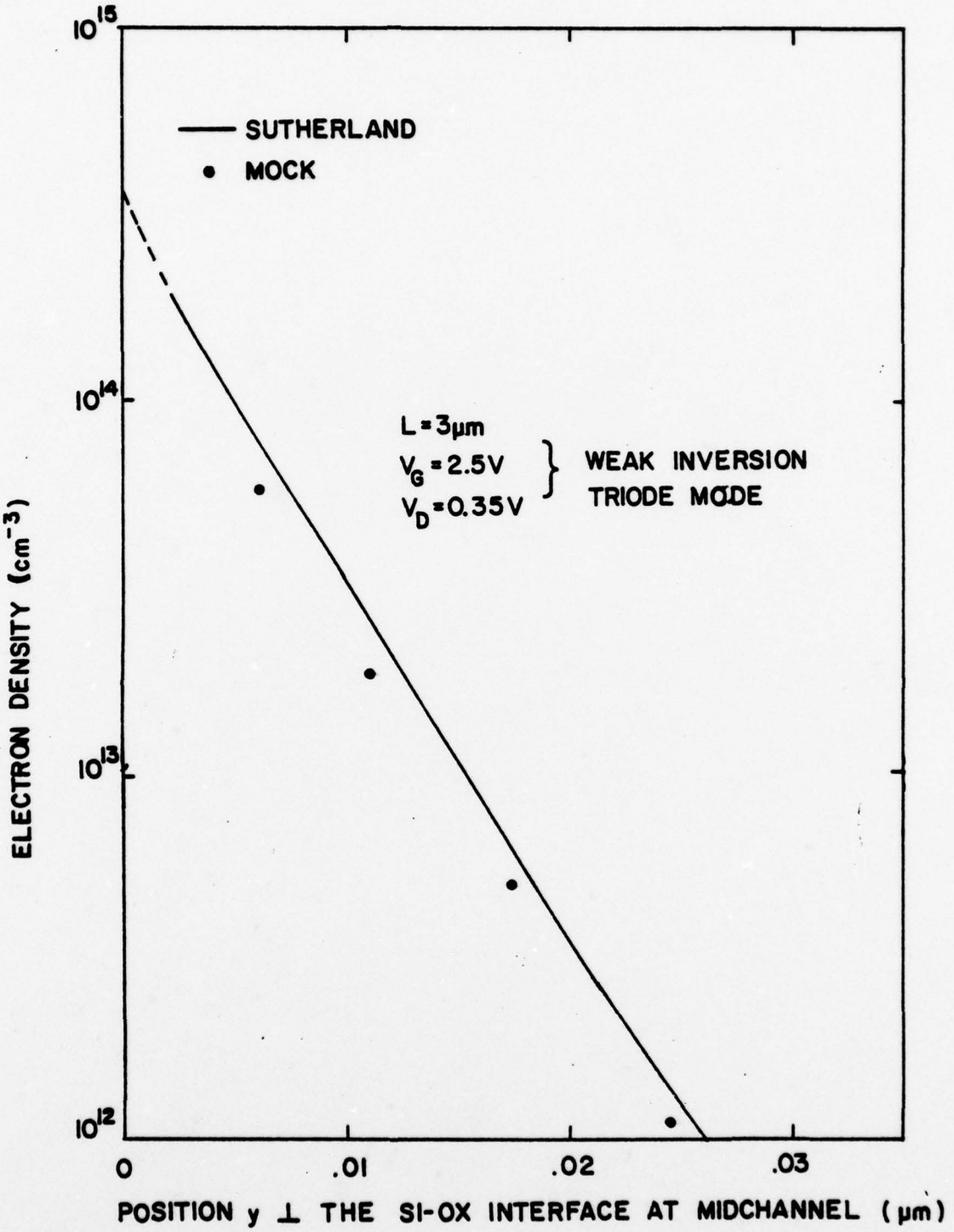


Figure 13 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

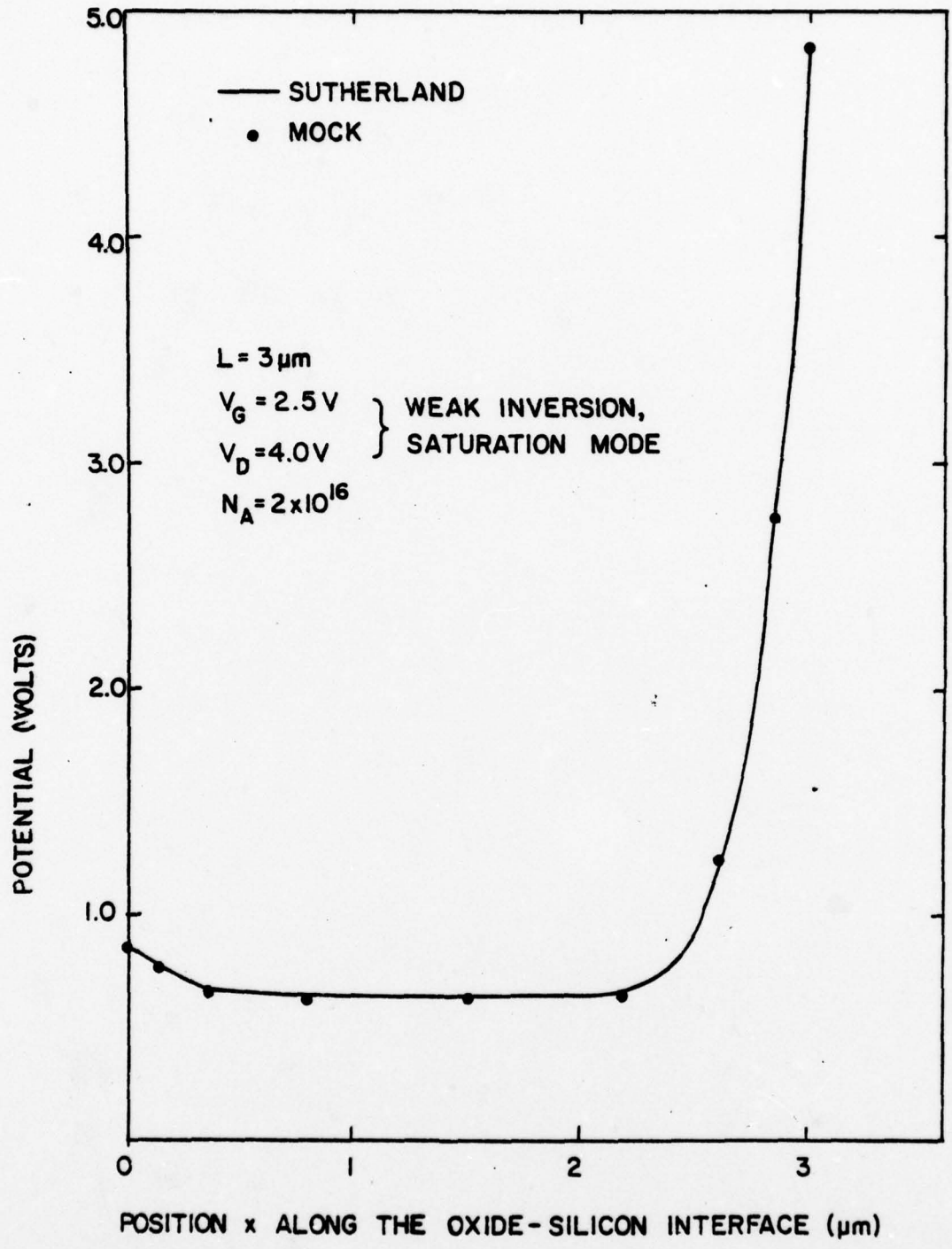


Figure 14 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

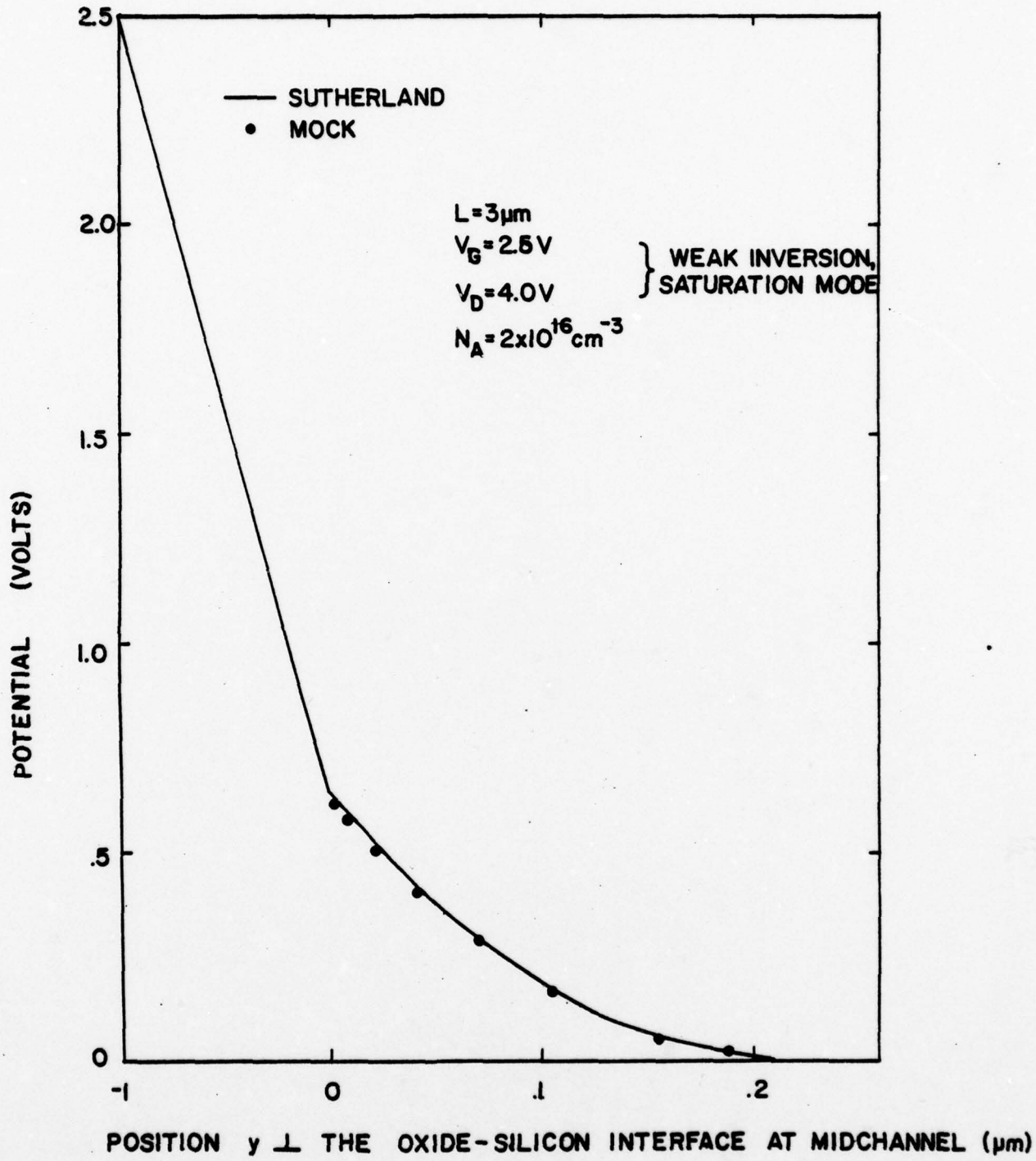


Figure 15 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

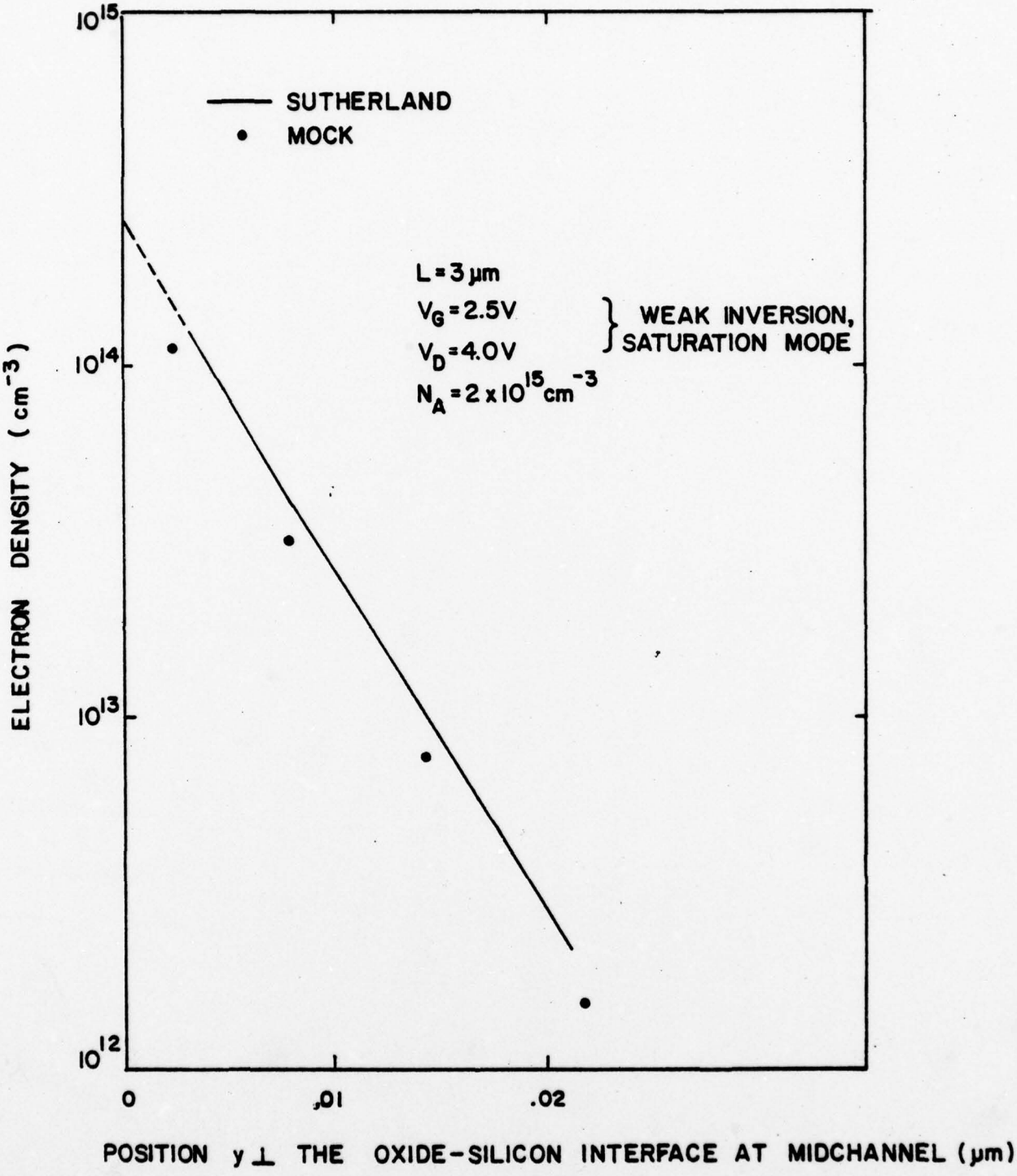


Figure 16 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

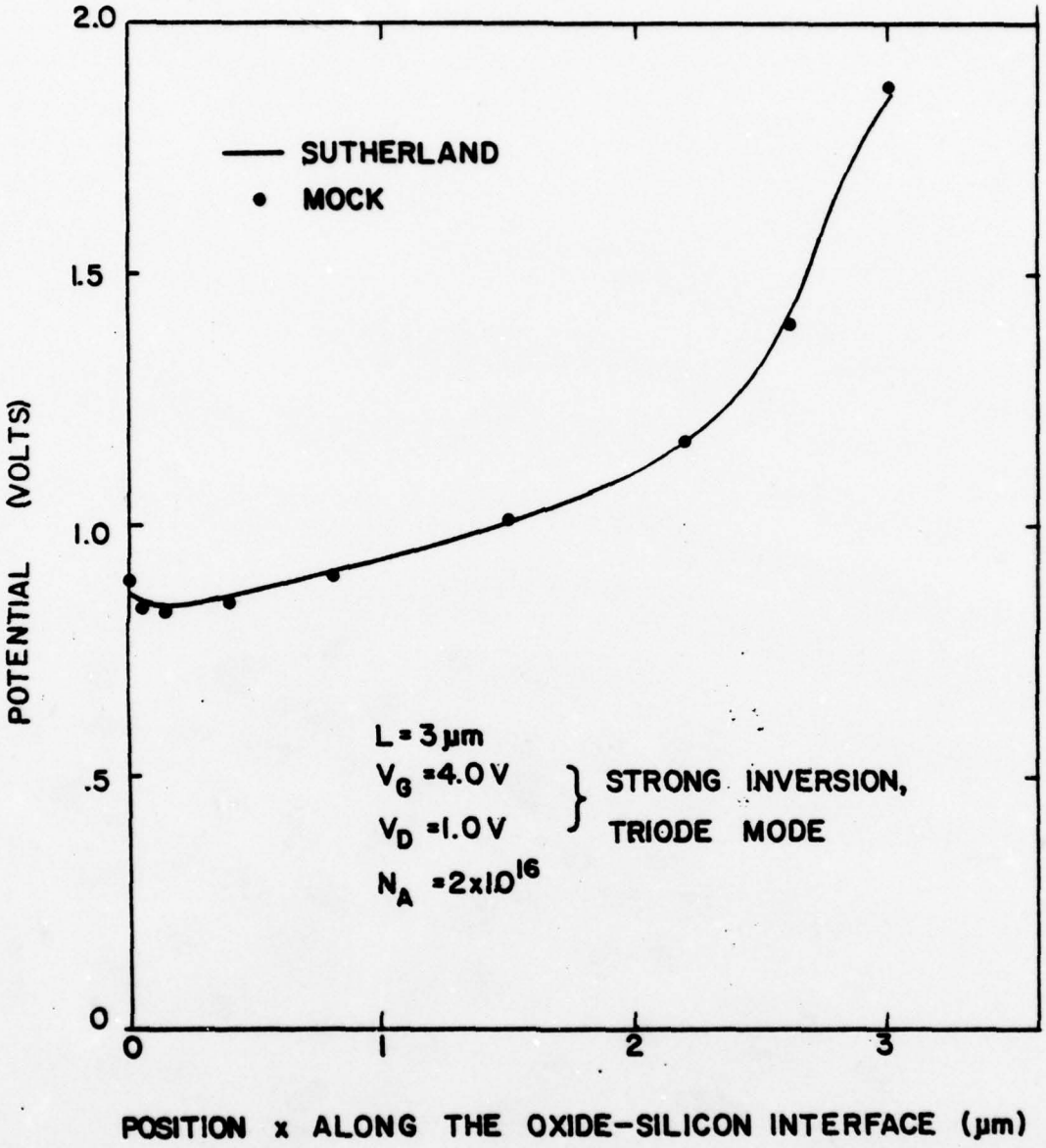


Figure 17 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

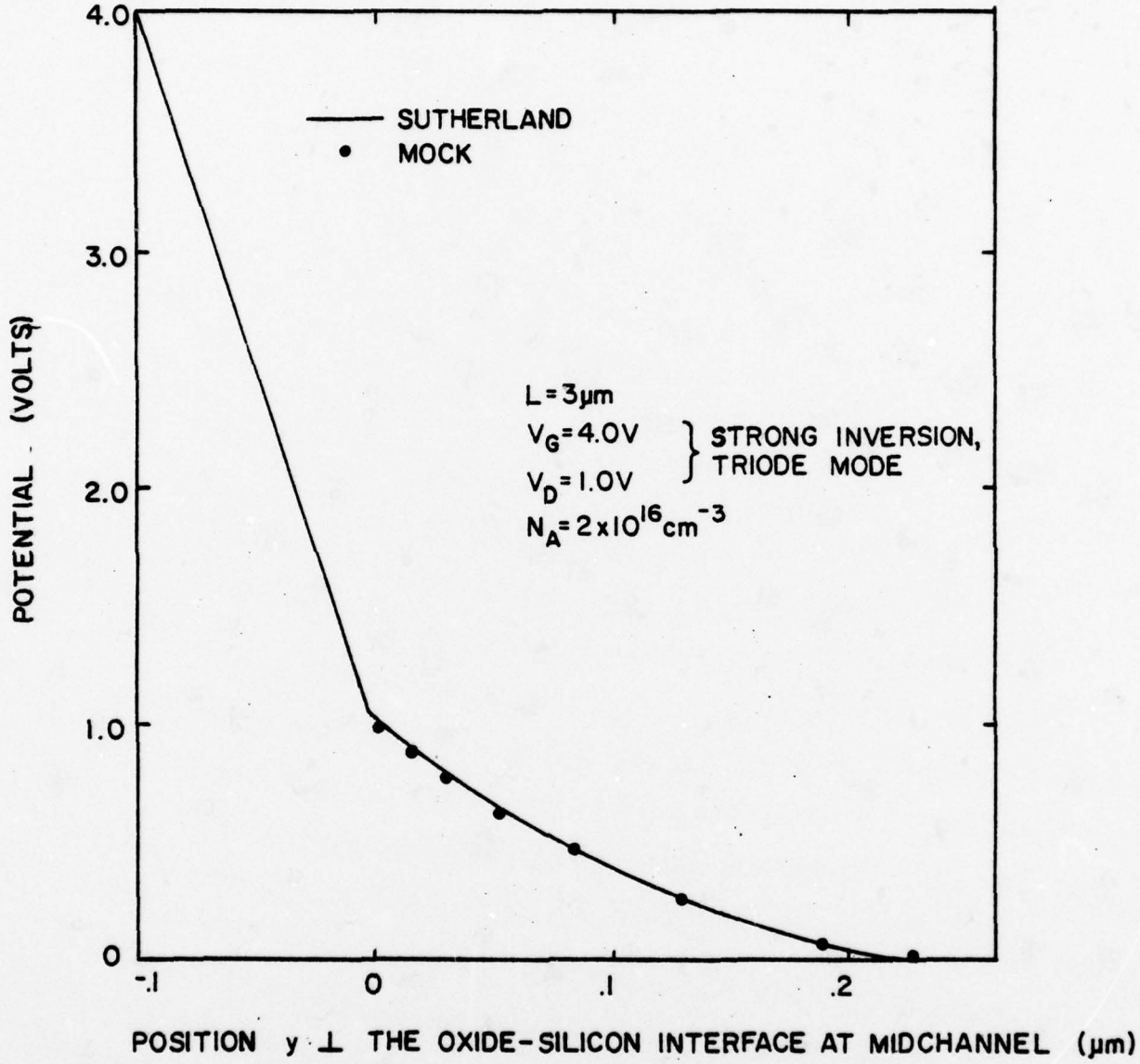


Figure 18 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

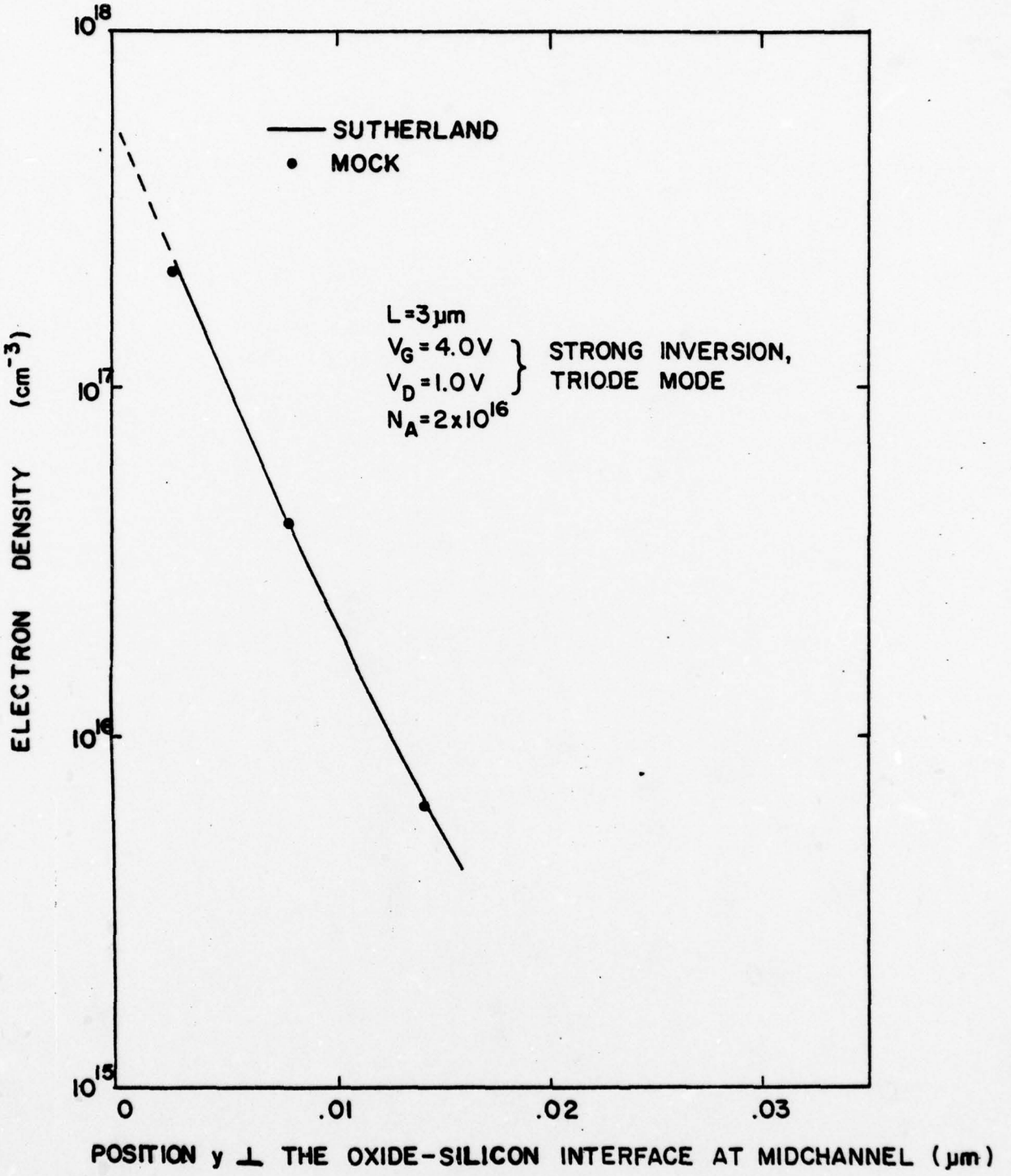


Figure 19 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

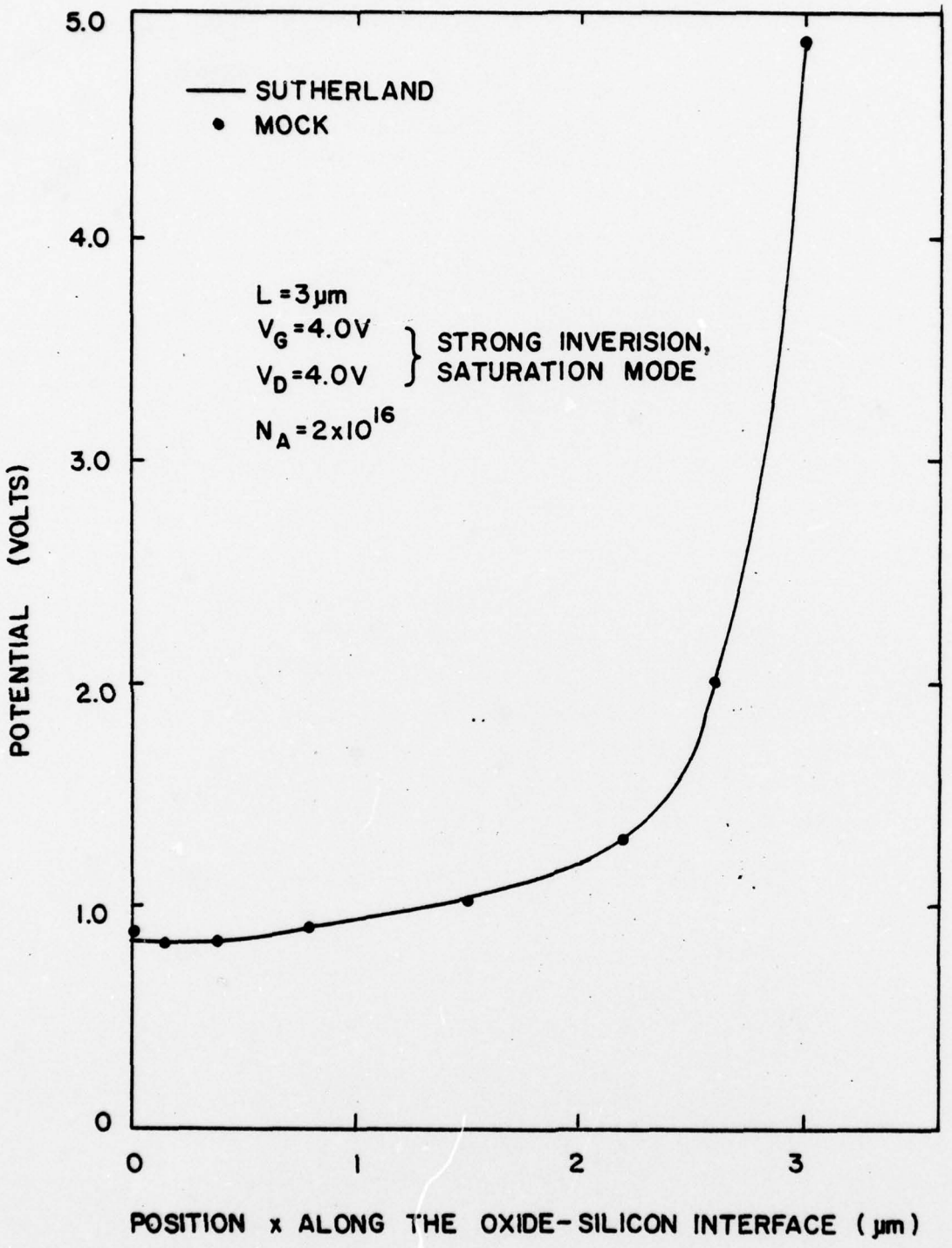


Figure 20 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

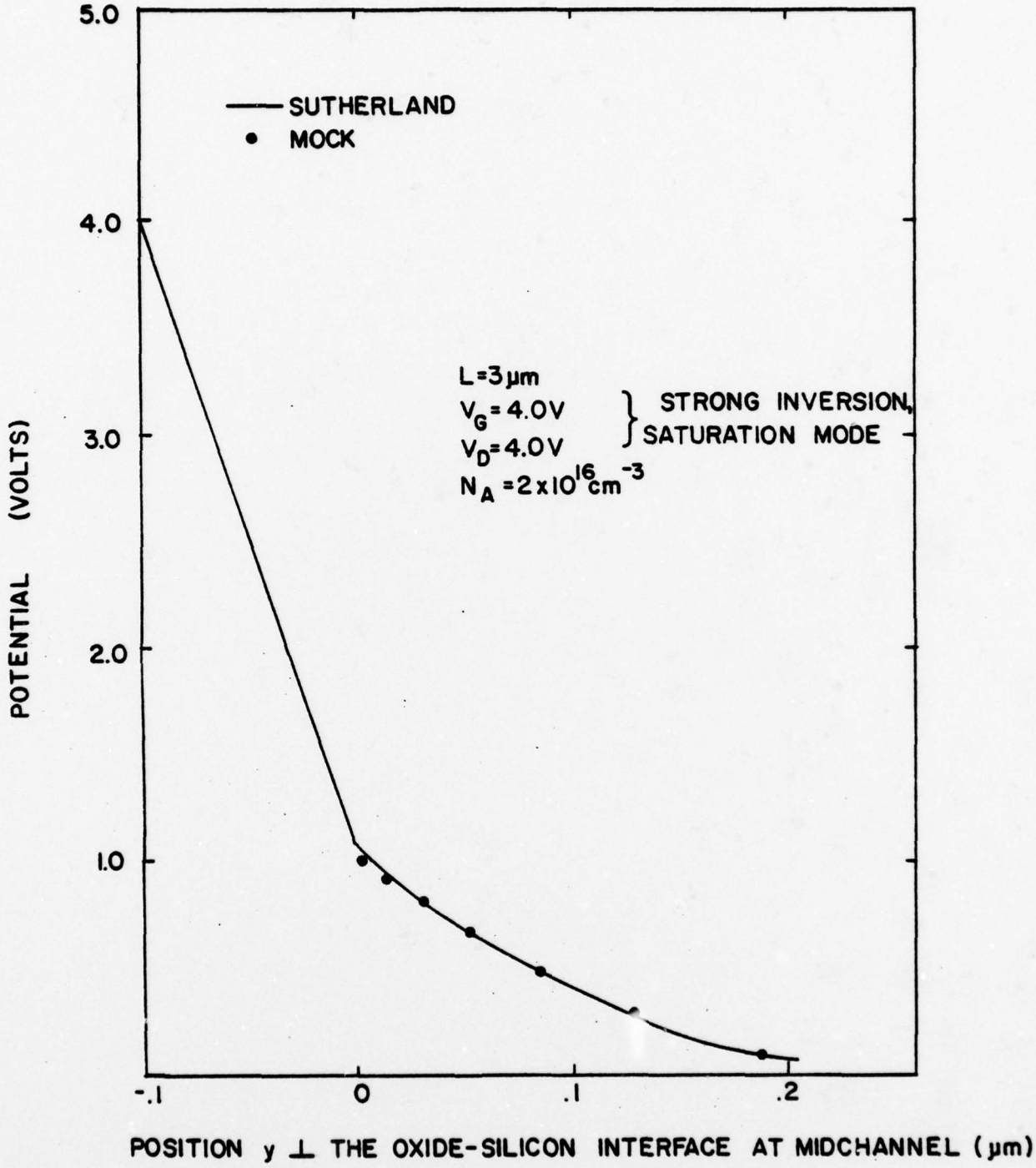


Figure 21 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

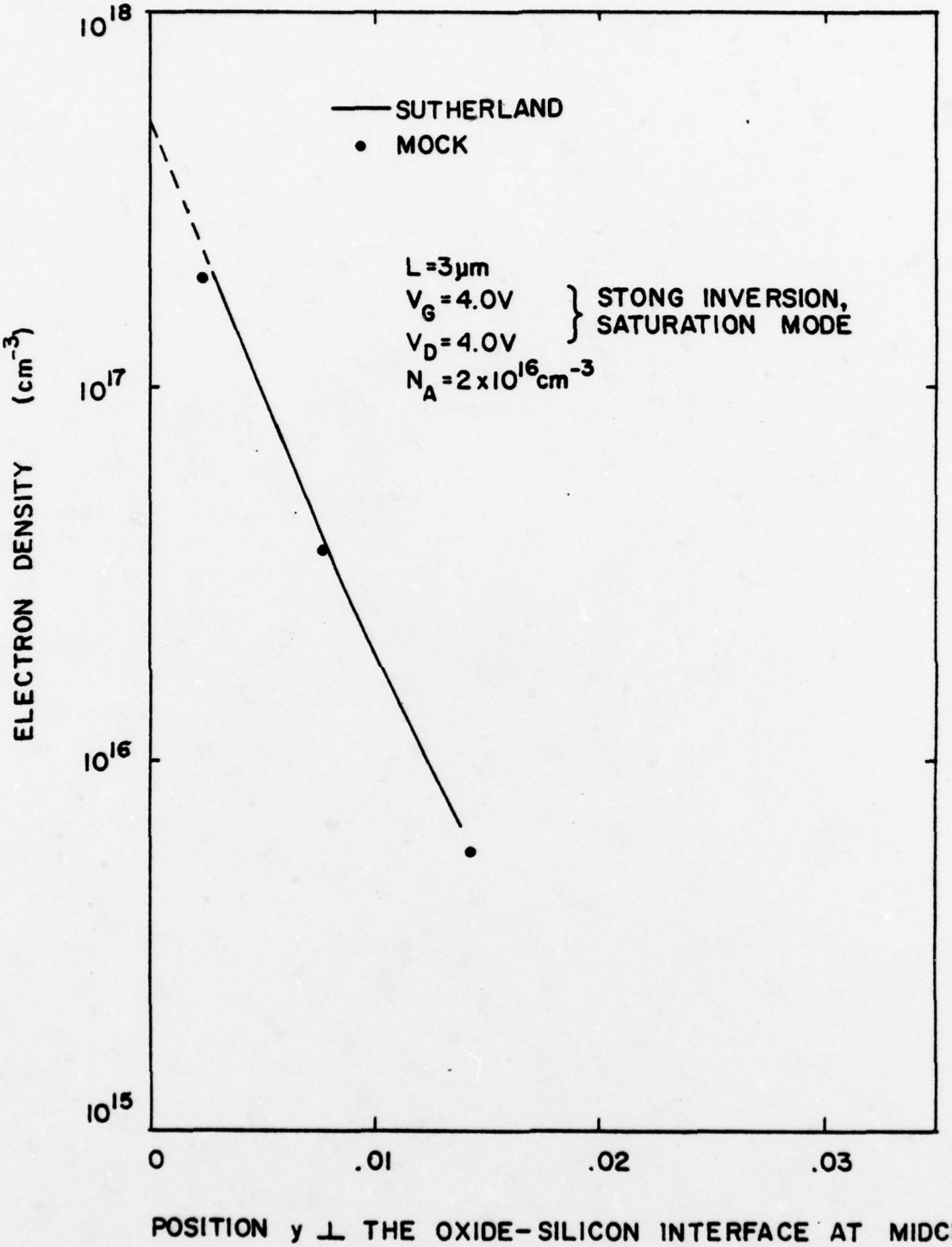


Figure 22 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

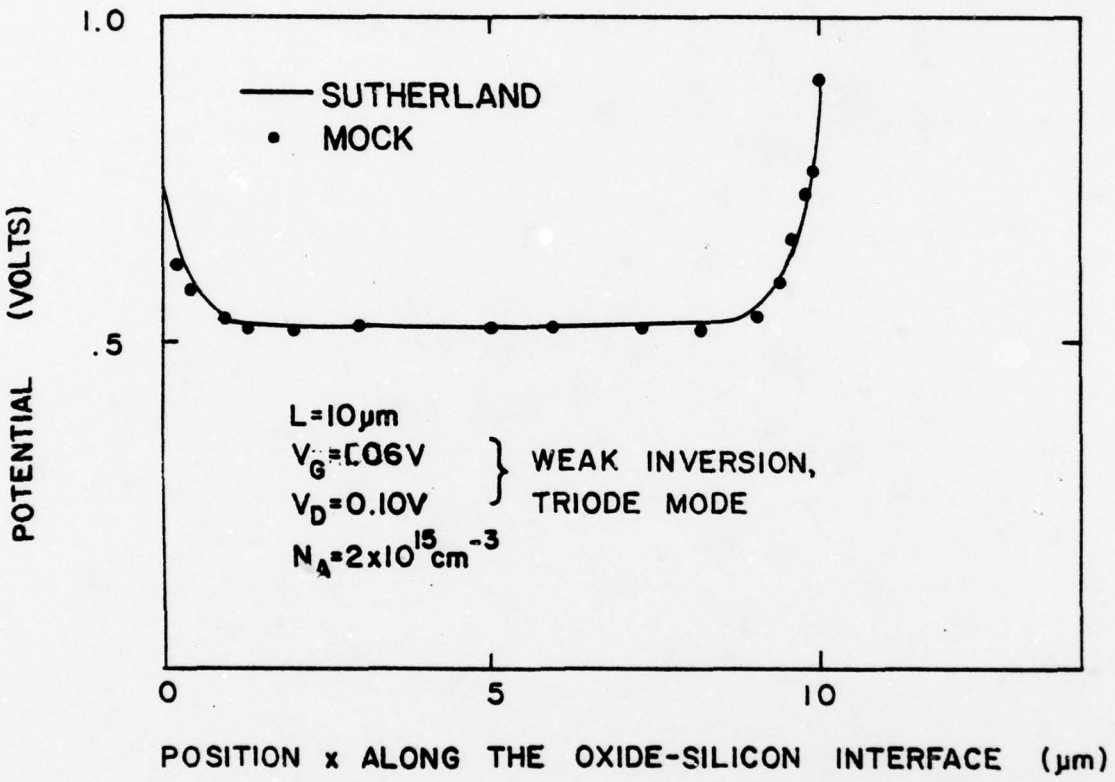


Figure 23 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

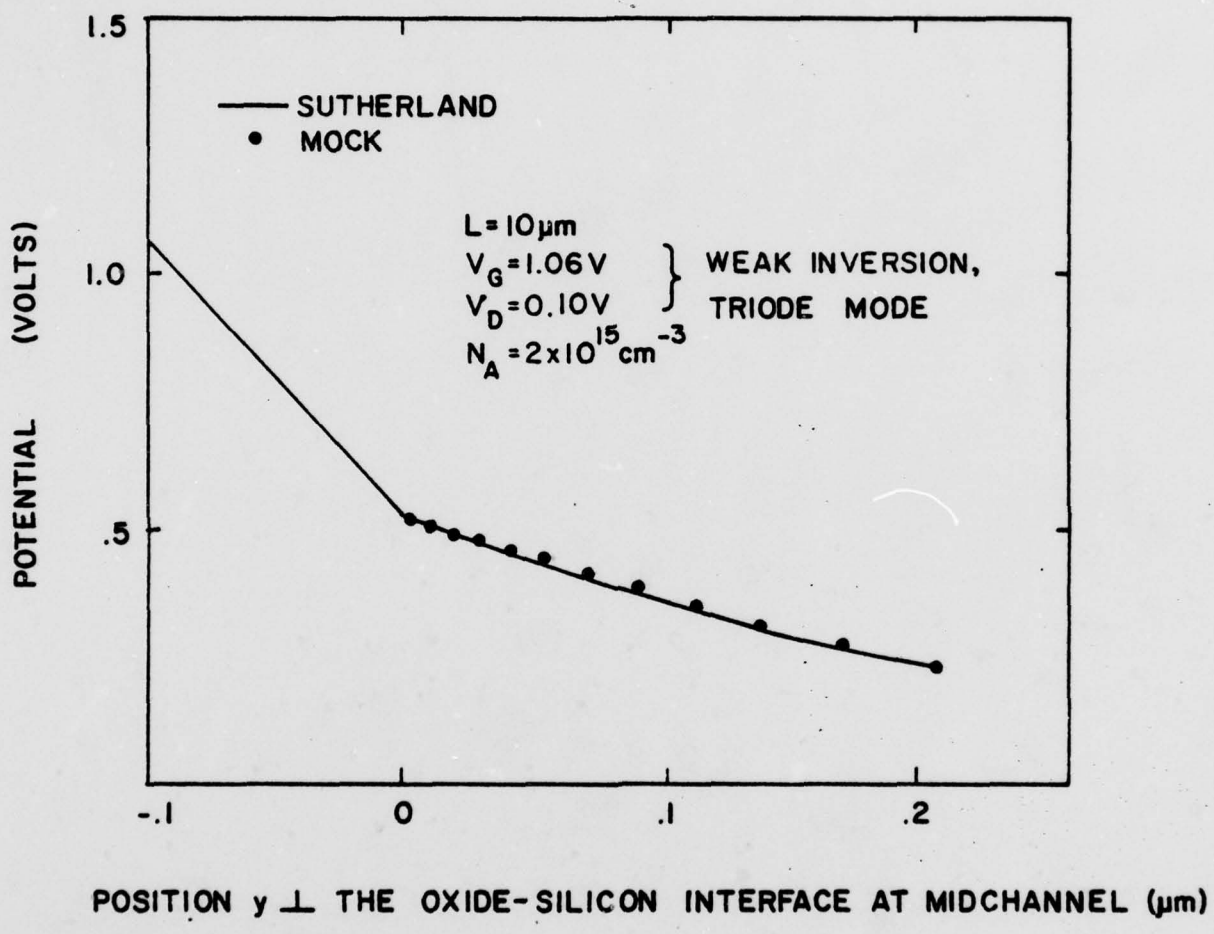


Figure 24 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

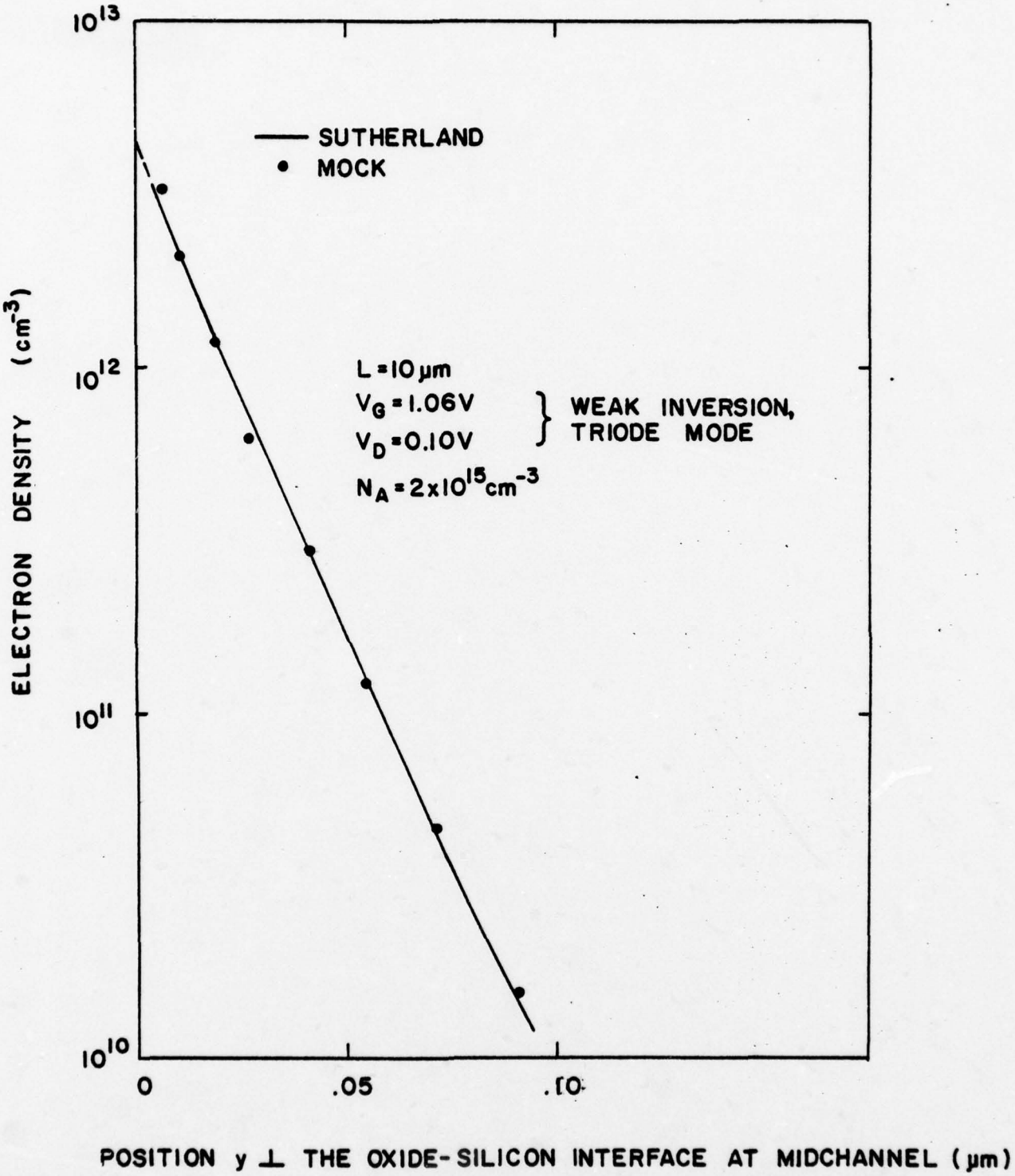


Figure 25 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

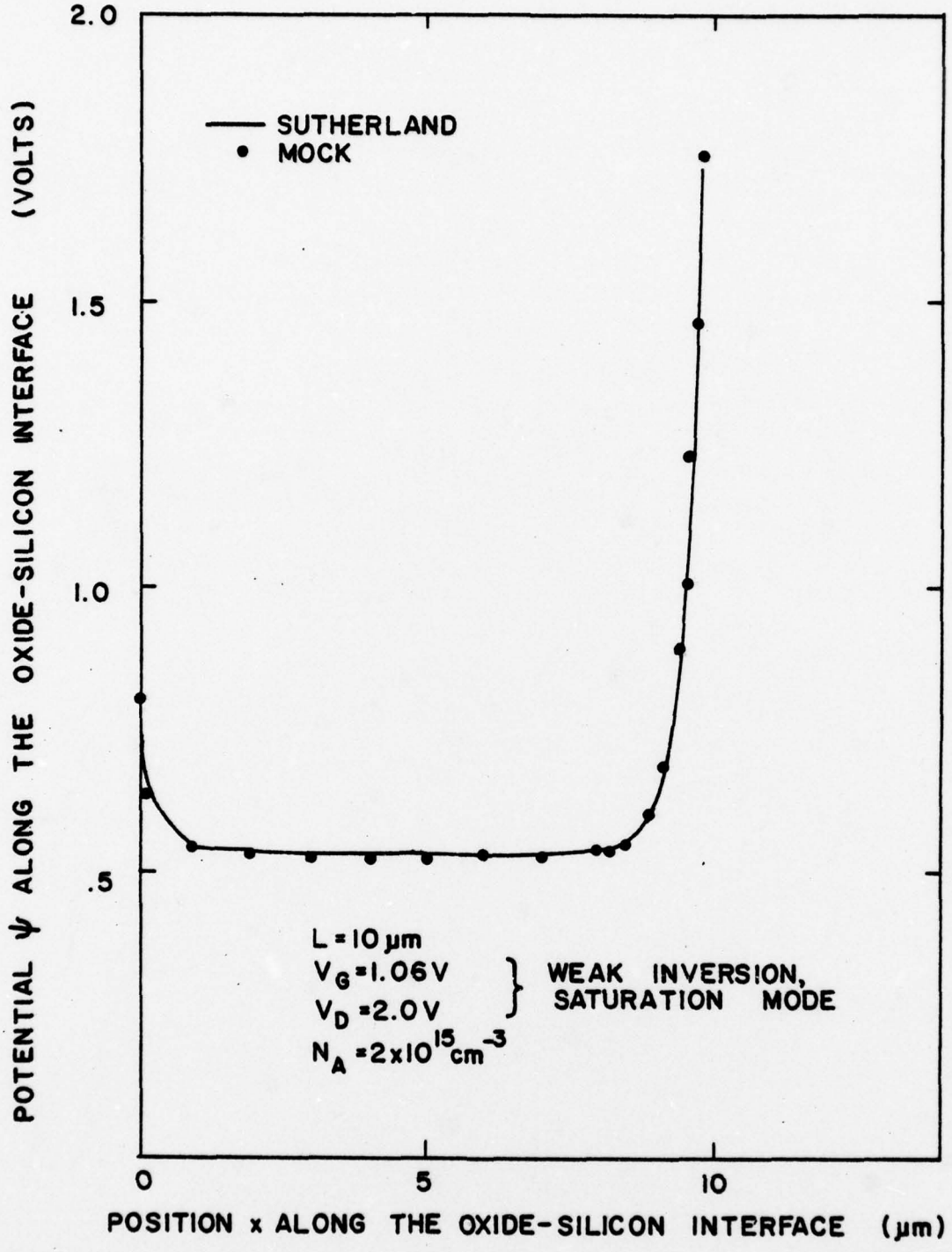


Figure 26 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

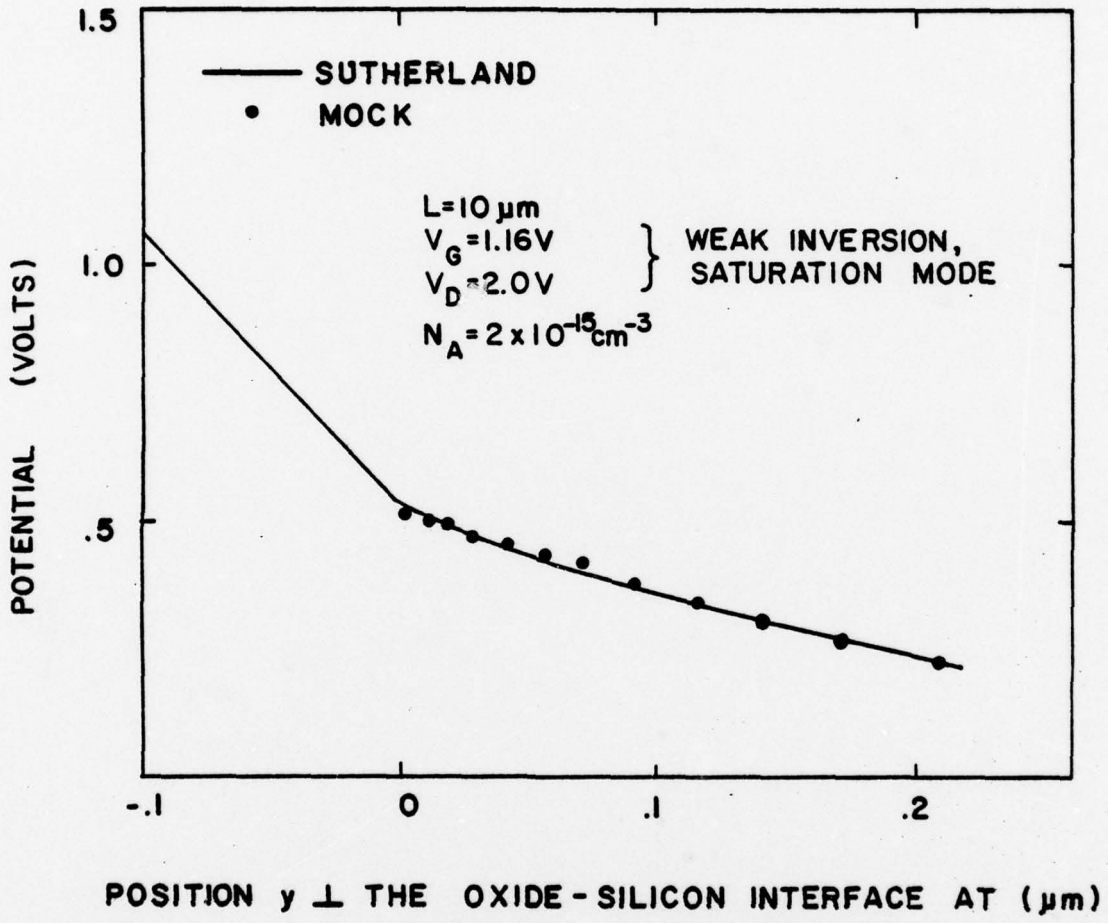


Figure 27 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

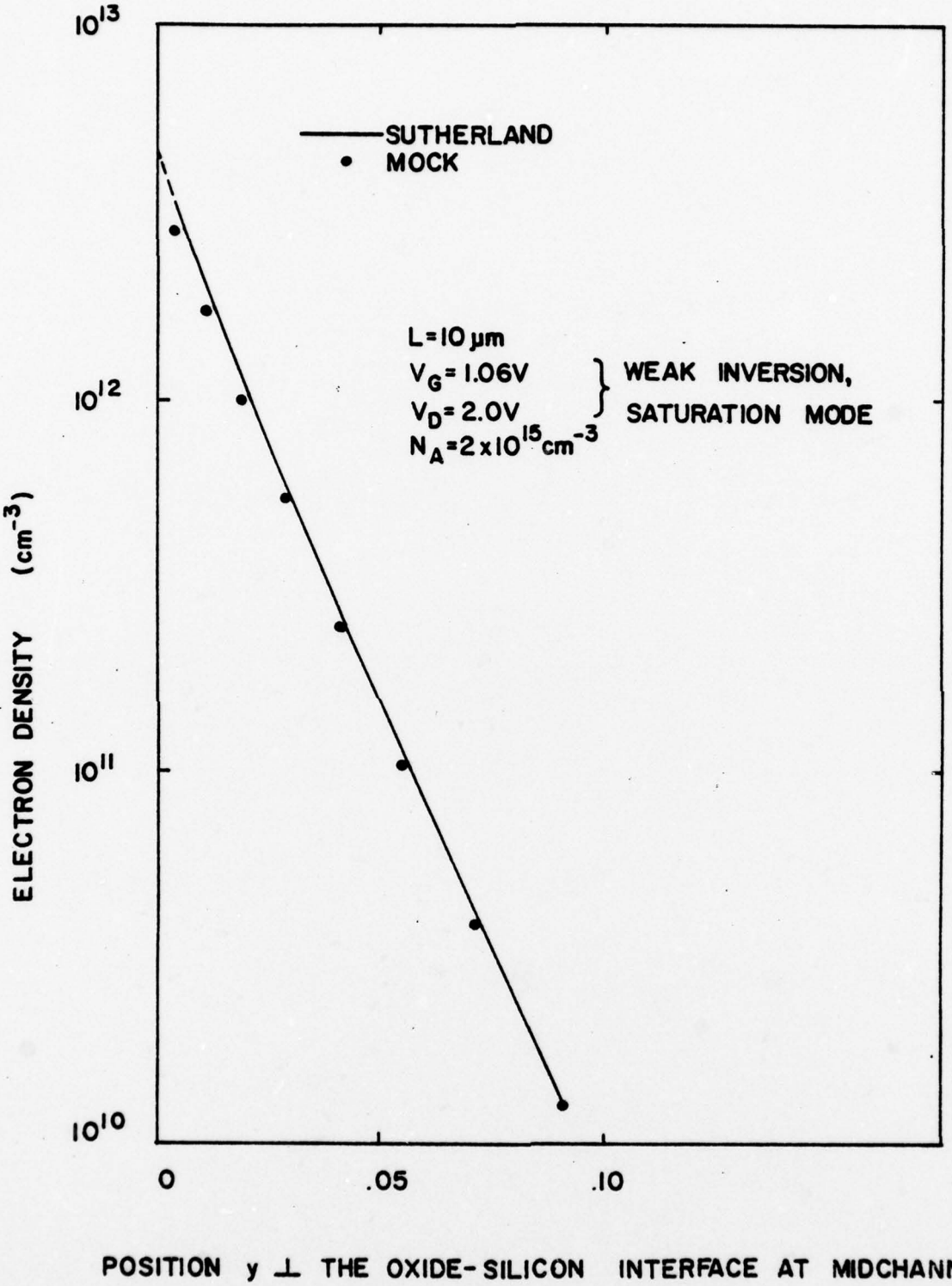


Figure 28 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

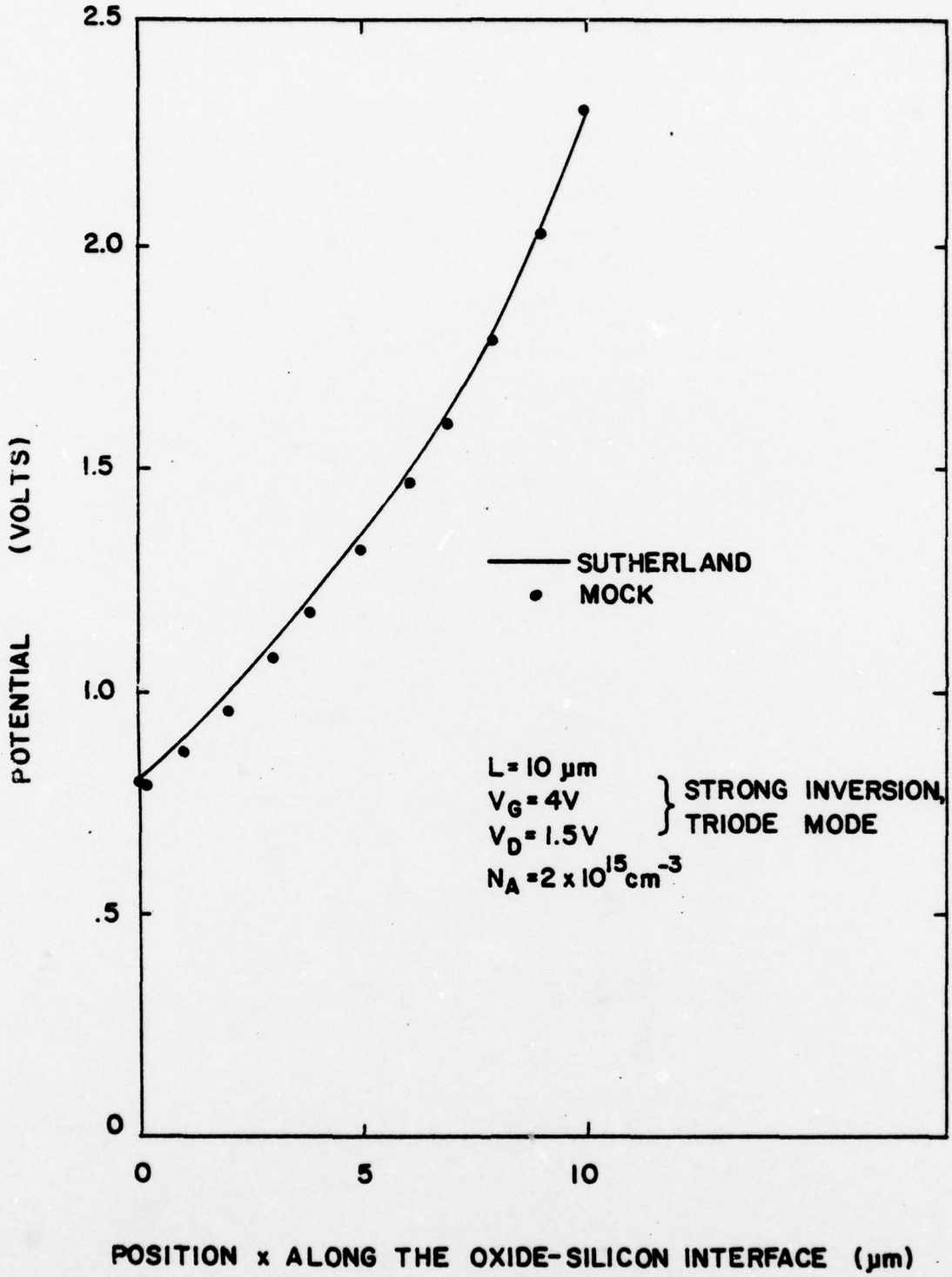


Figure 29 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

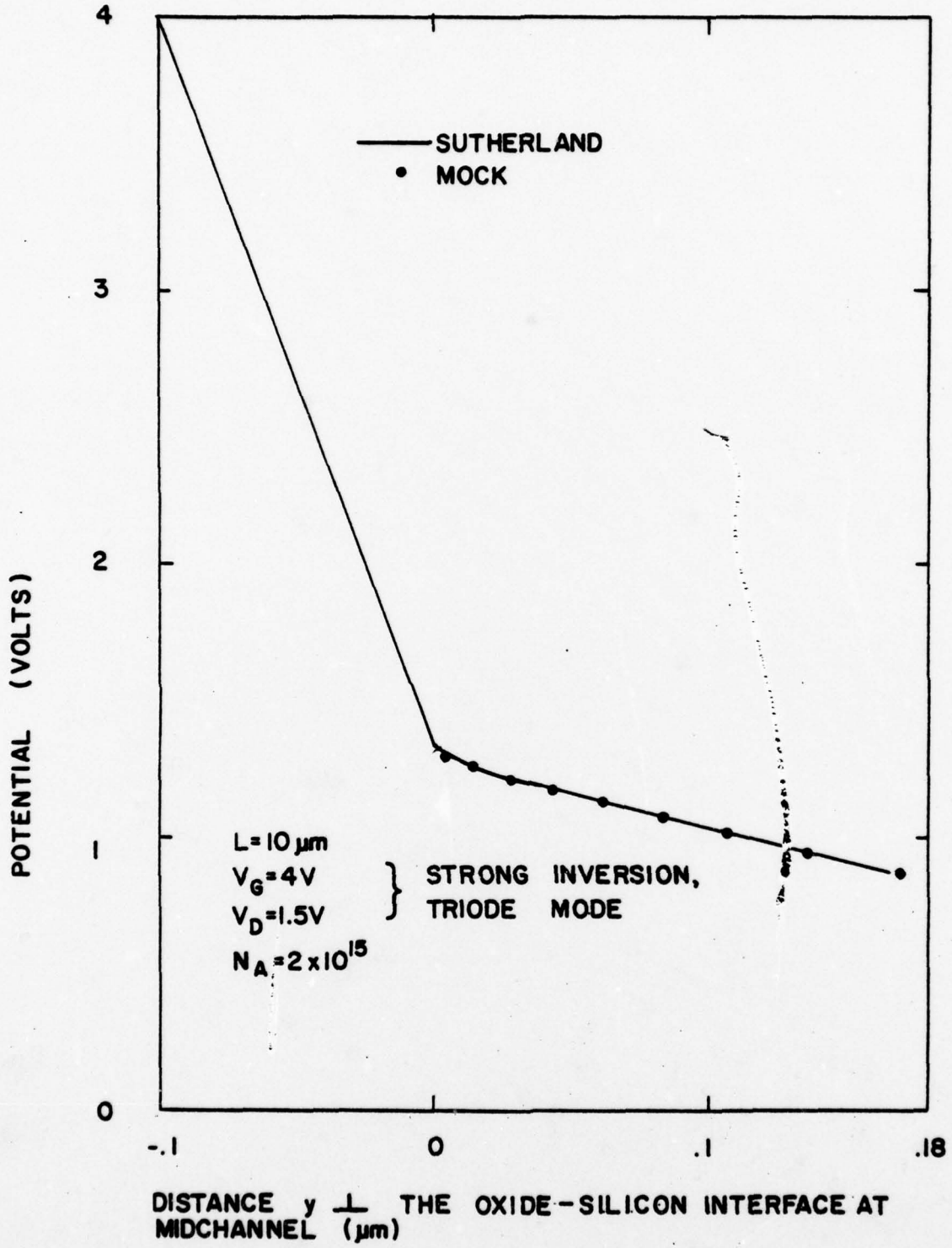


Figure 30 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

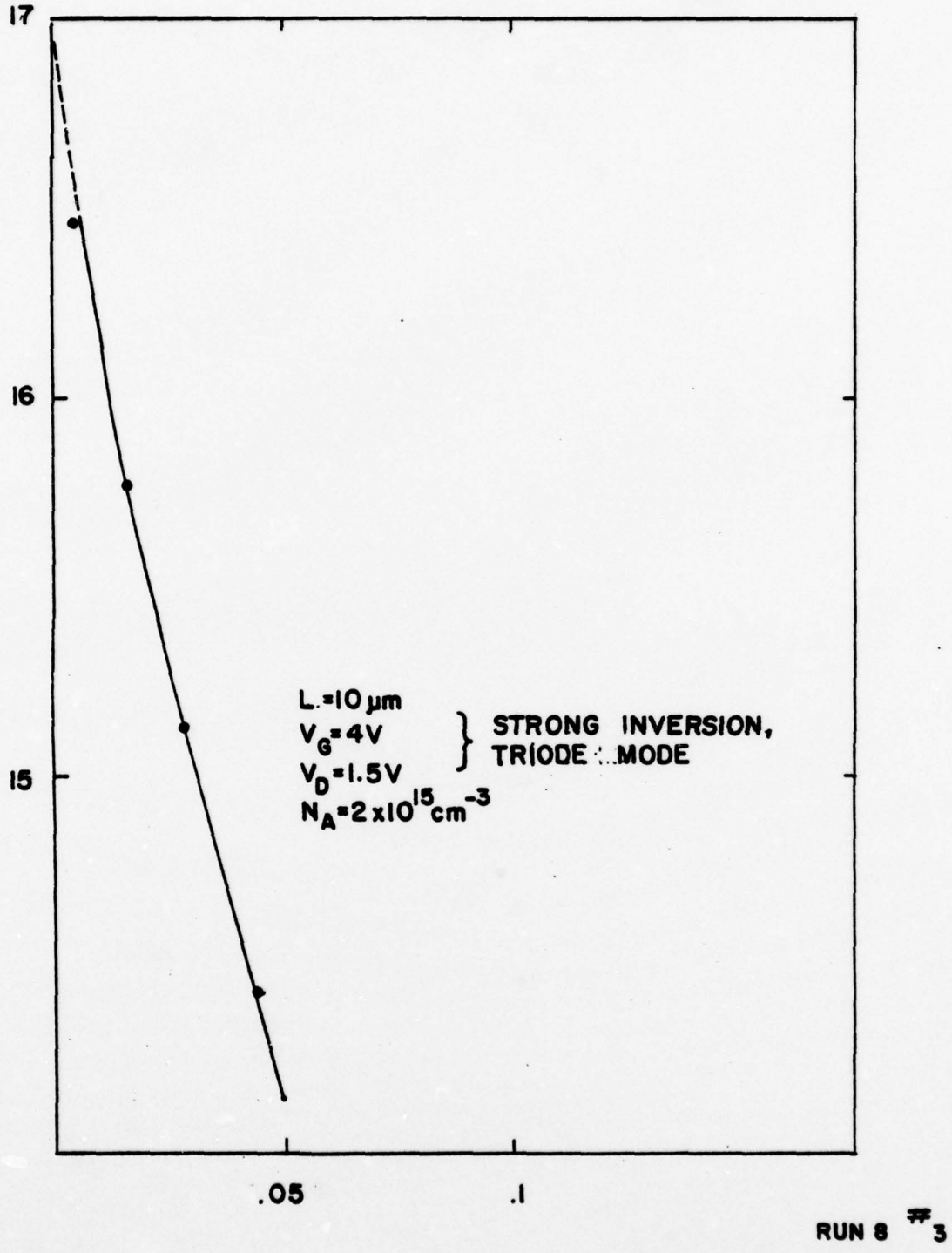


Figure 31 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

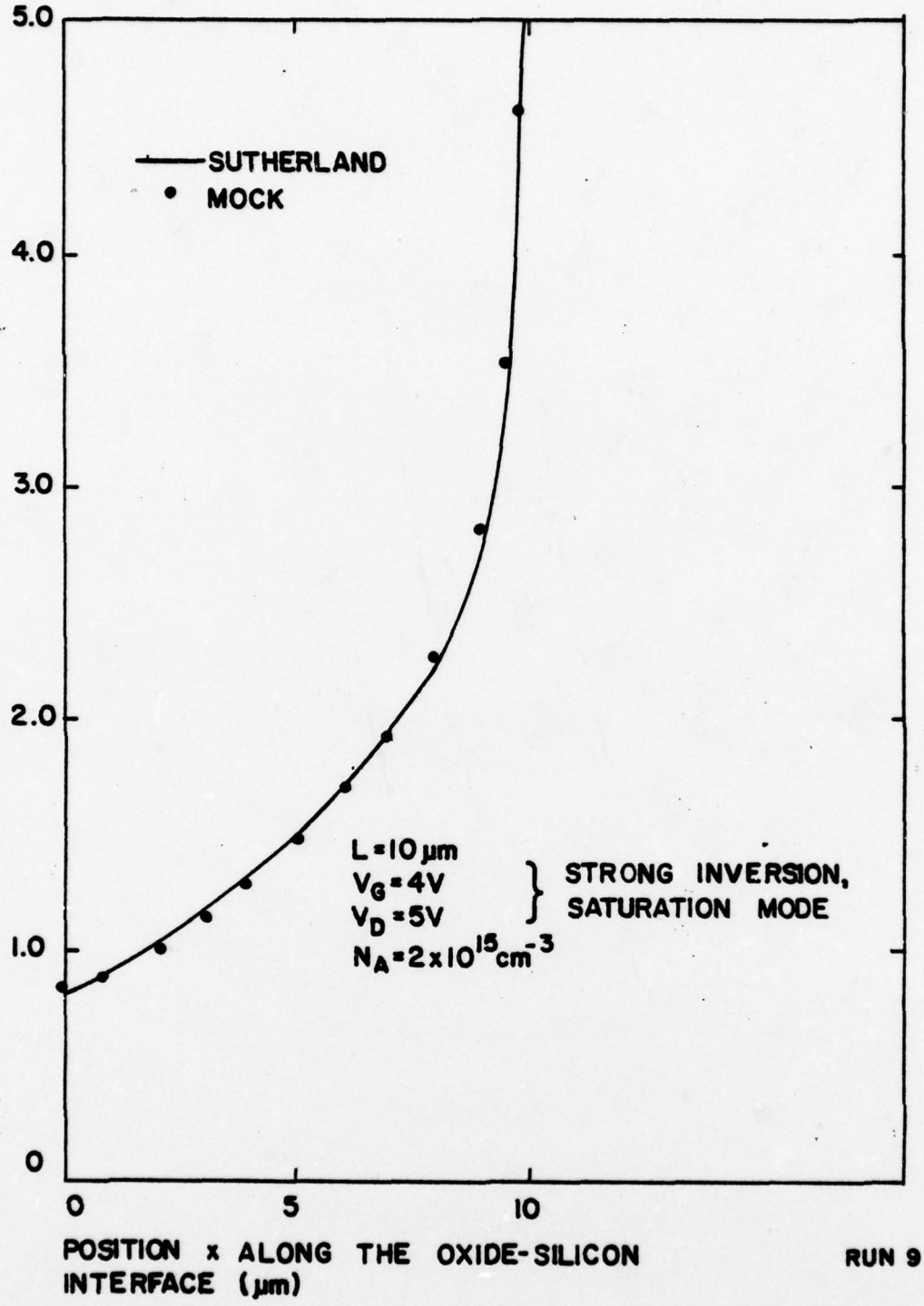


Figure 32 Comparison of the potential $\psi(x)$ along the oxide silicon interface, generated by this model and by Mock's model.

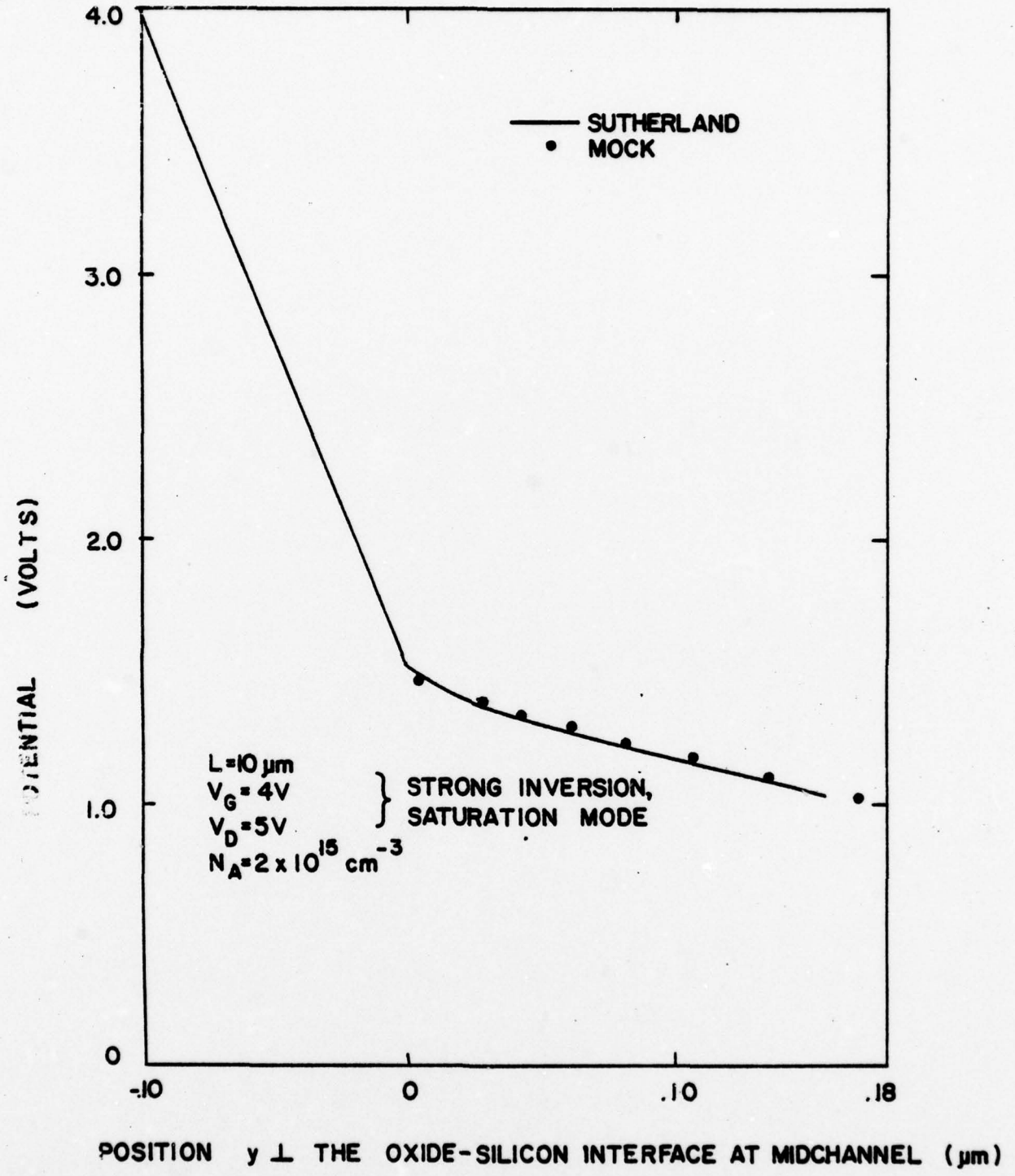


Figure 33 Comparison of the potential $\psi(y)$ along the plane midway between source and drain, generated by this model and by Mock's model.

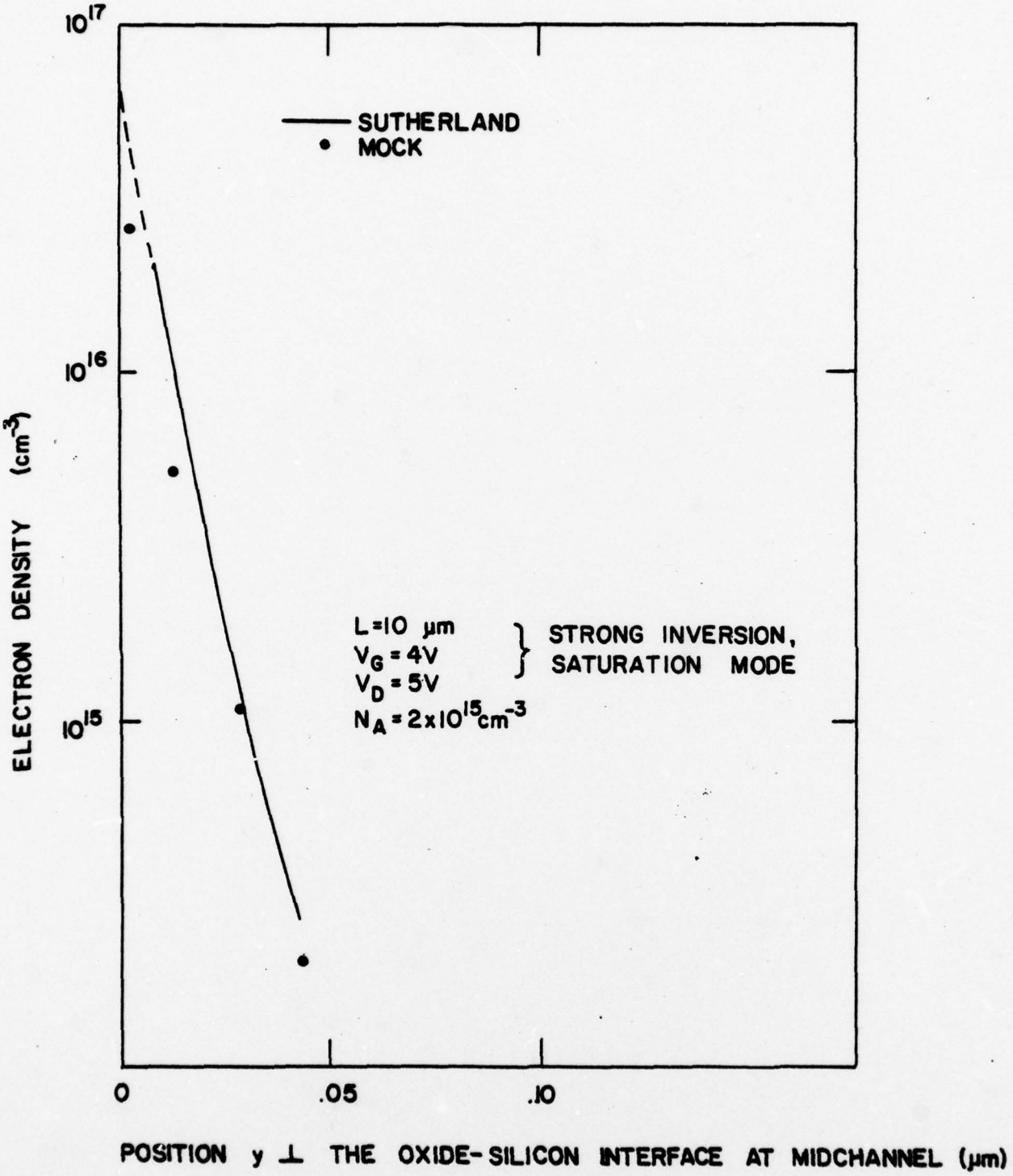


Figure 34 Comparison of the electron number density n(y) along the plane midway between source and drain, generated by this model and by Mock's model.

Table II compares the drain currents calculated by our model with those calculated by Mock's program for these eight cases

TABLE II -- COMPARISON OF DRAIN CURRENT CALCULATIONS
(ma/cm of channel width)

	Case	Our Model	Mock
L=3μm	Weak inversion, triode	.00164	.00127
	Weak inversion, saturation	.00184	.00148
	Strong inversion, triode	11.4	11.2
	Strong inversion, saturation	12.9	12.95
L=10μm	Weak inversion, triode	.000180	.000163
	Weak inversion, saturation	.000193	.000180
	Strong inversion, triode	23.4	22.9
	Strong inversion, saturation	27.8	29.0

It is seen that, in very weak inversion, the drain currents predicted by our model are only in fair agreement with those predicted by Mock's program, but in strong inversion the agreement is quite satisfactory. Insofar as the comparison of solution details is concerned, Figures 11 through 34 show essentially excellent agreement.

2.0 Comparison with Experiment

The experimental data with which we next compare our model were supplied to the author by Dr. D.L. Fraser, Jr. from measurements taken in conjunction with his Ph.D. dissertation [13], using the PMOS reliability test chips supplied

[13] D.L. Fraser, Jr., "Modeling and Optimization of MOSFET LSI Circuits," Ph.D. Dissertation, University of Florida, 1977. (This work was supported by the National Science Foundation, NSF Grant ENG-75-23078.)

by the National Security Agency, together with data on five additional test chips taken by NSA. The numerous test site configurations available on those chips, in addition to a silicon gate PMOS structure enabled Fraser to characterize the MOS device and its parameters in some detail. Full details are given in [13]. In particular such parameters as the device channel length and widch, the oxide thickness, the substrate doping, the source and drain doping, the gate threshold voltage V_T, and the flatband voltage V_{FB} are all deduced from such measurements. From these data, we deduced the effective mobility of holes in the inversion channel, (182 cm^2/volt-sec) as well. Thus, this appeared to the author to be an ideal structure for comparing the theoretical predictions of our model with experimental reality.

To utilize our model, which is for n-channel structures, to model the p-channel device, only two steps are necessary: (1) replace the electron mobility with the hole mobility, and (2) reverse the sign of Q_{ss}, since a positive Q_{ss} raises the threshold for a p-channel device, while lowering it for an n-channel device. Hence, to raise the threshold with an n-channel model, negative Q_{ss} should be used. We used an "effective" Q_{ss} ($Q_{ss_{eff}} = C_{ox} V_{FB}$), with V_{FB} the flatband voltage deduced from Fraser's measurements.

Figures 35 and 36 show comparisons of the drain current predicted with our model with Fraser's measurements. Figure 35 is for a fixed gate voltage putting the device into strong inversion (V_{GS} = 10V, with V_T = 2.18V). Figure 36 gives drain current versus V_{GS}, with the drain connected to the gate, so that $V_{DS} = V_{GS}$, and includes the weak inversion case as well. The results obtained are quite gratifying.

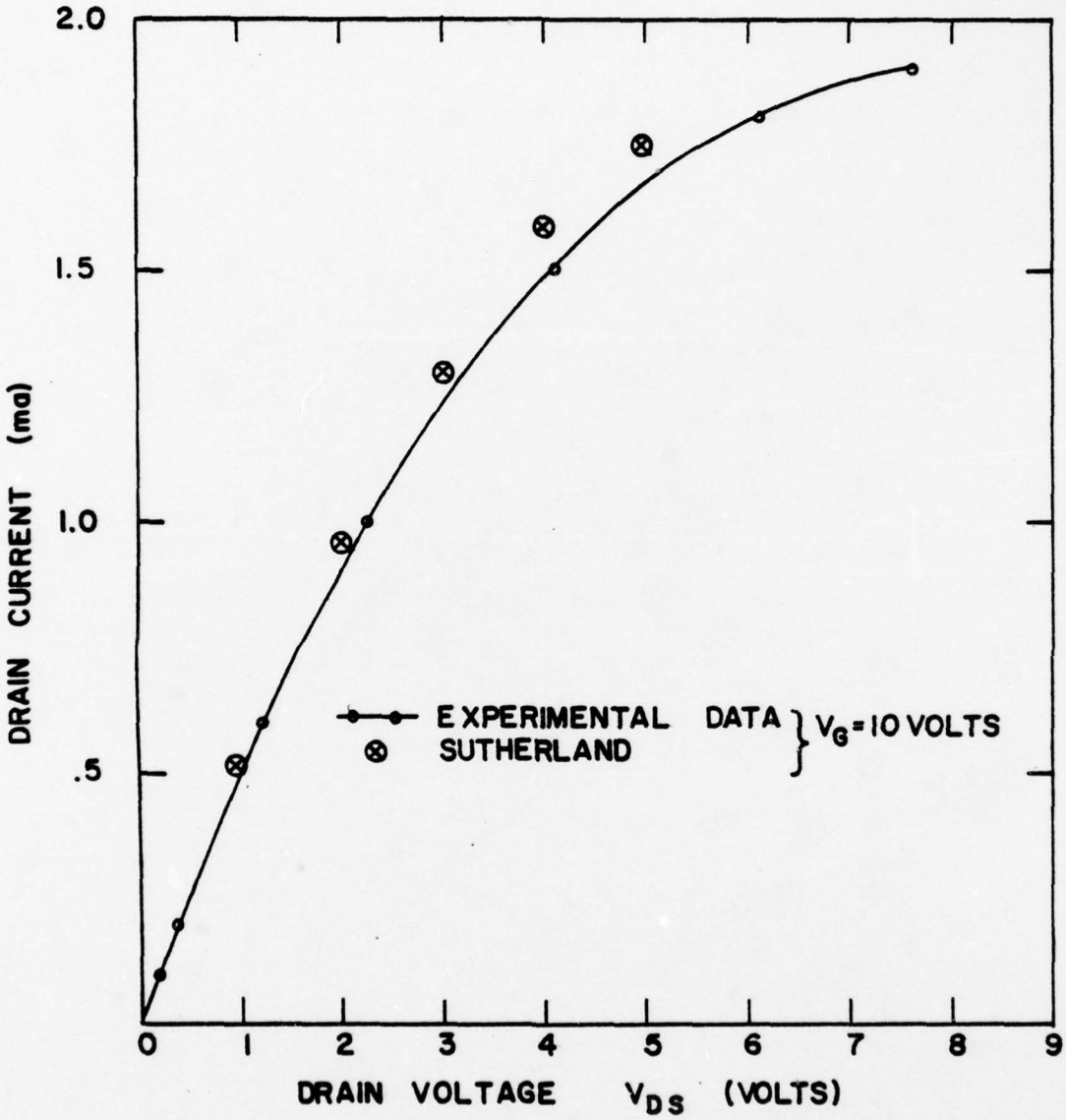


Figure 35 Comparison of the theory with experimental data obtained from a p-channel MOSFET device whose parameters were characterized by means of test sites on its chip. The curve shows I_D versus V_{DS}, with V_{GS} = 10 volts.

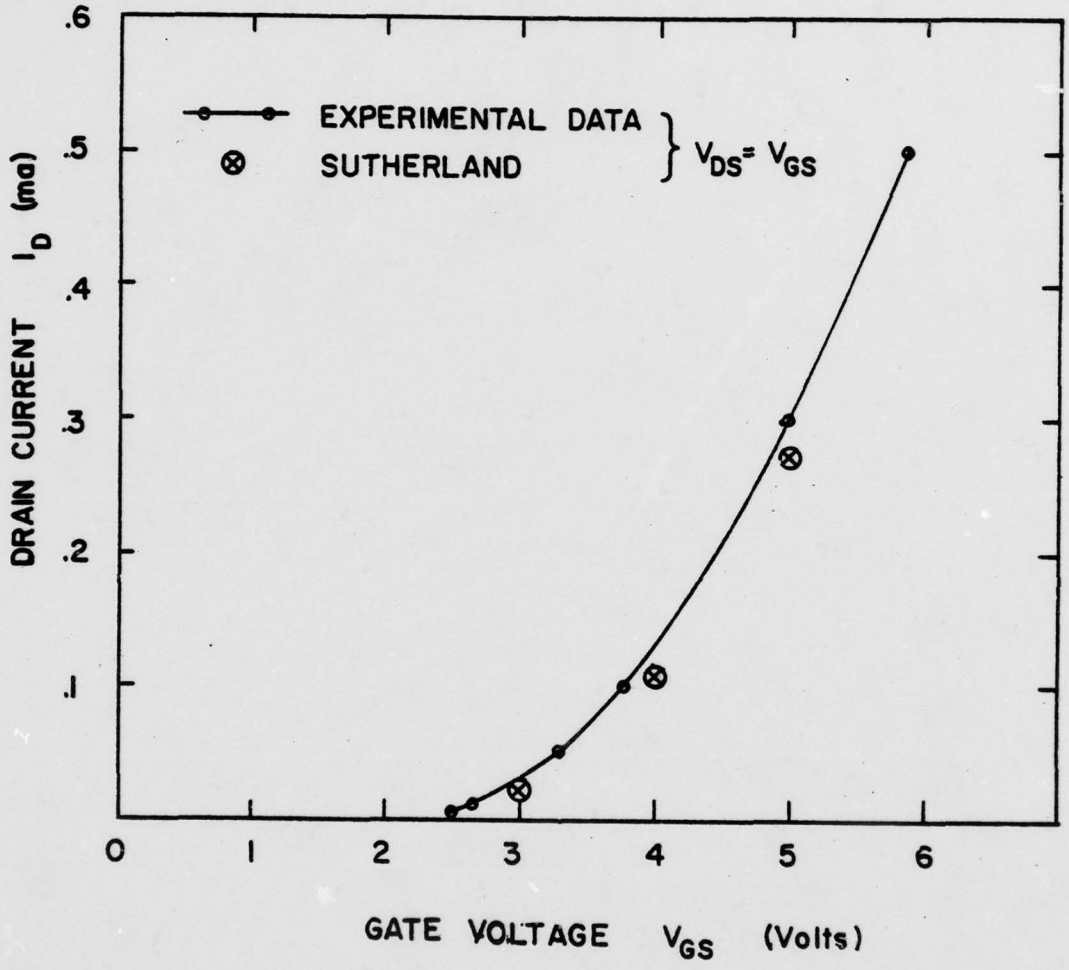


Figure 36 Comparison of the theory with experimental data obtained from a p-channel MOSFET device whose parameters were characterized by means of test sites on its chip. The curve shows I_D versus V_{GS}, with $V_{DS} = V_{GS}$.

CHAPTER III

Instructions for Using the Computer Program

1.0 Input Data

Input data for running the computer model are specified in the form of three NAMELIST data sets: NAMELIST/PRMTRS/, NAMELIST/VOLTGE/, and NAMELIST/CONTRL/. A sample input data set is given in Table III.

TABLE III -- SAMPLE INPUT DATA SET

```
&PRMTRS XCHNL=6.42,  YOXIDE=0.100, YJNCTN=0.500,  DONOR=9.07D19,
 ACCPTR=8.45D14,  QSS=-4.29D-8,  EMOB=182.0,  XDELTA=0.90,  YDELTA=0.85
 IMAX=41,  JMAX=25,  &END
&VOLTGE  VGDLT=1.00,  VGATE0=3.0,  VDDLT=1.00,  VDRN0=5.00, VSUB=-0.00,
NGATE=01,  NDRAIN=01,  &END
&CONTRL  CNVRG1=3.00,  CNVRG2=0.02,  SIDLT1=.01,  SIDLT2=.001,
THDLT1=.01,  THDLT2=.001,  ITMAX=200,  ITSIMX=200,  ITTHMX=200,
  KRSWCH=0,  OMEGA=1.6   &END
     EXPERIMENTAL CHECK OF MOSFET MODEL
       A.D.SUTHERLAND
            6/29/77
```

Note that the last 3 cards of the data set provide for a 3-line title by means of which the user may identify the purpose of that particular run. These 3 cards must be present in the data deck, even if left blank. Immediately following the input of these data, a listing of them is printed to provide a record for future reference.

1.01 NAMELIST/PRMTRS/

The data included in this set are:

XCHNL -- Channel length (μm)

YOXIDE -- Oxide thickness (μm)

YJNCTN -- Source and drain junction depth (μm)

DONOR -- Doping density of the source and drain (cm^{-3})

ACCPTR -- Doping density (uniform) of the p-type substrate (cm^{-3})

Q_{SS} -- Surface charge density (coul/cm^2)
EMOB -- Electron mobility (cm^2/volt-sec)
XDELTA -- Graded lattice parameter for the x coordinate
YDELTA -- Graded lattice parameter for the y cooordinate
IMAX -- # of lattice points in the x direction. This must be an odd integer $\leq$ 41
JMAX -- # of lattice points in the y direction. This can be any integer $\leq$ 25.

Regarding QSS, it is possible to account for work function differences between the gate metallization and the silicon substrate by defining an "effective" Q_{SS} given by $Q_{SS_{eff}} = C_{ox}V_{FB}$, where C_{ox} is the oxide layer capacitance, $C_{ox} \equiv \kappa_{ox}\varepsilon_o/y_{ox}$ (F/cm^2), and where V_{FB} is the flatband voltage of MOS capacitor theory.

The graded lattice parameters XDELTA and YDELTA determine the spacing between lattice points in the following manner. Let IMID by the I-index of the plane midway between the source and the drain. Let XDLT (IMID) be the spacing between that plane and the planes IMID±1 immediately to the right and left of the mid-plane. Then the algorithm for determining XDLT(I), given XDLT(IMID), is

```
XDLT(I-1) = XDELTA*XDLT(I),     I=3,IMID
XDLT(I+1) = XDELTA*XDLT(I),     I=IMID+1,IMAX-1
```

where XDELTA $\leq$ 1. The graded lattice illustrated in Figure 2, in Chapter I, was generated with XDELTA = Ø.9, the value suggested by Mock [1]. In a like manner, let YDLT(2) be the vertical lattice spacing at the substrate "contact." Then the algorithm employed for determining YDLT(J), given YDLT(2), is,

```
YDLT(J+1) = YDELTA*YDLT(J),     J=3,JMAX-1
```

where YDELTA $\leq$ 1. The graded lattice of Figure 2 was generated with YDELTA = 0.85, again the value suggested by Mock.

Note that the substrate thickness YSUB is not specified by the user. It is automatically adjusted to provide a spacing between the substrate contact and the lower edge of the source and drain "contacts" which is 50% greater than the drain space-charge layer width $W_{D_{drain}}$, calculated from abrupt junction depletion theory, and is readjusted each time the drain voltage changes. This is done to maximize the effectiveness of achieving small spacings between vertically adjacent lattice points in the vicinity of the oxide-silicon interface, while assuring charge neutrality at the substrate "contact."

1.02 NAMELIST/VOLTGE/

The data specified in this set are:

VGDLT -- Gate voltage increment (volts)
VGATEØ -- Starting gate voltage (volts)
VDDLT -- Drain voltage increment (volts)
VDRNØ -- Starting drain voltage (volts)
VSUB -- Applied substrate voltage (volts) with $V_{SUB} \leq 0$
NGATE -- # of gate voltage increments
NDRAIN -- # of drain voltage increments

The meaning of these parameters is as follows. The program has been written to function in the manner of a curve-tracer which plots families of curves of I_D versus V_{DS}, with V_{GS} varied as a parameter. The initial gate voltage is set equal to VGATEØ, and subsequent gate voltages are incremented upward in increments of VGDLT, until NGATE such voltages have been treated. The parameter NDRAIN determines the number of drain voltages at which the drain current I_D is to be determined. The lowest drain voltage used is VDRNØ. The drain voltage is then incremented upward in increments of VDDLT, until NDRAIN voltage values have been treated. The gate voltage is then incremented, and the process is repeated. During this sequence, the substrate voltage remains fixed. The user should always make an estimate of the threshold gate voltage, and specify VGATEØ $\geq V_T$.

1.03 NAMELIST/CONTRL/

The parameters in this data set provide the user a degree of control over the iteration process and, as described in Chapter I, allow him to specify whether the "Picard and Gummel" relaxation procedure devised by Mock, or the "Overrelaxed Gummel" procedure devised by the author is to be used. The parameters appearing in this data set are:

CNVRG1 -- "Coarse" and "Tight" convergence criteria for
CNVRG2 the Picard and Gummel algorithms.

SIDLT1 -- "Coarse" and "Tight" convergence criteria for
SIDLT2 Stone's method for finding ψ.

THDLT1 -- "Coarse" and "Tight" convergence criteria for
THDLT2 the Block-Line iteration for finding θ.

ITMAX -- Maximum number of iterations to be allowed with the Picard and Gummel algorithms.

ITSIMX -- Maximum number of iterations to be allowed with Stone's method.

ITTHMX -- Maximum number of iterations to be allowed with Block-Line iteration.

KRSWCH -- "Flag" to specify which iteration scheme is to be used. KRSWCH=Ø causes only Gummel's algorithm to be applied, with full donor density and "tight" convergence criteria. KRSWCH=1 causes Mock's approach to be implemented, viz. Picard iteration at reduced donor density until "coarse" convergence is achieved, followed by Gummel's algorithm at full donor density, with "tight" convergence criteria.

OMEGA -- Overrelaxation parameter used with the Gummel algorithm, as described in Section 4.03.

Table IV lists the values of the iteration control parameters which have been found to work well for gate and drain voltages $\leq$ 10 volts.*

TABLE IV -- RECOMMENDED CONTROL PARAMETERS

CNVRG1	= 3.0
CNVRG2	= .02
SIDLT1	= .01
SIDLT2	= .001
THDLT1	= .01
THDLT2	= .001
ITMAX	= 200
ITSIMX	= 100
ITTHMX	= 100
OMEGA	= 1.6

Concerning the three "maximum iterations" parameters, the program will stop, with a "MAXIMUM ITERATIONS EXCEEDED" message being printed, identifying why it stopped, in the event that this condition is encountered. These three parameters were installed by the author as "safety valves" to avoid excessive computation times when things are going awry. The author encountered program stops due to all three "safety valves" during the course of evaluating and improving the computer program and, with the exception of the "ITMAX EXCEEDED" criterion when using Mock's Picard + Gummel iterative procedure, was able to devise corrective measures to eliminate their occurrence. But the user may expect to encounter the "ITMAX EXCEEDED" exit message, especially under

*The range of gate and drain voltages explored with numerous computer runs was $V_T \leq V_G < 10V$ (V_T is the threshold gate voltage for inversion), and $1 \leq V_D \leq 9V$. The recommended value for ITMAX worked well over this range when the overrelaxed Gummel procedure was in use. However, the Picard + Gummel procedures frequently required excessive iterations, especially under conditions of strong inversion.

conditions of strong inversion, if he elects to use that procedure by setting KRSWCH=1. In the author's experience, both Stone's iterative procedure (limited by ITSIMX) and the block-line iterative procedure for the stream function (limited by ITTHMX), require only a small fraction of the "recommended" limits stated in Table IV.

2.0 Output Data

Table V (17 sheets) illustrates the output data printed by the program. A complete set of these data are printed for each pair of voltages V_G,V_D, and the user may not desire all of this data. In that event, he may wish to modify SUBROUTINE OUTPUT to suit his particular needs. Some plotting routines would also be helpful, but the author did not have sufficient time available to develop same.

All output data are listed in physical units, the conversion from normalized to unnormalized variables being implemented in SUBROUTINE OUTPUT. What these data represent will next be described.

Sheet 1 of Table V summarizes the three voltages V_G, V_D, and V_B, then gives the corresponding drain current I_D in milliamperes/cm of channel width. Next, is printed the substrate thickness which is automatically determined in the manner described above. Following this are listed in columns, the following results,

X -- The x coordinate (μm).

PSI -- The electrostatic potential at the silicon surface (volts).

PHIN -- The electron quasi-Fermi potential ϕ_n' along the row J=JMAX, just below the silicon surface.

QI -- The total inversion charge Q_i (coulombs/cm^2) at position x.

QD -- The total depletion charge Q_D (coulombs/cm^2) at position x.

YI -- The inversion layer thickness (μm) at position x.

YD -- The depletion layer thickness (μm) at position x.

Note that YI and YD are quantized because they can only be approximated, due to the quantized nature of the lattice of mesh points at which they can be determined.

The remaining data in Table V provide information about selected variables in the bulk silicon substrate. They are,

PSI(I,J) -- Electrostatic potential ψ' (volts) at the ψ-lattice points corresponding to the lattice indices (I,J). (Sheets 2 and 3)

EDENS(I,J) -- The electron density n' (cm^{-3}) at the ψ-lattice points corresponding to the lattice indices (I,J). (Sheets 4 and 5)

HDENS(I,J) -- The hole density p' (cm^{-3}) at the ψ-lattice points corresponding to the lattice indices (I,J). (Sheets 6 and 7)

THETA(I,J) -- The stream function θ (dimensionless) at the θ-lattice points corresponding to the lattice indices (I,J). (See Figure 4.) (Sheets 8 and 9)

ESUBX(I,J) -- The x-directed component of the electric field E'_x, (volts/cm) at the θ-lattice coordinates corresponding to the lattice index I, and the ψ-lattice coordinates corresponding to the lattice index J. (Sheets 10 and 11)

ESUBY(I,J) -- The y-directed component of the electric field, E'_y, (volts/cm) at the ψ lattice coordinates corresponding to the lattice index I, and the θ-lattice coordinates corresponding to the lattice index J. (Sheets 12 and 13)

JSUBX(I,J) -- The x-directed component of the electron current density, J'_x, ($amps/cm^2$) at the θ-lattice coordinates corresponding to the lattice index I, and the ψ-lattice coordinates corresponding to the lattice index J. (Sheets 14 and 15)

JSUBY(I,J) -- The y-directed component of the electron current density, J'_y, (amps/cm^2) at the ψ-lattice coordinates corresponding to the lattice index I, and the θ-lattice coordinates corresponding to the lattice index J. (Sheets 16 and 17)

In the case of Sheets 2 through 17 of Table IV, the data are formated in an identical manner. Across the top of the even numbered sheets is listed the x'-coordinate corresponding to the lattice index I. (ψ-lattice, or θ-lattice, as the case may be.) The first two columns list the lattice index J, and the corresponding y'-coordinate. (Again, ψ-lattice or θ-lattice, as the case may be.) Both coordinates are expressed in micrometers. The remaining columns give the values of the variable indicated in the legend printed just below the listing of the x'-coordinate values. One scans each horizontal group of data (correspoding to a given lattice index J) in the manner of reading a printed page. Scanning first the listing of X(I) at the top of the sheet, one finds the corresponding value of the variable of interest in the corresponding location in the listing for the value of Y(J) (Column 2). In this manner, the two-dimensional solution for these variables is obtained.

3.0 Glossary of the Variables Appearing in COMMON Statements

Five COMMON blocks are defined in the computer program: ARRAYS, COEFF, REAL, INTGR, NRMLZE. The FORTRAN variables contained therein are next defined.

3.01 COMMON/ARRAYS/

PSI(41,3Ø) -- The electrostatic potential ψ.
EDENS(41,25) -- The electron density n.
HDENS(41,25) -- The hole density p.
THETA(4Ø,25) -- The stream function θ.

B(41,3Ø)
D(41,3Ø)
E(41,3Ø)
F(41,3Ø)
H(41,3Ø)
Q(41,3Ø)
-- Stone's coefficients B,D,E,F,H, and Q. (Also used for the Block-line iteration for θ, in the same context.)

CURRENT VOLTAGE RESULT :

VG= 3.000+00 VOLTS
VD= 8.000+00 VOLTS
VB= 0.0 VOLTS
ID= -6.170-01 MILLIAMPS/CM OF CHANNEL WIDTH

SUBSTRATE THICKNESS (AUTOMATICALLY DETERMINED) IS YSUB= 1.636D+00 MICROMETERS

SUMMARY OF CONDITIONS ALONG THE SILICON-OXIDE INTERFACE :

X	PSI	PHIN	QI	QD	VI	YD
0.0	-2.5850-02	-5.2520-01	0.0	0.0	0.0	0.0
2.3070-02	-5.4020-02	-5.2530-01	2.1300-07	1.9540-07	.0520-01	7.1070-01
4.8700-02	-6.4160-02	-5.2540-01	6.4390-08	1.9290-07	.0520-01	7.1070-01
7.7180-02	-6.9580-02	-5.2540-01	4.8690-08	1.8840-07	.2460-01	7.1070-01
1.0880-01	-7.4580-02	-5.2490-01	3.7110-08	1.8060-07	.0750-01	7.1070-01
1.4400-01	-7.9440-02	-5.2460-01	2.8400-08	1.6810-07	.4160-01	7.1070-01
1.8300-01	-8.5650-02	-5.2410-01	2.1600-08	1.4580-07	.1590-01	5.9960-01
2.2640-01	-9.1330-02	-5.2340-01	1.6460-08	1.1470-07	.1880-02	5.9960-01
2.7470-01	-9.6420-02	-5.2240-01	1.2790-08	9.9860-08	.3390-02	5.0520-01
3.2830-01	-1.0030-01	-5.2100-01	1.0330-08	8.7750-08	.9580-02	3.5670-01
3.8740-01	-1.0260-01	-5.1920-01	8.7110-09	7.4090-08	.5390-02	3.5670-01
4.5240-01	-1.0520-01	-5.1680-01	7.8030-09	9.0980-08	.5390-02	3.5670-01
5.2370-01	-1.0250-01	-5.1410-01	7.1840-09	9.0230-08	.5390-02	3.5670-01
6.0910-01	-1.0070-01	-5.1070-01	6.1420-09	7.1550-08	.5390-02	3.9870-01
6.9990-01	-9.8170-02	-5.0670-01	6.3670-09	7.1600-08	.5390-02	3.9870-01
8.0070-01	-9.5020-02	-5.0200-01	5.9930-09	7.1720-08	.5390-02	3.9870-01
9.1280-01	-9.1270-02	-4.9640-01	5.5640-09	9.0470-08	.5390-02	3.5670-01
1.0370+00	-8.6430-02	-4.8960-01	5.1130-09	9.0670-08	.5390-02	3.5670-01
1.1760+00	-8.1550-02	-4.8140-01	8.5620-09	9.0920-08	.5390-02	3.5670-01
1.3290+00	-7.5140-02	-4.7090-01	3.9060-09	9.1220-08	.5390-02	3.5670-01
1.5000+00	-6.7100-02	-4.5700-01	3.1090-09	7.3640-08	.5390-02	3.5670-01
1.6710+00	-5.7240-02	-4.3860-01	2.2380-09	7.4180-08	.5390-02	3.5670-01
1.8240+00	-4.4810-02	-4.1470-01	1.4400-09	7.3030-08	.5390-02	3.5670-01
1.9630+00	-2.4920-02	-3.7990-01	6.1570-10	7.6790-08	.9580-02	3.5670-01
2.0870+00	1.2800-02	-3.2490-01	8.2260-10	8.1110-08	.9580-02	3.5670-01
2.1990+00	8.4890-02	-2.3550-01	2.2270-10	9.3040-08	.9580-02	2.2490-01
2.3000+00	2.0960-01	-0.3300-02	1.2520-10	1.2240-07	.9580-02	7.9960-01
2.3910+00	3.9930-01	1.0850-01	8.2520-11	1.6770-07	.9580-02	7.1070-01
2.4730+00	6.5830-01	2.8110-01	5.9540-11	2.0850-07	.9580-02	8.4150-01
2.5460+00	9.8490-01	5.2140-01	4.0770-11	2.3630-07	.5390-02	8.4150-01
2.6120+00	1.3740+00	8.1270+00	3.0620-11	2.5960-07	.3980-02	6.9520-01
2.6720+00	1.8200+00	1.1590+00	2.3830-11	2.7270-07	.3980-02	9.9520-01
2.7250+00	2.3150+00	1.5070+00	1.9130-11	2.8160-07	.1590-01	9.9520-01
2.7780+00	2.8520+00	1.8690+00	1.5170-11	2.8930-07	.4160-01	9.9520-01
2.8170+00	3.4260+00	2.2570+00	1.3240-11	2.9740-07	.4160-01	1.1760+00
2.8560+00	4.0320+00	2.6070+00	1.1270-11	3.0270-07	.7190-01	1.1760+00
2.8910+00	4.6700+00	3.0170+00	9.6930-12	3.0630-07	.7190-01	1.1760+00
2.9230+00	5.3410+00	3.4610+00	8.4410-12	3.0860-07	.7190-01	1.1760+00
2.9510+00	6.0540+00	3.9330+00	7.4910-12	3.0990-07	.7190-01	1.1760+00
2.9770+00	6.8600+00	4.4720+00	6.8600-12	3.1060-07	.7190-01	1.1760+00
3.0000+00	7.9740+00	7.4750+00	0.0	0.0	0.0	0.0

Table V sheet 1

```
X(I),I=1,IMAX

                 0.0       2.31D-02  4.87D-02  7.72D-02  1.09D-01  1.44D-01  1.83D-01  2.26D-01  2.75D-01  3.28D-01
                 3.88D-01  4.54D-01  5.27D-01  6.09D-01  7.00D-01  8.01D-01  9.13D-01  1.04D+00  1.18D+00  1.33D+00
                 1.50D+00  1.67D+00  1.82D+00  1.96D+00  2.09D+00  2.20D+00  2.30D+00  2.39D+00  2.47D+00  2.55D+00
                 2.61D+00  2.67D+00  2.73D+00  2.77D+00  2.82D+00  2.86D+00  2.89D+00  2.92D+00  2.95D+00  2.98D+00
                 3.00D+00
 J      Y(J) .              PSI(I,J),I=1,IMAX

30  -1.000D-01   2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00
                 2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00
                 2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00
                 2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00  2.11D+00
                 2.11D+00
29  -7.530D-02   1.58D+00  1.58D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.56D+00
                 1.56D+00  1.56D+00  1.56D+00  1.56D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00  1.57D+00
                 1.57D+00  1.58D+00  1.58D+00  1.58D+00  1.59D+00  1.61D+00  1.64D+00  1.69D+00  1.76D+00  1.84D+00
                 1.94D+00  2.05D+00  2.18D+00  2.31D+00  2.46D+00  2.61D+00  2.77D+00  2.92D+00  3.06D+00  3.15D+00
                 3.19D+00
28  -5.060D-02   1.04D+00  1.04D+00  1.04D+00  1.03D+00  1.03D+00  1.03D+00  1.03D+00  1.02D+00  1.02D+00  1.02D+00
                 1.02D+00  1.02D+00  1.02D+00  1.02D+00  1.02D+00  1.02D+00  1.02D+00  1.03D+00  1.03D+00  1.03D+00
                 1.04D+00  1.04D+00  1.05D+00  1.06D+00  1.08D+00  1.11D+00  1.18D+00  1.27D+00  1.40D+00  1.57D+00
                 1.76D+00  1.99D+00  2.24D+00  2.51D+00  2.79D+00  3.10D+00  3.41D+00  3.73D+00  4.02D+00  4.25D+00
                 4.35D+00
27  -2.590D-02   5.14D-01  5.08D-01  5.02D-01  4.97D-01  4.93D-01  4.89D-01  4.84D-01  4.80D-01  4.77D-01  4.74D-01
                 4.72D-01  4.71D-01  4.72D-01  4.73D-01  4.75D-01  4.77D-01  4.80D-01  4.83D-01  4.87D-01  4.92D-01
                 4.98D-01  5.05D-01  5.15D-01  5.30D-01  5.59D-01  6.14D-01  7.08D-01  8.50D-01  1.04D+00  1.29D+00
                 1.58D+00  1.91D+00  2.28D+00  2.69D+00  3.12D+00  3.57D+00  4.05D+00  4.53D+00  5.02D+00  5.46D+00
                 5.75D+00
26  -1.270D-03  -1.05D-03 -2.61D-02 -3.64D-02 -4.19D-02 -4.69D-02 -5.22D-02 -5.78D-02 -6.34D-02 -6.84D-02 -7.23D-02
                -7.45D-02 -7.51D-02 -7.44D-02 -7.27D-02 -7.02D-02 -6.71D-02 -6.34D-02 -5.90D-02 -5.38D-02 -4.75D-02
                -3.95D-02 -2.98D-02 -1.75D-02  2.18D-03  3.95D-02  1.11D-01  2.34D-01  4.21D-01  6.77D-01  1.00D+00
                 1.38D+00  1.83D+00  2.31D+00  2.84D+00  3.41D+00  4.01D+00  4.64D+00  5.30D+00  6.01D+00  6.80D+00
                 7.80D+00
25   3.800D-03  -2.58D-02 -8.19D-02 -9.19D-02 -9.72D-02 -1.02D-01 -1.08D-01 -1.13D-01 -1.19D-01 -1.24D-01 -1.28D-01
                -1.31D-01 -1.31D-01 -1.30D-01 -1.29D-01 -1.26D-01 -1.23D-01 -1.19D-01 -1.15D-01 -1.09D-01 -1.03D-01
                -9.47D-02 -8.48D-02 -7.21D-02 -5.20D-02 -1.39D-02  5.90D-02  1.85D-01  3.77D-01  6.39D-01  9.70D-01
                 1.36D+00  1.81D+00  2.32D+00  2.86D+00  3.44D+00  4.05D+00  4.70D+00  5.38D+00  6.10D+00  6.92D+00
                 7.97D+00
24   9.760D-03  -2.58D-02 -1.01D-01 -1.17D-01 -1.25D-01 -1.33D-01 -1.41D-01 -1.49D-01 -1.56D-01 -1.63D-01 -1.67D-01
                -1.70D-01 -1.71D-01 -1.71D-01 -1.69D-01 -1.66D-01 -1.63D-01 -1.60D-01 -1.55D-01 -1.50D-01 -1.44D-01
                -1.36D-01 -1.26D-01 -1.13D-01 -9.30D-02 -5.45D-02  1.95D-02  1.48D-01  3.43D-01  6.10D-01  9.46D-01
                 1.35D+00  1.81D+00  2.32D+00  2.87D+00  3.46D+00  4.09D+00  4.74D+00  5.43D+00  6.17D+00  6.99D+00
                 7.97D+00
23   1.680D-02  -2.58D-02 -1.14D-01 -1.37D-01 -1.51D-01 -1.63D-01 -1.75D-01 -1.86D-01 -1.96D-01 -2.05D-01 -2.11D-01
                -2.15D-01 -2.16D-01 -2.16D-01 -2.14D-01 -2.12D-01 -2.09D-01 -2.05D-01 -2.01D-01 -1.96D-01 -1.90D-01
                -1.82D-01 -1.72D-01 -1.60D-01 -1.40D-01 -1.01D-01 -2.59D-02  1.03D-01  3.04D-01  5.76D-01  9.19D-01
                 1.33D+00  1.80D+00  2.32D+00  2.88D+00  3.49D+00  4.12D+00  4.79D+00  5.49D+00  6.24D+00  7.06D+00
                 7.97D+00
22   2.500D-02  -2.58D-02 -1.23D-01 -1.55D-01 -1.77D-01 -1.95D-01 -2.12D-01 -2.28D-01 -2.42D-01 -2.52D-01 -2.60D-01
                -2.65D-01 -2.67D-01 -2.67D-01 -2.66D-01 -2.63D-01 -2.60D-01 -2.57D-01 -2.53D-01 -2.48D-01 -2.42D-01
                -2.35D-01 -2.25D-01 -2.13D-01 -1.93D-01 -1.54D-01 -7.76D-02  5.53D-02  2.59D-01  5.37D-01  8.88D-01
                 1.31D+00  1.79D+00  2.32D+00  2.90D+00  3.51D+00  4.16D+00  4.84D+00  5.55D+00  6.31D+00  7.12D+00
                 7.97D+00
21   3.470D-02  -2.58D-02 -1.29D-01 -1.72D-01 -2.03D-01 -2.30D-01 -2.53D-01 -2.74D-01 -2.92D-01 -3.06D-01 -3.15D-01
                -3.21D-01 -3.24D-01 -3.24D-01 -3.23D-01 -3.21D-01 -3.18D-01 -3.15D-01 -3.11D-01 -3.07D-01 -3.01D-01
                -2.94D-01 -2.85D-01 -2.73D-01 -2.53D-01 -2.14D-01 -1.36D-01 -7.92D-04  2.07D-01  4.92D-01  8.52D-01
                 1.28D+00  1.77D+00  2.32D+00  2.91D+00  3.54D+00  4.20D+00  4.90D+00  5.62D+00  6.38D+00  7.17D+00
                 7.97D+00
20   4.610D-02  -2.58D-02 -1.35D-01 -1.89D-01 -2.31D-01 -2.67D-01 -2.98D-01 -3.26D-01 -3.48D-01 -3.65D-01 -3.77D-01
                -3.84D-01 -3.87D-01 -3.88D-01 -3.87D-01 -3.85D-01 -3.83D-01 -3.80D-01 -3.76D-01 -3.72D-01 -3.66D-01
                -3.60D-01 -3.51D-01 -3.40D-01 -3.20D-01 -2.81D-01 -2.02D-01 -6.43D-02  1.49D-01  4.41D-01  8.11D-01
                 1.25D+00  1.76D+00  2.32D+00  2.92D+00  3.57D+00  4.25D+00  4.95D+00  5.69D+00  6.44D+00  7.21D+00
                 7.97D+00
19   5.960D-02  -2.58D-02 -1.40D-01 -2.06D-01 -2.60D-01 -3.07D-01 -3.47D-01 -3.81D-01 -4.09D-01 -4.30D-01 -4.44D-01
                -4.53D-01 -4.57D-01 -4.58D-01 -4.57D-01 -4.56D-01 -4.54D-01 -4.51D-01 -4.47D-01 -4.43D-01 -4.38D-01
                -4.32D-01 -4.24D-01 -4.13D-01 -3.94D-01 -3.55D-01 -2.76D-01 -1.36D-01  8.25D-02  3.83D-01  7.64D-01
                 1.22D+00  1.74D+00  2.31D+00  2.94D+00  3.60D+00  4.29D+00  5.01D+00  5.75D+00  6.50D+00  7.25D+00
                 7.97D+00
18   7.540D-02  -2.58D-02 -1.45D-01 -2.23D-01 -2.90D-01 -3.49D-01 -3.99D-01 -4.41D-01 -4.75D-01 -5.00D-01 -5.17D-01
                -5.27D-01 -5.32D-01 -5.33D-01 -5.33D-01 -5.32D-01 -5.30D-01 -5.27D-01 -5.24D-01 -5.20D-01 -5.16D-01
                -5.10D-01 -5.03D-01 -4.93D-01 -4.74D-01 -4.36D-01 -3.57D-01 -2.15D-01  8.42D-03  3.17D-01  7.09D-01
                 1.18D+00  1.71D+00  2.31D+00  2.95D+00  3.63D+00  4.34D+00  5.07D+00  5.81D+00  6.55D+00  7.28D+00
                 7.97D+00
17   9.400D-02  -2.58D-02 -1.50D-01 -2.41D-01 -3.21D-01 -3.92D-01 -4.53D-01 -5.04D-01 -5.45D-01 -5.74D-01 -5.93D-01
                -6.04D-01 -6.10D-01 -6.12D-01 -6.12D-01 -6.11D-01 -6.09D-01 -6.07D-01 -6.04D-01 -6.01D-01 -5.97D-01
                -5.92D-01 -5.85D-01 -5.76D-01 -5.59D-01 -5.22D-01 -4.44D-01 -3.01D-01 -7.37D-02  2.44D-01  6.47D-01
                 1.13D+00  1.68D+00  2.30D+00  2.96D+00  3.65D+00  4.38D+00  5.12D+00  5.86D+00  6.59D+00  7.30D+00
                 7.97D+00
16   1.160D-01  -2.58D-02 -1.56D-01 -2.59D-01 -3.52D-01 -4.36D-01 -5.08D-01 -5.69D-01 -6.16D-01 -6.49D-01 -6.71D-01
                -6.83D-01 -6.89D-01 -6.91D-01 -6.91D-01 -6.90D-01 -6.89D-01 -6.87D-01 -6.85D-01 -6.82D-01 -6.79D-01
```

Table V sheet 2

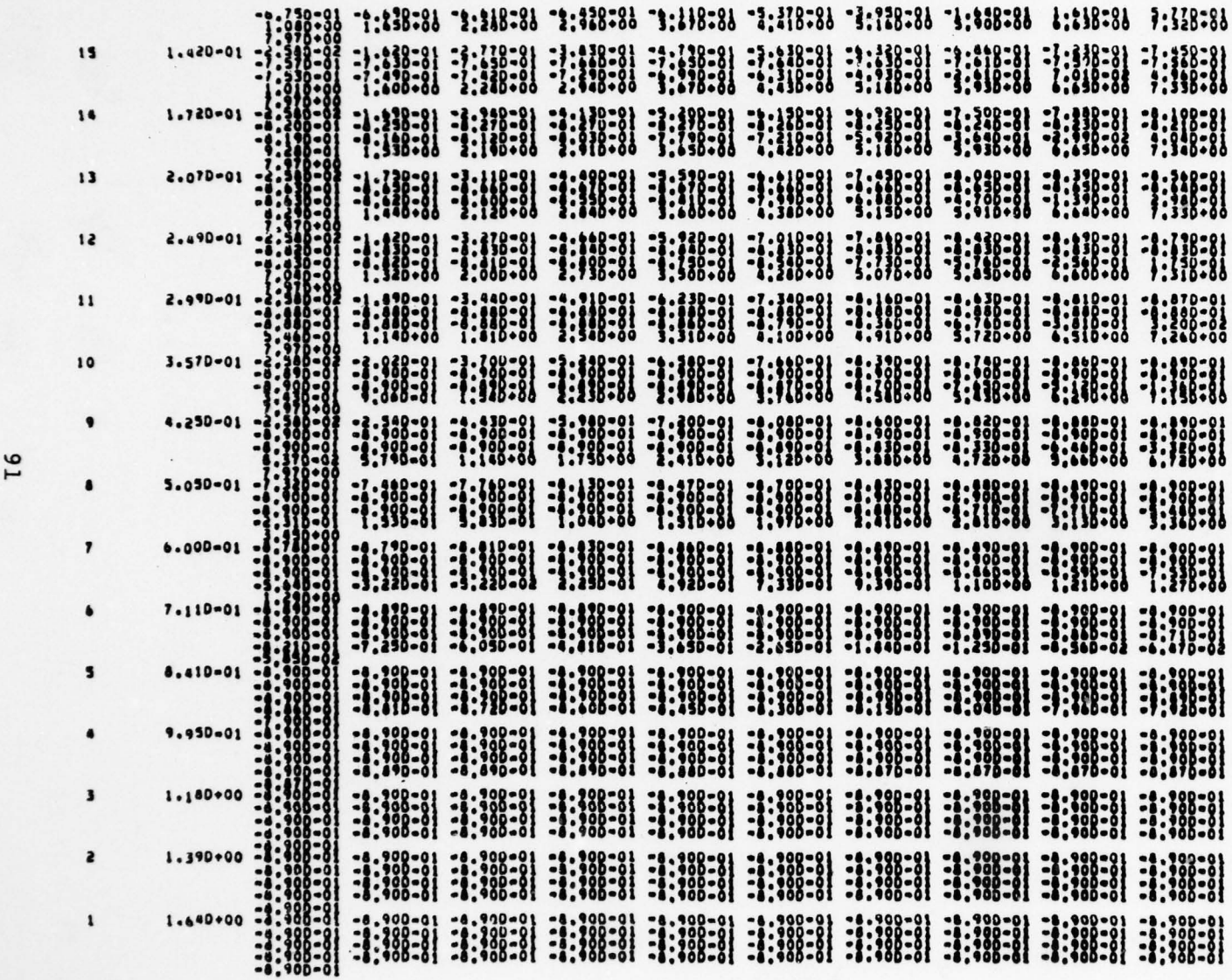


Table V sheet 3

X(I),I=1,IMAX

0.0	2.310-02	4.870-02	7.720-02	1.090-01	1.440-01	1.830-01	2.260-01	2.730-01	3.280-01
3.880-01	4.540-01	5.270-01	6.090-01	7.000-01	8.010-01	9.130-01	1.040+00	1.180+00	1.330+00
1.500+00	1.670+00	1.820+00	1.960+00	2.090+00	2.200+00	2.300+00	2.390+00	2.470+00	2.550+00
2.610+00	2.670+00	2.730+00	2.770+00	2.820+00	2.860+00	2.890+00	2.920+00	2.950+00	2.980+00
3.000+00									

J	Y(J)	EDENS(I,J),I=1,IMAX									
25	3.800-03	3.680+18	4.230+17	2.870+17	2.320+17	1.890+17	1.510+17	1.190+17	9.240+16	7.280+16	5.980+16
		5.070+16	4.520+16	4.170+16	3.920+16	3.710+16	3.500+16	3.260+16	2.990+16	2.670+16	2.290+16
		1.830+16	1.320+16	8.540+15	4.850+15	2.520+15	1.330+15	7.710+14	4.680+14	3.260+14	[illegible]
		1.450+14	8.890+13	4.910+13	2.410+13	1.070+13	4.350+12	1.640+12	6.380+11	2.410+11	9.580+10
		3.680+18									
24	9.760-03	3.680+18	1.980+17	1.100+17	7.930+16	5.840+16	4.250+16	3.070+16	2.230+16	1.670+16	1.310+16
		1.100+16	9.650+15	8.840+15	8.270+15	7.800+15	7.330+15	6.820+15	6.240+15	5.560+15	4.750+15
		3.170+15	2.710+15	1.740+15	9.920+14	5.240+14	2.680+14	1.810+14	1.300+14	1.030+14	8.690+13
		7.360+13	5.910+13	4.270+13	2.710+13	1.520+13	7.690+12	3.640+12	1.670+12	7.680+11	3.920+11
		3.680+18									
23	1.680-02	3.680+18	1.210+17	4.990+16	2.920+16	1.810+16	1.130+16	7.140+15	4.680+15	3.230+15	2.430+15
		[illegible]	1.700+15	1.540+15	1.430+15	1.340+15	1.260+15	1.180+15	1.090+15	[illegible]	[illegible]
		[illegible]	1.460+14	8.850+14	1.630+14	4.690+13	4.980+13	1.400+13	8.810+12	2.710+12	2.890+12
		1.190+13	3.380+13	3.180+13	2.570+13	1.790+13	1.110+13	6.350+12	3.440+12	1.940+12	1.210+12
		3.680+18									
22	2.500-02	3.680+18	8.630+16	2.460+16	1.080+16	5.230+15	2.650+15	1.480+15	8.170+14	5.190+14	3.620+14
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		1.150+13	1.620+13	1.440+13	2.050+13	1.770+13	1.330+13	9.160+12	6.000+12	3.960+12	2.900+12
		3.680+18									
21	3.470-02	3.680+18	6.710+16	1.270+16	3.860+15	1.370+15	5.400+14	2.360+14	1.140+14	6.380+13	4.280+13
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		3.370+12	6.390+12	1.030+13	1.350+13	1.450+13	1.320+13	1.090+13	8.390+12	6.450+12	5.330+12
		3.680+18									
20	4.610-02	3.680+18	5.450+16	6.670+15	1.320+15	3.250+14	9.460+13	3.230+13	1.330+13	6.630+12	3.980+12
		2.820+12	2.260+12	1.970+12	1.780+12	1.640+12	1.500+12	1.360+12	1.210+12	1.040+12	8.560+11
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		7.700+11	1.990+12	4.270+12	7.150+12	9.520+12	1.050+13	1.030+13	9.260+12	8.150+12	7.330+12
		3.680+18									
19	5.960-02	3.680+18	4.490+16	3.490+15	4.280+14	6.970+13	1.440+13	3.750+12	1.250+12	5.360+11	2.940+11
		1.970+11	1.540+11	1.310+11	1.180+11	1.070+11	9.760+10	8.770+10	7.710+10	[illegible]	[illegible]
		1.940+10	1.630+10	1.540+10	[illegible]	[illegible]	3.070+09	[illegible]	[illegible]	[illegible]	[illegible]
		1.320+11	8.720+11	1.360+12	2.930+12	4.850+12	6.500+12	7.450+12	7.730+12	7.580+12	7.290+12
		3.680+18									
18	7.540-02	3.680+18	3.700+16	1.800+15	1.340+14	1.380+13	1.940+12	3.700+11	9.760+10	3.570+10	1.770+10
		1.130+10	8.330+09	7.170+09	6.330+09	5.730+09	5.160+09	4.590+09	3.980+09	3.340+09	2.650+09
		1.850+09	1.250+09	7.360+08	3.930+08	2.070+08	1.330+08	1.330+08	2.340+08	[illegible]	[illegible]
		1.620+10	6.090+10	3.150+11	8.840+11	1.830+12	2.940+12	3.940+12	4.630+12	4.970+12	5.040+12
		3.680+18									
17	9.400-02	3.680+18	3.010+16	9.090+14	4.050+13	2.590+12	2.390+11	3.230+10	6.660+09	2.080+09	9.240+08
		5.580+08	4.120+08	3.420+08	2.990+08	2.670+08	2.380+08	2.090+08	1.790+08	1.470+08	1.140+08
		8.100+07	5.120+07	2.910+07	1.480+07	7.340+06	4.450+06	4.440+06	8.990+06	3.130+07	1.430+08
		1.350+09	9.410+09	5.000+10	1.830+11	4.740+11	9.180+11	1.420+12	1.860+12	2.160+12	2.300+12
		3.680+18									
16	1.160-01	3.680+18	2.400+16	4.520+14	1.210+13	4.750+11	2.840+10	2.710+09	4.260+08	1.120+08	4.610+07
		2.690+07	1.960+07	1.610+07	1.400+07	1.230+07	1.090+07	9.380+06	7.880+06	6.360+06	4.820+06
		3.300+06	2.010+06	1.090+06	5.210+05	2.330+05	1.240+05	1.140+05	2.240+05	6.610+05	[illegible]
		7.070+07	6.860+08	4.960+09	2.340+10	7.760+10	1.800+11	3.200+11	4.650+11	5.780+11	6.460+11
		3.680+18									
15	1.420-01	3.680+18	1.890+16	2.250+14	3.680+12	8.900+10	3.460+09	2.320+08	2.870+07	6.600+06	2.600+06
		1.510+06	1.110+06	9.090+05	7.860+05	6.870+05	5.960+05	5.060+05	4.170+05	3.280+05	2.420+05
		1.600+05	9.280+04	4.750+04	2.060+04	7.820+03	3.330+03	2.440+03	4.270+03	1.860+04	1.660+05
		2.150+06	2.800+07	2.720+04	1.710+09	6.990+09	1.950+10	3.990+10	6.420+10	8.640+10	1.030+11
		3.680+18									
14	1.720-01	3.680+18	1.470+16	1.140+14	1.170+12	1.800+10	4.670+08	2.310+07	2.380+06	5.230+05	2.130+05
		1.320+05	1.000+05	8.330+04	7.200+04	6.300+04	5.390+04	4.490+04	3.610+04	2.760+04	1.970+04
		1.280+04	6.870+03	3.270+03	1.260+03	3.870+02	1.190+02	5.850+01	6.910+01	2.490+02	2.230+03
		1.370+04	5.470+05	6.800+06	5.880+07	2.790+08	9.430+08	2.260+09	4.160+09	6.280+09	8.280+09
		3.680+18									
13	2.070-01	3.680+18	1.140+16	5.910+13	4.000+11	4.130+09	7.680+07	3.050+06	3.000+05	7.440+04	3.630+04
		2.570+04	2.100+04	1.810+04	1.610+04	1.380+04	1.170+04	9.580+03	7.550+03	5.640+03	3.900+03
		2.390+03	1.270+03	5.840+02	2.170+02	6.600+01	2.100+01	9.040+00	6.080+00	2.340+01	[illegible]
		2.360+02	4.070+03	5.760+04	5.520+05	3.430+06	1.430+07	4.250+07	9.660+07	1.790+08	2.860+08
		3.680+18									
12	2.490-01	3.680+18	8.840+15	3.180+13	1.500+11	1.120+09	1.650+07	6.110+05	7.020+04	2.380+04	1.510+04
		1.220+04	1.060+04	9.350+03	8.620+03	7.440+03	6.280+03	5.130+03	4.030+03	3.000+03	2.080+03
		1.280+03	7.080+02	3.560+02	1.620+02	6.750+01	2.520+01	1.010+01	6.470+00	5.140+00	4.560+00
		4.760+00	1.210+01	1.070+02	9.830+02	6.700+03	3.450+04	1.410+05	4.800+05	1.390+06	3.450+06
		3.680+18									
11	2.990-01	3.680+18	6.630+15	1.670+13	5.710+10	3.410+08	4.620+06	1.910+05	3.090+04	1.470+04	1.110+04
		9.680+03	8.650+03	7.640+03	7.350+03	6.340+03	5.420+03	4.470+03	3.550+03	2.690+03	1.910+03

Table V sheet 4

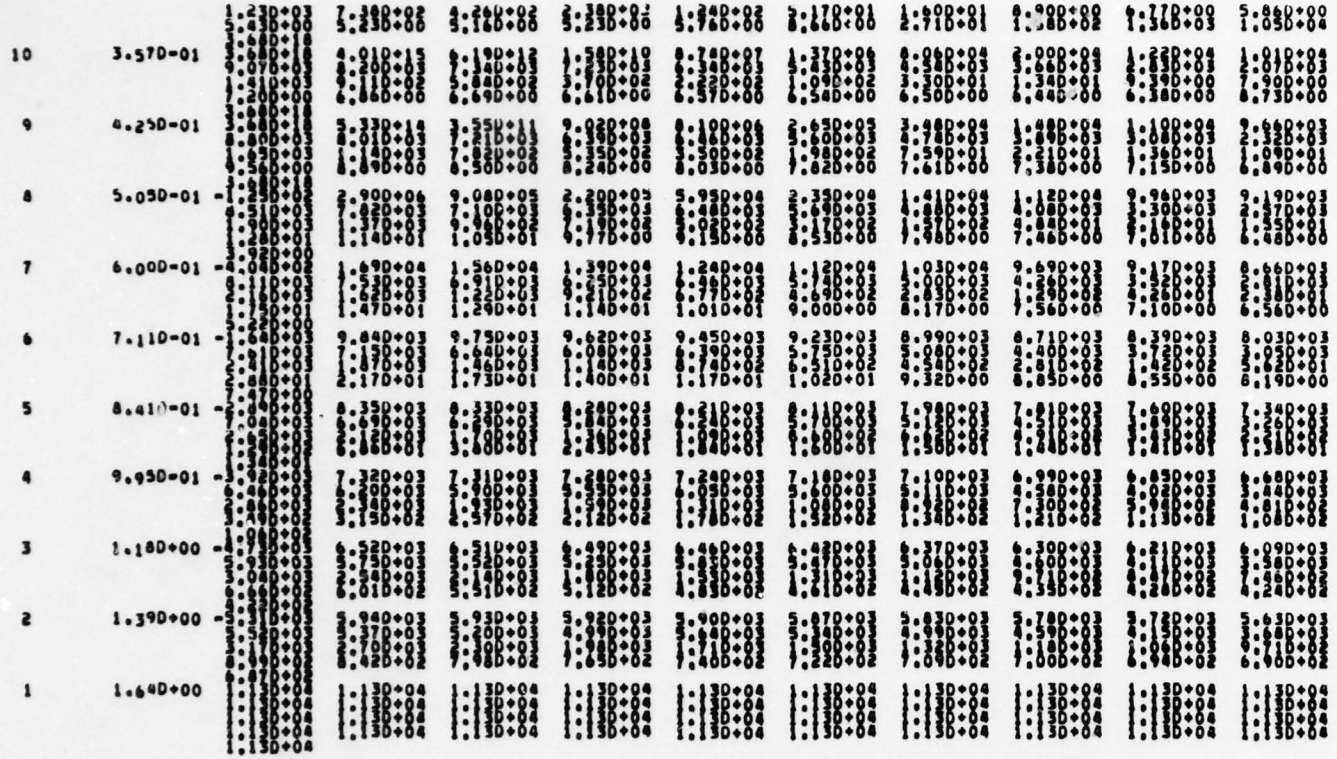


Table V sheet 5

X(I),I=1,IMAX

0.0	2.31D-02	4.87D-02	7.72D-02	1.09D-01	1.44D-01	1.83D-01	2.26D-01	2.75D-01	3.28D-01
3.68D-01	4.54D-01	5.27D-01	6.09D-01	6.00D-01	1.01D-01	1.13D-01	1.94D+00	1.10D+00	1.33D+00
1.56D+00	1.67D+00	1.82D+00	1.96D+00	2.09D+00	2.20D+00	2.30D+00	2.39D+00	2.47D+00	2.55D+00
2.61D+00	2.67D+00	2.73D+00	2.77D+00	2.82D+00	2.86D+00	2.89D+00	2.92D+00	2.95D+00	2.98D+00
3.00D+00									

J	Y(J)	MDENS(I,J),I=1,IMAX									
25	3.80D-03	6.12D+01	5.32D+02	7.81D+02	9.41D+04	1.17D+03	1.44D+03	1.81D+03	2.26D+03	2.77D+03	3.28D+03
		1.52D+03	1.61D+03	3.50D+03	3.27D+03	4.46D+03	5.96D+03	1.06D+03	1.54D+03	1.20D+03	1.00D+03
		1.76D+02	3.97D+02	3.67D+02	1.68D+02	6.86D+01	2.96D+00	1.35D-02	1.04D-03	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
24	9.76D-03	6.12D+01	1.13D+03	2.03D+03	2.81D+03	3.79D+03	5.14D+03	7.00D+03	9.39D+03	1.21D+04	1.46D+04
		1.63D+04	1.69D+04	1.65D+04	1.55D+04	1.41D+04	1.25D+04	1.08D+04	9.11D+03	1.44D+03	1.81D+03
		1.26D+03	1.92D+03	1.80D+03	1.22D+02	1.83D+02	1.06D+01	1.08D-02	1.88D-03	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
23	1.68D-02	6.12D+01	1.85D+03	4.49D+03	7.63D+04	1.22D+04	1.94D+04	3.01D+04	4.47D+04	6.20D+04	7.88D+04
		9.08D+04	9.49D+04	9.96D+04	8.96D+04	1.18D+04	1.28D+04	1.37D+04	1.41D+04	1.41D+04	1.37D+04
		1.27D+04	1.10D+04	8.03D+03	1.22D+03	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
22	2.50D-02	6.12D+01	2.41D+03	9.10D+03	2.07D+04	4.33D+04	8.24D+04	1.51D+05	2.34D+05	3.31D+05	5.24D+05
		6.84D+05	8.85D+05	9.51D+05	8.68D+04	6.35D+04	4.55D+04	3.01D+04	1.91D+04	1.01D+04	5.61D+03
		1.37D+05	1.50D+04	1.04D+04	1.64D+03	1.22D+03	1.01D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
21	3.47D-02	6.12D+01	3.35D+03	1.77D+04	5.78D+04	1.41D+05	3.05D+05	9.12D+05	1.81D+06	3.98D+06	9.47D+06
		2.20D+06	6.30D+06	8.04D+06	8.04D+06	6.72D+06	5.40D+06	4.13D+06	3.34D+06	2.45D+06	1.81D+06
		1.37D+06	8.70D+05	3.01D+05	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
20	4.61D-02	6.12D+01	9.13D+03	3.37D+04	1.70D+05	4.82D+05	2.31D+06	4.82D+06	1.57D+07	3.05D+07	4.81D+07
		7.21D+07	7.43D+07	1.20D+07	6.71D+07	6.06D+07	5.81D+07	4.26D+07	3.96D+07	3.24D+07	3.24D+07
		1.79D+07	1.15D+07	3.36D+06	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
19	5.96D-02	6.12D+01	5.01D+03	6.44D+04	5.22D+05	3.18D+06	1.32D+07	5.74D+07	1.48D+08	3.74D+08	6.32D+08
		1.06D+09	1.11D+09	1.09D+09	8.05D+08	5.05D+07	3.78D+08	2.27D+08	1.37D+08	8.28D+06	4.98D+06
		1.09D+08	1.47D+08	3.36D+07	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
18	7.54D-02	6.12D+01	6.08D+03	1.25D+05	1.68D+06	1.41D+07	1.13D+08	5.83D+08	2.13D+09	5.44D+09	1.08D+10
		1.91D+10	2.04D+10	2.24D+09	2.07D+10	1.92D+10	1.77D+10	1.61D+10	1.43D+10	1.23D+10	1.04D+10
		8.30D+09	6.24D+09	3.07D+09	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
17	9.40D-02	6.12D+01	7.48D+03	2.48D+05	5.55D+06	8.62D+07	9.22D+08	6.43D+09	3.16D+10	9.65D+10	1.98D+11
		3.96D+11	4.28D+11	4.28D+11	4.11D+11	3.84D+11	3.62D+11	3.26D+11	2.91D+11	2.61D+11	2.31D+11
		1.54D+11	1.08D+11	5.51D+10	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
16	1.16D-01	6.12D+01	9.37D+03	4.97D+05	1.86D+07	4.71D+08	7.40D+09	8.04D+10	6.96D+11	4.17D+12	1.93D+13
		6.35D+12	9.10D+12	1.19D+12	1.57D+12	1.61D+12	1.57D+12	1.43D+12	1.26D+12	1.08D+12	9.00D+11
		7.93D+12	5.87D+12	3.09D+12	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
15	1.42D-01	6.12D+01	1.19D+04	9.99D+05	6.12D+07	2.52D+09	6.45D+10	9.43D+11	7.42D+12	3.09D+13	7.41D+13
		1.48D+14	1.61D+14	1.63D+14	1.60D+14	1.54D+14	1.46D+14	1.36D+14	1.24D+14	1.11D+14	9.80D+13
		8.53D+13	6.64D+13	4.02D+13	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
14	1.72D-01	6.12D+01	1.53D+04	1.98D+06	1.93D+08	1.25D+10	4.79D+11	9.37D+12	9.03D+13	3.24D+14	9.03D+14
		1.63D+15	1.79D+15	1.77D+15	1.78D+15	1.73D+15	1.64D+15	1.53D+15	1.40D+15	1.25D+15	1.10D+15
		9.17D+14	6.63D+14	4.64D+14	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
13	2.07D-01	6.12D+01	1.97D+04	3.81D+06	5.42D+08	5.44D+10	2.92D+12	7.30D+13	7.64D+14	2.61D+15	5.34D+15
		7.81D+15	8.10D+15	8.18D+15	8.14D+15	8.04D+15	7.85D+15	7.60D+15	7.24D+15	6.78D+15	6.23D+15
		5.23D+15	4.16D+15	3.16D+15	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
12	2.49D-01	6.12D+01	2.54D+04	7.07D+06	1.50D+09	2.01D+11	1.34D+13	3.97D+14	3.14D+15	8.43D+15	1.30D+16
		1.54D+16	1.57D+16	1.57D+16	1.51D+16	1.44D+16	1.36D+16	1.27D+16	1.16D+16	1.04D+16	9.18D+15
		7.90D+15	6.45D+15	5.01D+15	1.22D+04	1.64D+03	1.13D+01	1.08D-01	1.77D-04	1.09D+01	1.09D+01
		1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01	1.09D+01
		1.09D+01									
11	2.94D-01	6.12D+01	3.38D+04	1.35D+07	3.94D+09	6.60D+11	4.87D+13	1.17D+15	7.14D+15	1.44D+16	1.77D+16
		1.86D+16	1.89D+16	1.90D+16	1.90D+16	1.90D+16	1.90D+16	1.90D+16	1.90D+16	1.90D+16	1.90D+16

Table V sheet 6

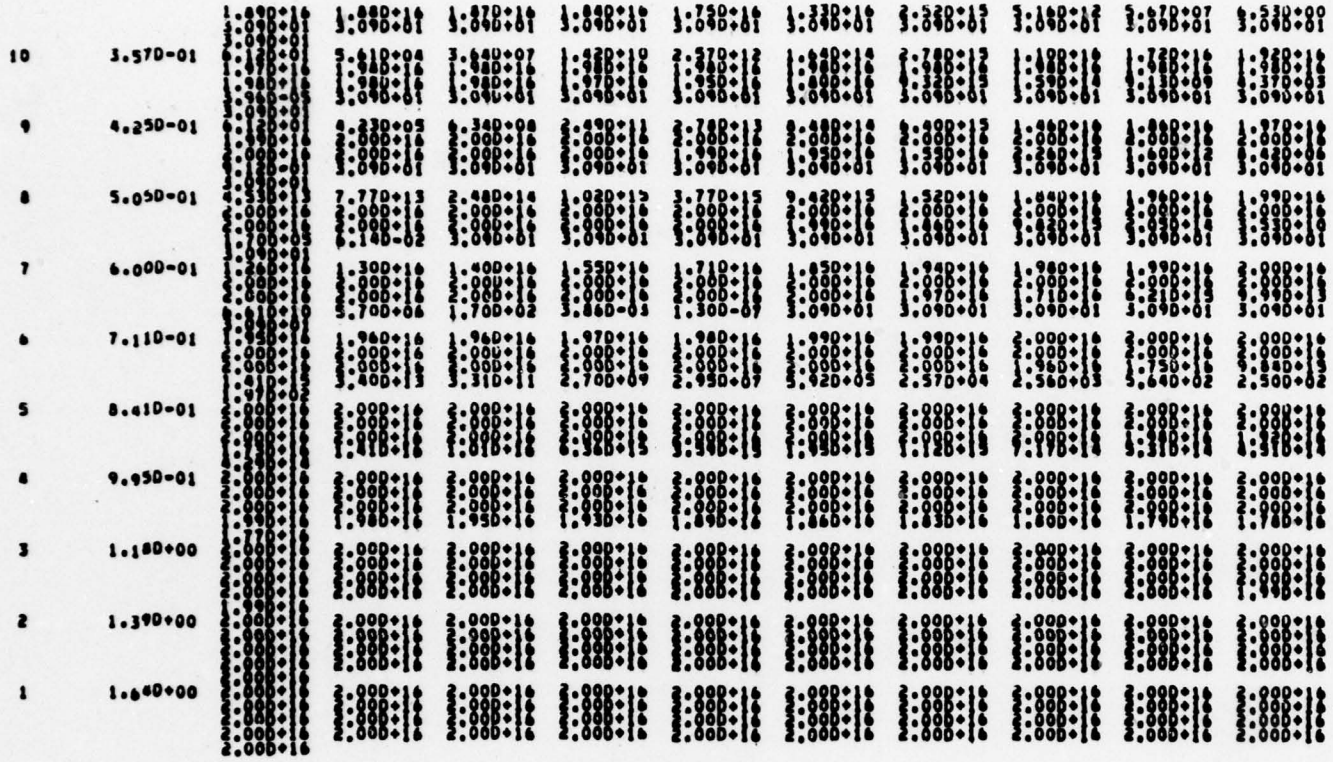


Table V sheet 7

XTH(I),I=1,ITHMAX

		1.150-02	3.570-02	6.290-02	9.100-02	1.200-01	1.640-01	4.030-01	2.510-01	3.010-01	3.500-01
		1.170-01	2.010-01	3.660-01	5.550-01	7.500-01	9.270-01	1.060+00	1.118+00	1.250+00	1.410+00
		1.530+00	1.730+00	1.930+00	2.130+00	2.140+00	2.250+00	2.350+00	2.430+00	2.510+00	2.580+00
		2.640+00	2.700+00	2.750+00	2.800+00	2.840+00	2.870+00	2.910+00	2.940+00	2.960+00	2.980+00

J	YTH(J)	THETA(I,J),I=1,ITHMAX									
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	6.780-03	4.020-01	5.100-01	6.020-01	6.640-01	7.070-01	7.370-01	7.590-01	7.750-01	7.850-01	7.920-01
		7.960-01	7.990-01	8.010-01	8.020-01	8.010-01	8.010-01	8.000-01	8.000-01	8.000-01	8.000-01
		8.000-01	8.000-01	8.000-01	8.000-01	8.000-01	8.000-01	8.000-01	8.000-01	8.000-01	8.000-01
		5.210-01	3.950-01	2.630-01	1.350-01	7.070-02	3.800-02	1.720-02	7.730-03	3.640-03	2.080-03
23	1.330-02	4.130-01	7.150-01	8.110-01	8.710-01	9.060-01	9.270-01	9.410-01	9.490-01	9.540-01	9.570-01
		9.590-01	9.600-01	9.610-01	9.620-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01
		9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01	9.630-01
		7.940-01	6.360-01	4.650-01	3.210-01	1.980-01	1.030-01	5.330-02	2.740-02	1.480-02	8.660-03
22	2.090-02	7.610-01	8.360-01	9.110-01	9.520-01	9.720-01	9.820-01	9.880-01	9.900-01	9.920-01	9.930-01
		9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01
		9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01	9.940-01
		6.140-01	4.880-01	3.260-01	1.780-01	8.460-02	2.160-01	1.240-01	7.640-02	4.740-02	3.540-02
21	2.990-02	8.530-01	9.120-01	9.400-01	9.840-01	9.930-01	9.960-01	9.980-01	9.990-01	9.990-01	9.990-01
		9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01
		9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01	9.990-01
		6.720-01	5.270-01	3.390-01	1.790-01	8.370-02	3.720-02	1.740-02	9.980-01	1.260-01	1.030-01
20	4.080-02	9.290-01	9.570-01	9.840-01	9.950-01	9.980-01	9.990-01	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		8.930-01	7.790-01	6.240-01	4.480-01	7.190-01	5.710-01	4.420-01	3.410-01	2.740-01	2.410-01
19	5.290-02	9.680-01	9.820-01	9.940-01	9.990-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		9.740-01	9.140-01	8.240-01	6.940-01	5.370-01	7.560-01	6.440-01	5.550-01	4.870-01	4.520-01
18	6.750-02	9.880-01	9.940-01	9.980-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		9.990-01	9.940-01	9.760-01	9.390-01	8.790-01	8.025-01	8.270-01	7.630-01	7.140-01	6.860-01
17	8.470-02	9.960-01	9.980-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		9.990-01	9.940-01	9.790-01	9.490-01	8.990-01	8.350-01	8.090-01	8.040-01	8.040-01	8.040-01
16	1.050-01	9.990-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		9.990-01	9.990-01	9.940-01	9.770-01	9.470-01	9.030-01	8.850-01	8.770-01	8.690-01	8.630-01
15	1.290-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
14	1.570-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	9.990-01
13	1.900-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
12	2.260-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
11	2.700-01	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00
		1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00

Table V sheet 8

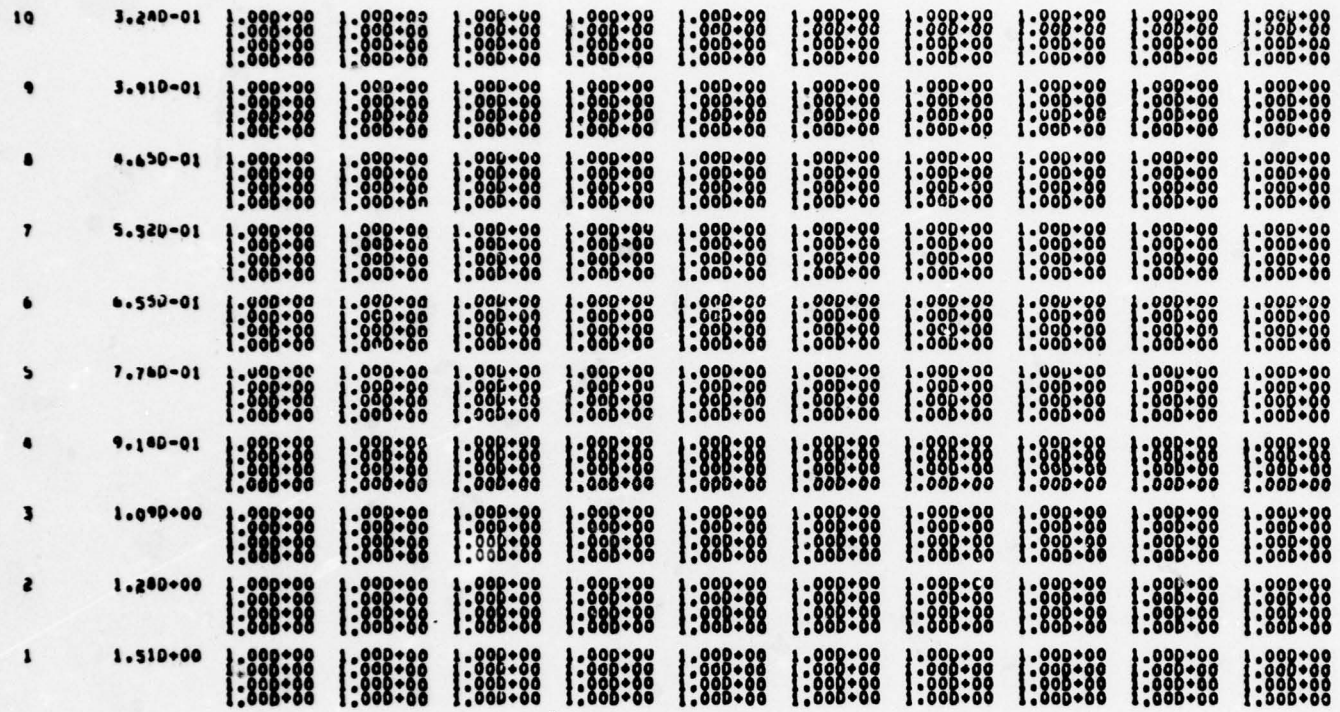


Table V sheet 9

ATM(I),I=1,ITMMAX

	1.150-02	3.590-02	6.290-02	9.300-02	1.260-01	1.640-01	2.050-01	2.510-01	3.010-01	3.580-01
	4.210-01	4.910-01	5.680-01	6.350-01	7.500-01	8.570-01	9.750-01	1.110+00	1.250+00	1.410+00
	1.560+00	1.750+00	1.890+00	2.030+00	2.140+00	2.250+00	2.350+00	2.430+00	2.510+00	2.580+00
	2.640+00	2.700+00	2.750+00	2.800+00	2.840+00	2.870+00	2.910+00	2.940+00	2.960+00	2.990+00

J	Y(J)	ESUBX(I,J),I=1,ITMMAX									
25	3.800-03	4.430+04	3.900+03	1.880+03	1.390+03	1.560+03	1.480+03	1.33D+03	1.07D+03	7.320+02	3.850+02
		9.380+01	-1.030+02	-2.190+02	-3.220+02	-4.160+02	-5.110+02	-6.010+02	-6.870+02	-4.220+02	-4.770+02
		-7.810+04	-8.950+04	-1.150+05	-1.360+05	-1.510+05	-1.630+05	-2.120+05	-2.210+05	-3.500+04	-5.960+04
		-7.580+04	-8.950+04	-1.130+05	-1.340+05	-1.510+05	-1.630+05	-2.150+05	-2.340+05	-3.140+05	-4.370+05
24	9.740D-03	4.280+04	3.920+03	2.950+03	2.430+03	2.240+03	2.030+03	1.73D+03	1.340+03	9.010+02	4.400+02
		1.360+02	-7.680+01	-2.050+02	-3.100+02	-4.090+02	-5.030+02	-5.950+02	-6.800+02	-4.150+02	-4.690+02
		-7.730+04	-8.940+04	-1.140+05	-1.360+05	-1.540+05	-1.670+05	-2.180+05	-2.270+05	-4.360+04	-6.070+04
		-7.710+04	-8.920+04	-1.150+05	-1.360+05	-1.540+05	-1.670+05	-2.180+05	-2.590+05	-3.200+05	-4.260+05
23	1.68D-02	3.830+04	3.950+03	2.820+03	2.470+03	2.380+03	2.090+03	2.350+03	1.730+03	1.130+03	6.170+02
		1.180+02	-3.960+01	-1.850+02	-2.990+02	-3.980+02	-4.940+02	-5.440+02	-6.690+02	-4.020+02	-4.360+02
		-7.690+04	-8.950+04	-1.140+05	-1.340+05	-1.700+05	-1.830+05	-2.190+05	-2.330+05	-4.070+04	-5.190+04
		-7.870+04	-8.710+04	-1.170+05	-1.340+05	-1.530+05	-1.660+05	-2.220+05	-2.220+05	-3.160+05	-3.460+05
22	2.50D-02	3.210+04	1.260+04	7.460+03	5.860+03	4.890+03	4.010+03	3.130+03	2.230+03	1.430+03	7.830+02
		1.840+02	-2.490+01	-1.660+02	-2.730+02	-3.740+02	-4.740+02	-5.300+02	-6.540+02	-3.970+02	-4.390+02
		-7.820+04	-8.930+04	-1.140+05	-1.340+05	-1.660+05	-1.800+05	-2.250+05	-2.410+05	-3.740+04	-4.930+04
		-8.050+04	-8.930+04	-1.200+05	-1.380+05	-1.560+05	-1.690+05	-2.250+05	-2.350+05	-3.150+05	-3.720+05
21	3.47D-02	4.490+04	1.680+04	1.080+04	8.380+03	6.770+03	5.360+03	4.060+03	2.650+03	1.600+03	9.740+02
		4.020+02	-1.480+01	-1.320+02	-2.230+02	-3.170+02	-4.080+02	-4.940+02	-6.370+02	-3.690+02	-4.200+02
		-8.210+04	-9.020+04	-1.150+05	-1.350+05	-1.650+05	-1.790+05	-2.240+05	-2.490+05	-3.900+04	-4.890+04
		-8.260+04	-9.020+04	-1.230+05	-1.450+05	-1.700+05	-1.870+05	-2.490+05	-2.660+05	-3.040+05	-3.300+05
20	4.61D-02	4.720+04	2.120+04	1.470+04	1.140+04	9.380+03	7.950+03	6.140+03	4.290+03	2.470+03	1.190+03
		6.080+02	-1.100+02	-1.000+02	-2.010+02	-2.990+02	-3.970+02	-4.950+02	-5.950+02	-3.470+02	-3.940+02
		-8.480+04	-1.040+05	-1.230+05	-1.440+05	-1.740+05	-1.870+05	-2.310+05	-2.580+05	-3.010+04	-3.760+04
		-8.480+04	-1.050+05	-1.260+05	-1.480+05	-1.740+05	-2.010+05	-2.310+05	-2.580+05	-3.010+05	-3.300+05
19	5.96D-02	4.940+04	2.580+04	1.900+04	1.480+04	1.150+04	8.780+03	6.390+03	4.320+03	2.650+03	1.420+03
		8.230+02	-1.170+02	-1.440+01	-1.740+02	-2.660+02	-3.540+02	-4.720+02	-5.950+02	-3.440+02	-3.880+02
		-8.640+04	-1.060+05	-1.240+05	-1.460+05	-1.760+05	-1.910+05	-2.400+05	-2.680+05	-2.980+04	-3.870+04
		-8.730+04	-1.080+05	-1.290+05	-1.530+05	-1.780+05	-2.040+05	-2.330+05	-2.580+05	-2.920+05	-3.140+05
18	7.54D-02	5.160+04	3.050+04	2.360+04	1.850+04	1.430+04	1.080+04	7.740+03	5.180+03	3.120+03	1.660+03
		1.160+03	-3.240+01	-1.140+01	-1.440+02	-2.410+02	-3.370+02	-4.440+02	-5.520+02	-3.310+02	-3.780+02
		-9.040+04	-1.090+05	-1.290+05	-1.510+05	-1.810+05	-2.010+05	-2.460+05	-2.760+05	-2.980+04	-3.880+04
		-9.010+04	-1.110+05	-1.330+05	-1.570+05	-1.810+05	-2.070+05	-2.340+05	-2.610+05	-2.940+05	-3.010+05
17	9.40D-02	5.390+04	3.530+04	2.820+04	2.280+04	1.740+04	1.310+04	9.290+03	6.070+03	3.600+03	1.880+03
		1.320+03	-1.730+01	-1.770+00	-1.480+02	-2.480+02	-3.460+02	-4.530+02	-5.620+02	-3.360+02	-3.850+02
		-9.240+04	-1.060+05	-1.250+05	-1.480+05	-1.800+05	-2.020+05	-2.510+05	-2.820+05	-3.040+04	-3.950+04
		-9.290+04	-1.140+05	-1.370+05	-1.610+05	-1.850+05	-2.160+05	-2.390+05	-2.680+05	-2.990+05	-2.960+05
16	1.16D-01	5.640+04	4.000+04	3.290+04	2.640+04	2.060+04	1.540+04	1.080+04	6.970+03	4.000+03	2.030+03
		1.690+03	-3.030+02	-4.970+01	-1.950+02	-3.000+02	-4.000+02	-5.180+02	-6.330+02	-3.730+02	-4.270+02
		-9.520+04	-1.080+05	-1.290+05	-1.540+05	-1.840+05	-2.110+05	-2.540+05	-2.930+05	-3.330+04	-4.330+04
		-9.580+04	-1.180+05	-1.410+05	-1.650+05	-1.890+05	-2.150+05	-2.390+05	-2.710+05	-2.990+05	-2.830+05
15	1.42D-01	5.910+04	4.470+04	3.730+04	3.040+04	2.380+04	1.780+04	1.230+04	7.660+03	4.210+03	2.040+03
		1.650+03	-2.990+02	-4.660+01	-2.070+02	-3.040+02	-4.130+02	-5.260+02	-6.480+02	-3.830+02	-4.310+02
		-9.520+04	-1.120+05	-1.310+05	-1.560+05	-1.870+05	-2.170+05	-2.550+05	-2.990+05	-3.400+04	-4.740+04
		-9.860+04	-1.210+05	-1.450+05	-1.640+05	-1.920+05	-2.150+05	-2.560+05	-2.930+05	-2.670+05	-2.780+05
14	1.72D-01	6.190+04	4.900+04	4.150+04	3.410+04	2.680+04	1.980+04	1.44D+04	7.900+03	4.000+03	1.780+03
		1.080+03	-3.400+02	-4.380+01	-2.230+02	-3.400+02	-4.790+02	-6.040+02	-7.430+02	-4.030+02	-4.540+02
		-9.670+04	-1.210+05	-1.480+05	-1.670+05	-2.200+05	-2.280+05	-2.520+05	-4.090+04	-5.900+04	-7.920+04
		-1.010+05	-1.240+05	-1.480+05	-1.720+05	-1.960+05	-2.170+05	-2.370+05	-2.930+05	-2.660+05	-2.760+05
13	2.07D-01	6.470+04	5.310+04	4.530+04	3.740+04	2.930+04	2.130+04	1.470+04	7.230+03	4.120+03	1.170+03
		9.130+02	-1.320+02	-4.760+01	-2.200+02	-3.730+02	-5.420+01	-6.430+01	-4.220+01	-4.150+01	-4.660+01
		-1.010+05	-1.220+05	-1.530+05	-1.700+05	-2.200+05	-2.400+05	-4.060+04	-5.940+04	-8.040+04	-8.920+04
		-1.030+05	-1.260+05	-1.510+05	-1.750+05	-1.990+05	-2.210+05	-2.400+05	-2.560+05	-2.690+05	-2.780+05
12	2.49D-01	6.760+04	5.680+04	4.860+04	4.000+04	3.100+04	2.180+04	1.280+04	5.560+03	1.840+03	3.540+02
		1.440+02	-4.830+01	-1.040+01	-2.060+02	-3.760+02	-5.870+02	-8.110+01	-1.390+01	-1.390+01	-1.670+01
		-1.030+05	-1.280+05	-1.160+05	-1.750+05	-2.250+05	-8.030+03	-2.170+04	-5.920+04	-8.170+04	-8.000+04
		-1.030+05	-1.270+05	-1.520+05	-1.770+05	-2.020+05	-2.250+05	-2.460+05	-2.630+05	-2.770+05	-2.870+05
11	2.99D-01	7.080+04	6.040+04	5.150+04	4.180+04	3.160+04	2.110+04	1.080+04	3.740+03	9.890+02	2.360+02
		5.660+01	-1.410+01	-2.070+00	-4.210+01	-1.400+00	-2.040+00	-2.360+00	-2.620+00	-2.960+00	-3.570+00
		-1.050+05	-1.270+05	-1.510+05	-1.790+05	-2.330+05	-2.550+05	-2.760+05	-2.910+05	-2.820+05	-2.770+05
		-1.010+05	-1.250+05	-1.500+05	-1.770+05	-2.040+05	-2.300+05	-2.550+05	-2.770+05	-2.950+05	-3.080+05

Table V sheet 10

10	3.570-01	7.440+04 1.080+01 -1.030+01 -9.470+04	8.330+04 -1.030+00 -1.670+00 -1.180+05	5.420+04 -1.300-01 -4.010+00 -1.440+05	4.250+04 -2.430-02 -2.430+01 -1.710+05	3.040+04 -1.110-01 -1.840+01 -2.010+05	1.470+04 -1.490-01 -1.030+01 -2.330+05	8.140+03 -1.170-01 -5.160+01 -2.680+05	2.490+03 -4.310-01 -3.040+01 -3.020+05	5.450+02 -3.380-01 -1.120+01 -3.350+05	-1.090+02 -4.970-01 -7.240+01 -3.580+05
9	4.250-01	9.910+04 8.540+00 -1.190+01 -1.310+04	7.380+04 -1.330+00 -4.870+01 -1.040+04	5.420+04 -1.480-01 -1.810+01 -1.270+05	3.830+04 -4.830-02 -4.030+00 -1.520+05	2.510+04 -1.040-01 -1.160+01 -1.810+05	1.340+04 -1.110-01 -6.240+01 -2.170+05	6.910+03 -2.240-01 -2.110+01 -2.640+05	1.290+03 -3.310-01 -2.190+01 -3.280+05	2.770+02 -3.240-01 -6.260+01 -4.170+05	5.140+01 -7.600-01 -6.280+01 -5.420+05
8	5.050-01	2.020+03 2.660+00 -1.110+00 -2.450+04	1.470+04 -1.870-01 -3.040-02 -3.020+04	1.290+04 -3.280-02 -1.100+01 -1.480+04	1.070+04 -1.700-01 -1.040+00 -1.080+05	6.750+03 -2.670-01 -1.180+01 -1.160+05	3.180+03 -4.170-01 -1.280+01 -1.250+05	1.140+03 -4.930-01 -1.420+01 -1.420+05	3.260+02 -5.520-01 -1.730+01 -1.180+05	7.730+01 -4.160-01 -3.040+01 -8.830+04	-1.550+01 -1.520-01 -4.790+01 -4.050+04
7	6.000-01	3.010+02 5.130-01 -2.420+04 -2.060+04	7.470+02 -4.210-02 -3.810-01 -3.040+04	9.230+02 -1.110-02 -1.430+01 -3.750+04	8.260+04 -1.170-01 -4.050+01 -2.150+04	5.690+02 -1.080-01 -1.910-06 -2.240+04	3.080+02 -1.920-04 -2.780+01 -5.030+04	1.280+02 -2.610-04 -3.590+02 -5.070+04	4.320+01 -4.040-04 -3.220+03 -3.930+04	1.190+01 -4.610-04 -1.490+04 -2.480+04	2.720+00 -5.650-04 -2.850+04 -8.450+03
6	7.110-01	1.120+01 6.220-02 -2.230-04 -2.230-04	3.060+01 1.090-02 -1.820-04 -2.350-04	4.120+01 -1.340-04 -2.920-04 -2.340-04	4.050+01 -2.840-04 -2.160+04 -2.670+04	3.100+01 -6.310-06 -4.470-01 -2.970-01	1.890+01 -1.580-05 -2.330+00 -2.500+04	9.140+00 -2.010-05 -4.240-01 -1.440+04	3.570+00 -2.260-05 -1.670+02 -1.370+04	1.130+00 -2.540-05 -2.020+03 -8.160+03	2.930-01 -3.190-05 -7.690+03 -2.680+03
5	8.410-01	3.380-01 -1.020-01 -2.040-02 -2.040-02	9.590-01 -1.160-03 -1.600-03 -1.600-03	1.370+00 -1.470-04 -2.440-03 -2.440-03	1.450+00 -1.880-05 -3.410-03 -3.410-03	1.210+00 -1.300-06 -4.040-03 -4.040-03	8.160-01 -1.490-07 -4.550-01 -4.110-03	4.440-01 -8.240-07 -2.790+00 -3.620-03	1.960-01 -9.190-07 -2.140-01 -2.740-03	6.990-02 -1.080-06 -1.170-02 -1.660-03	2.030-02 -1.130-06 -3.670-02 -3.470-02
4	9.950-01	7.950-03 -1.440-04 -1.440-04 -1.440-04	3.100-02 -1.140-01 -1.140-01 -1.140-01	3.420-02 -2.080-06 -2.080-06 -2.080-06	3.410-02 -2.080-06 -2.080-06 -2.080-06	3.420-02 -2.080-06 -2.080-06 -2.080-06	3.310-02 -2.080-06 -2.080-06 -2.080-06	3.300-02 -2.080-06 -2.080-06 -2.080-06	3.320-02 -2.080-06 -2.080-06 -2.080-06	3.310-02 -2.080-06 -2.080-06 -2.080-06	3.330-02 -2.080-06 -2.080-06 -2.080-06
3	1.180+00	1.430-04 -1.440-06 -3.710-01 -3.710-01	1.190-04 -1.870-06 -1.200-06 -1.200-06	9.380-05 -1.780-07 -1.780-06 -1.780-06	7.410-05 -5.340-06 -1.310-06 -1.310-06	7.030-05 -6.240-06 -1.210-06 -1.210-06	5.310-05 -1.010-10 -2.210-06 -2.210-06	3.570-05 -5.440-10 -2.440-03 -2.290-06	1.900-05 -6.010-10 -2.760-06 -1.760-06	8.280-03 -7.990-10 -2.330-01 -1.030-06	2.940-03 -1.030-09 -3.980-01 -3.380-01
2	1.390+00	1.920-04 -1.070-07 -1.230-10 -1.230-10	3.440-04 -1.010-06 -1.440-02 -1.440-02	8.440-04 -1.010-06 -1.440-02 -1.440-02	1.040-03 -1.010-06 -1.070-02 -1.070-02	1.050-03 -1.010-06 -1.130-02 -1.130-02	8.710-04 0.0 -1.440-02 -1.440-02	6.030-04 0.0 -1.480-02 -1.480-02	-1.440-04 -1.010-06 -2.300-02 -2.300-02	1.040-04 -1.010-06 -2.340-02 -2.330-02	4.310-07 -1.010-06 -3.140-01 -3.140-01

Table V sheet 11

X(I),I=1,IMAX

0.0	2.310-02	4.870-02	7.720-04	1.090-01	1.440-01	1.810-01	2.260-01	2.750-01	3.280-01
3.880-01	4.540-01	5.270-01	6.060-01	7.000-01	8.010-01	9.130-01	1.040+00	1.180+00	1.330+00
1.500+00	1.670+00	1.850+00	1.960+00	2.090+00	2.200+00	2.300+00	2.390+00	2.470+00	2.550+00
2.610+00	2.670+00	2.730+00	2.770+00	2.820+00	2.860+00	2.890+00	2.920+00	2.950+00	2.980+00
3.000+00									

J	YTH(J)	ESUBY(I,J),I=1,IMAX								
25	0.0	0.0	7.340+04	7.300+04	7.280+04	7.290+04	7.300+04	7.320+04	7.340+04	7.360+04	7.380+04
		7.380+04	7.380+04	7.380+04	7.370+04	7.360+04	7.350+04	7.340+04	7.320+04	7.310+04	7.280+04
		7.260+04	7.230+04	7.190+04	7.130+04	7.050+04	6.940+04	6.820+04	6.680+04	6.490+04	6.290+04
		2.750+04	1.320+04	-2.480+03	-1.450+04	-3.750+04	-5.640+04	-7.630+04	-9.820+04	-1.240+05	-1.570+05
		0.0									
24	6.780-03	0.0	3.280+04	4.140+04	4.940+04	5.110+04	5.320+04	5.880+04	6.170+04	6.490+04	6.550+04
		6.440+04	6.190+04	6.130+04	6.750+04	6.160+04	6.770+04	6.740+04	6.600+04	6.820+04	6.630+04
		6.210+04	6.380+04	6.040+04	6.480+04	6.810+04	6.130+04	6.240+04	5.720+04	4.930+04	3.920+04
		2.710+04	1.330+04	-2.020+03	-1.850+04	-3.570+04	-5.370+04	-7.210+04	-9.120+04	-1.100+05	-1.200+05
		0.0									
23	1.330-02	0.0	1.810+04	2.910+04	3.670+04	4.320+04	4.890+04	5.370+04	5.750+04	6.030+04	6.220+04
		6.320+04	6.400+04	6.440+04	6.470+04	6.480+04	6.500+04	6.520+04	6.540+04	6.560+04	6.540+04
		6.520+04	6.480+04	6.380+04	6.190+04	5.880+04	5.460+04	5.140+04	5.600+04	6.030+04	6.450+04
		2.680+04	1.330+04	-1.530+03	-1.780+04	-3.600+04	-5.100+04	-6.600+04	-8.440+04	-9.770+04	-1.360+04
		0.0									
22	2.090-02	0.0	1.040+04	2.210+04	3.120+04	3.870+04	4.530+04	5.090+04	5.470+04	5.770+04	5.960+04
		6.080+04	6.150+04	6.160+04	6.210+04	6.230+04	6.270+04	6.240+04	6.310+04	6.340+04	6.340+04
		6.410+04	6.430+04	6.450+04	6.450+04	6.410+04	6.370+04	5.960+04	5.460+04	6.730+04	6.780+04
		2.640+04	1.340+04	-8.930+02	-1.620+04	-3.410+04	-4.980+04	-6.300+04	-7.640+04	-8.410+04	-7.160+04
		0.0									
21	2.990-02	0.0	4.720+03	1.760+04	2.740+04	3.540+04	4.240+04	4.740+04	5.200+04	5.500+04	5.690+04
		5.810+04	5.870+04	5.910+04	5.940+04	5.950+04	5.970+04	5.990+04	6.010+04	6.030+04	6.060+04
		6.100+04	6.140+04	6.170+04	6.180+04	6.160+04	6.090+04	6.310+04	6.010+04	6.010+04	6.100+04
		2.610+04	1.360+04	-9.560+01	-1.460+04	-3.240+04	-4.930+04	-5.780+04	-6.790+04	-7.010+04	-5.140+04
		0.0									
20	4.040-02	0.0	4.710+03	1.440+04	2.430+04	3.240+04	3.940+04	4.490+04	4.900+04	5.170+04	5.340+04
		5.450+04	5.530+04	5.580+04	5.610+04	5.630+04	5.650+04	5.660+04	5.680+04	5.710+04	5.740+04
		5.770+04	5.810+04	5.850+04	5.870+04	5.860+04	5.790+04	5.360+04	5.130+04	5.340+04	5.480+04
		2.440+04	1.340+04	6.780+02	-1.270+04	-3.040+04	-4.540+04	-5.060+04	-5.780+04	-5.890+04	-3.980+04
		0.0									
19	5.290-02	0.0	3.790+03	1.230+04	2.160+04	2.960+04	3.620+04	4.160+04	4.560+04	4.840+04	5.020+04
		5.130+04	5.210+04	5.260+04	5.290+04	5.310+04	5.330+04	5.340+04	5.360+04	5.390+04	5.430+04
		5.470+04	5.510+04	5.540+04	5.560+04	5.550+04	5.490+04	5.100+04	4.880+04	5.090+04	5.230+04
		2.440+04	1.340+04	2.350+03	-1.030+04	-2.770+04	-4.210+04	-4.720+04	-5.360+04	-5.470+04	-3.700+04
		0.0									
18	6.750-02	0.0	3.140+03	1.080+04	1.910+04	2.630+04	3.240+04	3.730+04	4.120+04	4.430+04	4.630+04
		4.760+04	4.850+04	4.910+04	4.940+04	4.970+04	4.990+04	5.000+04	5.020+04	5.050+04	5.090+04
		5.130+04	5.170+04	5.200+04	5.220+04	5.210+04	5.150+04	4.800+04	4.590+04	4.800+04	4.940+04
		2.440+04	1.340+04	3.060+03	-7.370+03	-2.440+04	-3.910+04	-4.410+04	-5.040+04	-5.140+04	-3.420+04
		0.0									
17	8.470-02	0.0	2.880+03	9.470+03	1.680+04	2.330+04	2.910+04	3.380+04	3.760+04	4.070+04	4.270+04
		4.410+04	4.500+04	4.560+04	4.590+04	4.620+04	4.640+04	4.650+04	4.670+04	4.700+04	4.740+04
		4.780+04	4.820+04	4.850+04	4.870+04	4.860+04	4.800+04	4.450+04	4.240+04	4.450+04	4.590+04
		2.440+04	1.350+04	3.260+03	-3.840+03	-1.980+04	-3.440+04	-4.110+04	-4.760+04	-4.870+04	-3.140+04
		0.0									
16	1.050-01	0.0	2.630+03	8.220+03	1.430+04	2.010+04	2.520+04	2.960+04	3.330+04	3.640+04	3.840+04
		3.980+04	4.070+04	4.130+04	4.160+04	4.190+04	4.210+04	4.220+04	4.240+04	4.270+04	4.310+04
		4.350+04	4.390+04	4.420+04	4.440+04	4.430+04	4.370+04	4.030+04	3.820+04	4.030+04	4.170+04
		2.440+04	1.350+04	3.430+03	-7.000+03	-2.410+04	-3.880+04	-4.390+04	-5.020+04	-5.130+04	-3.410+04
		0.0									
15	1.290-01	0.0	2.410+03	7.020+03	1.200+04	1.680+04	2.130+04	2.510+04	2.830+04	3.090+04	3.270+04
		3.400+04	3.490+04	3.550+04	3.590+04	3.620+04	3.650+04	3.670+04	3.700+04	3.730+04	3.770+04
		3.810+04	3.850+04	3.880+04	3.900+04	3.900+04	3.850+04	3.530+04	3.330+04	3.540+04	3.680+04
		2.440+04	1.350+04	3.590+03	-6.200+03	-2.380+04	-3.850+04	-4.380+04	-5.010+04	-5.120+04	-3.410+04
		0.0									
14	1.570-01	0.0	2.140+03	5.850+03	9.800+03	1.370+04	1.710+04	1.980+04	2.190+04	2.350+04	2.460+04
		2.540+04	2.600+04	2.640+04	2.660+04	2.680+04	2.700+04	2.710+04	2.730+04	2.760+04	2.800+04
		2.840+04	2.880+04	2.910+04	2.930+04	2.930+04	2.890+04	2.670+04	2.540+04	2.690+04	2.790+04
		2.440+04	1.360+04	3.740+03	-5.420+03	-2.350+04	-3.830+04	-4.370+04	-5.000+04	-5.120+04	-3.410+04
		0.0									
13	1.900-01	0.0	1.840+03	4.740+03	7.770+03	1.070+04	1.310+04	1.490+04	1.610+04	1.690+04	1.730+04
		1.740+04	1.740+04	1.740+04	1.740+04	1.750+04	1.760+04	1.770+04	1.790+04	1.820+04	1.860+04
		1.900+04	1.940+04	1.970+04	1.990+04	1.990+04	1.960+04	1.810+04	1.730+04	1.830+04	1.890+04
		2.440+04	1.360+04	3.880+03	-5.200+03	-2.330+04	-3.820+04	-4.370+04	-5.000+04	-5.120+04	-3.410+04
		0.0									
12	2.280-01	0.0	1.570+03	3.820+03	6.060+03	8.060+03	9.690+03	9.960+03	9.930+03	7.090+03	5.470+03
		4.140+03	3.060+03	2.280+03	1.740+03	1.370+03	1.130+03	9.810+02	8.880+02	8.360+02	8.190+02
		8.270+02	8.540+02	8.980+02	9.600+02	1.040+03	1.130+03	1.180+03	1.240+03	1.280+03	1.310+03
		2.440+04	1.360+04	3.960+03	-5.180+03	-2.320+04	-3.810+04	-4.370+04	-5.000+04	-5.120+04	-3.410+04
		0.0									
11	2.740-01	0.0	1.390+03	3.330+03	5.080+03	6.240+03	6.690+03	6.970+03	6.320+03	6.340+03	6.490+03
		1.21n+03	1.070+03	1.020+03	1.000+03	1.010+03	1.020+03	1.040+03	1.060+03	1.090+03	1.120+03

Table V sheet 12

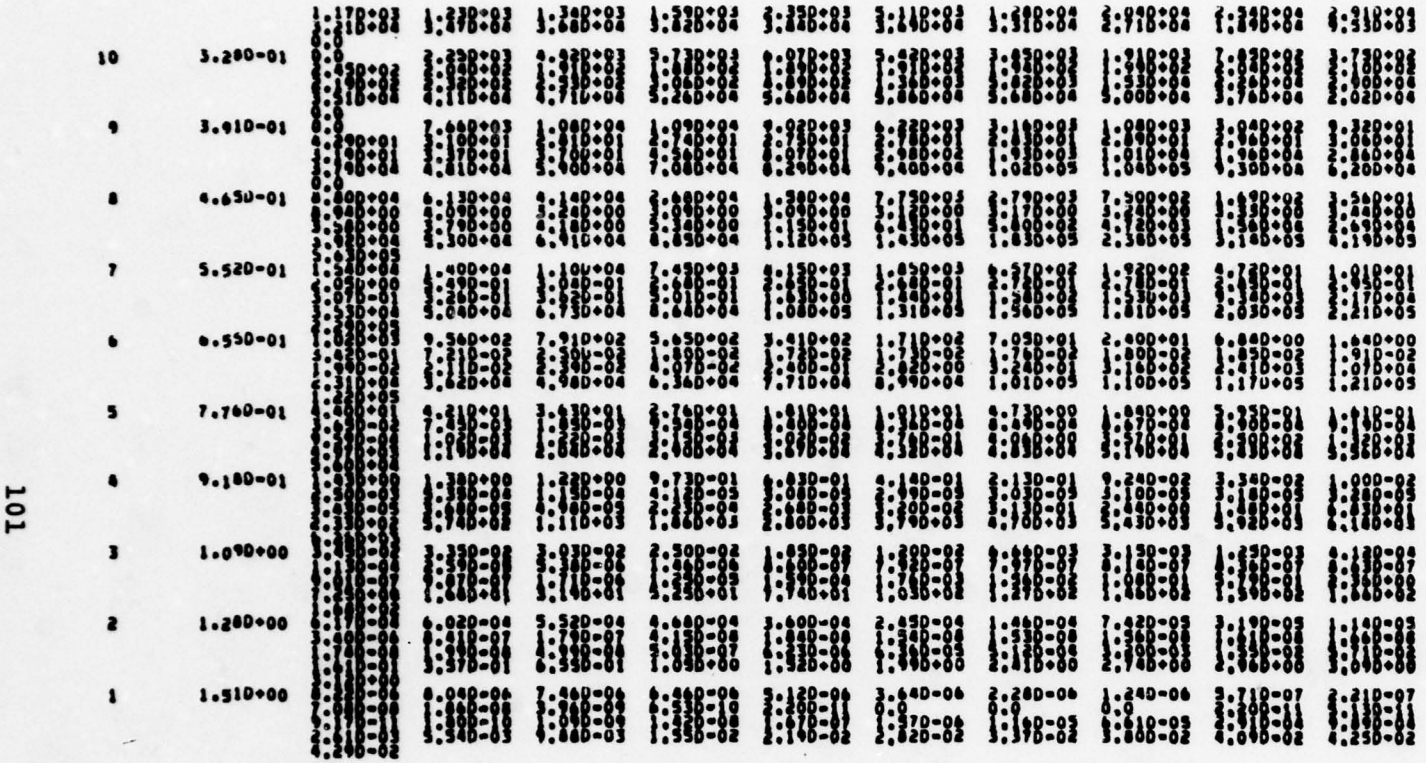


Table V sheet 13

XTH(I),I=1,ITHMAX

		1.150-02	3.590-02	6.290-02	9.300-02	1.260-01	1.640-01	2.050-01	2.510-01	3.010-01	3.580-01
		4.210-01	4.910-01	5.640-01	6.550-01	7.500-01	8.570-01	9.750-01	1.110+00	1.250+00	1.410+00
		1.540+00	1.740+00	1.890+00	2.030+00	2.140+00	2.250+00	2.350+00	2.430+00	2.510+00	2.580+00
		2.640+00	2.700+00	2.750+00	2.800+00	2.840+00	2.870+00	2.910+00	2.940+00	2.960+00	2.990+00
J	Y(J)	JSUBX(I,J),I=1,ITHMAX									
25	3.800-03	-4.660+02	-4.640+02	-5.480+02	-6.050+02	-6.440+02	-6.710+02	-6.910+02	-7.060+02	-7.130+02	-7.210+02
		-3.380+02	-3.370+02	-3.280+02	-2.660+02	-2.290+02	-2.100+02	-2.000+02	-1.910+02	-1.820+02	-1.730+02
		-1.280+02	-1.240+02	-1.180+02	-1.130+02	-1.080+02	-1.030+02	-9.860+01	-9.410+01	-8.960+01	-8.540+01
		-8.150+01	-7.790+01	-7.460+01	-7.150+01	-6.860+01	-6.590+01	-6.330+01	-6.080+01	-5.840+01	-5.610+01
24	9.760-03	-4.010+02	-1.950+02	-1.990+02	-1.960+02	-1.890+02	-1.810+02	-1.720+02	-1.660+02	-1.610+02	-1.580+02
		-1.560+02	-1.540+02	-1.540+02	-1.530+02	-1.510+02	-1.480+02	-1.520+02	-1.520+02	-1.540+02	-1.510+02
		-1.510+02	-1.480+02	-1.500+02	-1.510+02	-1.480+02	-1.380+02	-1.440+02	-1.460+02	-1.490+02	-1.540+02
		-1.460+02	-1.430+02	-2.120+02	-1.600+02	-1.050+02	-6.210+01	-3.440+01	-1.870+01	-1.060+01	-7.210+00
23	1.680-02	-1.190+02	-9.800+01	-8.100+01	-6.620+01	-5.400+01	-4.470+01	-3.800+01	-3.350+01	-3.030+01	-2.870+01
		-2.740+01	-2.700+01	-2.670+01	-2.650+01	-2.630+01	-2.610+01	-2.590+01	-2.570+01	-2.580+01	-2.520+01
		-2.480+01	-2.460+01	-2.450+01	-2.480+01	-2.540+01	-2.560+01	-2.550+01	-2.530+01	-2.540+01	-2.740+01
		-3.040+01	-3.420+01	-4.610+01	-7.540+01	-1.260+02	-2.120+02	-4.120+01	-6.100+01	-3.960+01	-2.080+01
22	2.500-02	-7.010+01	-5.200+01	-3.400+01	-2.140+01	-1.400+01	-9.420+00	-7.060+00	-5.560+00	-4.690+00	-4.200+00
		-3.880+00	-3.770+00	-3.690+00	-3.630+00	-3.580+00	-3.540+00	-3.500+00	-3.450+00	-3.380+00	-3.310+00
		-3.230+00	-3.170+00	-3.140+00	-3.190+00	-3.280+00	-3.330+00	-3.870+00	-3.710+00	-3.540+00	-3.140+00
		-3.020+01	-6.480+01	-1.030+02	-1.260+02	-1.270+02	-1.120+02	-8.940+01	-6.870+01	-5.340+01	-4.670+01
21	3.470-02	-3.890+01	-2.670+01	-1.390+01	-6.540+00	-3.220+00	-1.760+00	-1.080+00	-7.430-01	-5.740-01	-4.850-01
		-4.380-01	-4.130-01	-3.990-01	-3.900-01	-3.830-01	-3.750-01	-3.660-01	-3.590-01	-3.490-01	-3.380-01
		-3.280-01	-3.190-01	-3.090-01	-3.140-01	-3.380-01	-4.020-01	-5.690-01	-9.580-01	-1.870+00	-3.780+00
		-2.200+01	-2.810+01	-3.470+01	-3.480+01	-3.060+01	-2.130+01	-1.070+02	-4.530+01	-1.810+01	-8.060+01
20	4.610-02	-1.930+01	-1.240+01	-5.170+00	-1.800+00	-6.490-01	-2.710-01	-1.350-01	-8.000-02	-5.370-02	-4.410-02
		-3.880-02	-3.530-02	-3.370-02	-3.260-02	-3.170-02	-3.090-02	-3.000-02	-2.900-02	-2.790-02	-2.660-02
		-2.550-02	-2.480-02	-2.330-02	-2.330-02	-2.360-02	-2.460-02	-2.830-02	-2.720-02	-2.530-02	-2.180-02
		-2.830+00	-5.010+00	-2.320+01	-4.610+01	-7.150+01	-9.160+01	-1.030+02	-1.060+02	-1.060+02	-1.050+02
19	5.960-02	-8.390+00	-4.920+00	-1.680+00	-5.410-01	-1.150-01	-3.550-02	-1.390-02	-6.950-03	-4.310-03	-3.170-03
		-2.680-03	-2.380-03	-2.240-03	-2.150-03	-2.070-03	-2.000-03	-1.920-03	-1.840-03	-1.740-03	-1.630-03
		-1.490-03	-1.420-03	-1.350-03	-1.340-03	-1.460-03	-1.690-03	-2.060-03	-2.210-03	-2.410-03	-3.060-03
		-1.000-01	-2.200+00	-7.500+00	-1.440+01	-1.730+01	-1.760+01	-1.510+01	-8.800+00	-7.500+00	-4.900+00
18	7.540-02	-4.970+00	-1.590+00	-4.320-01	-9.270-02	-1.790-02	-4.040-03	-1.220-03	-5.010-04	-2.730-04	-1.860-04
		-1.490-04	-1.310-04	-1.210-04	-1.130-04	-1.100-04	-1.030-04	-1.000-04	-9.430-05	-8.790-05	-8.090-05
		-7.380-05	-6.670-05	-6.180-05	-5.880-05	-5.980-05	-6.380-05	-7.410-05	-8.980-05	-1.410-04	-2.400-04
		-2.310-02	-5.880-01	-1.810+00	-3.940+00	-7.230+01	-2.640+01	-4.000+01	-5.220+01	-6.110+01	-6.350+01
17	9.400-02	-8.300-01	-3.980-01	-9.420-02	-1.610-02	-2.410-03	-4.080-04	-9.320-05	-3.140-05	-1.510-05	-9.530-06
		-7.310-06	-6.300-06	-5.760-06	-5.410-06	-5.110-06	-4.920-06	-4.820-06	-4.760-06	-3.840-06	-3.450-06
		-3.040-06	-2.680-06	-2.390-06	-2.210-06	-2.240-06	-2.730-06	-4.810-06	-1.440-05	-7.440-05	-5.790-04
		-5.410-03	-2.650-02	-2.960-01	-1.280+00	-1.800+00	-8.350+00	-1.440+01	-2.080+01	-2.590+01	-2.880+01
16	1.160-01	-1.750-01	-7.510-02	-1.570-02	-2.250-03	-2.790-04	-3.760-05	-6.830-06	-1.490-06	-7.990-07	-4.730-07
		-3.310-07	-2.940-07	-2.700-07	-2.510-07	-2.340-07	-2.180-07	-2.010-07	-1.830-07	-1.640-07	-1.430-07
		-1.220-07	-1.030-07	-8.690-08	-7.450-08	-6.660-08	-7.590-08	-1.250-07	-2.900-06	-2.300-06	-2.300-05
		-2.930-04	-3.490-03	-3.020-02	-1.710-01	-6.340-01	-1.660+00	-3.260+00	-5.130+00	-6.780+00	-7.840+00
15	1.420-01	-4.720-02	-1.040-02	-1.930-03	-2.460-04	-2.720-05	-3.220-06	-5.070-07	-1.230-07	-4.770-08	-2.720-08
		-2.000-08	-1.690-08	-1.530-08	-1.400-08	-1.300-08	-1.190-08	-1.070-08	-9.540-09	-8.340-09	-7.030-09
		-5.750-09	-4.600-09	-3.600-09	-2.750-09	-2.160-09	-1.870-09	-2.700-09	-7.440-09	-4.670-08	-5.360-07
		-9.170-06	-1.470-04	-1.700-03	-1.250-02	-5.810-02	-1.810-01	-4.060-01	-7.030-01	-1.000+00	-1.240+00
14	1.720-01	-3.020-03	-1.040-03	-1.770-04	-2.120-05	-2.240-06	-2.630-07	-4.190-08	-1.040-08	-4.100-09	-2.400-09
		-1.610-09	-1.560-09	-1.410-09	-1.290-09	-1.180-09	-1.060-09	-9.400-10	-8.180-10	-6.840-10	-5.590-10
		-4.350-10	-3.260-10	-2.310-10	-1.500-10	-1.010-10	-1.370-10	-2.220-10	-4.240-10	-7.630-09	
		-1.470-07	-2.640-06	-4.350-05	-4.080-04	-2.360-03	-8.870-03	-2.320-02	-4.560-02	-7.240-02	-9.840-02
13	2.070-01	-4.190-04	-7.300-05	-1.220-05	-1.450-06	-1.420-07	-2.190-08	-4.190-09	-1.420-09	-6.930-10	-4.630-10
		-3.760-10	-3.350-10	-3.070-10	-2.810-10	-2.440-10	-2.160-10	-1.960-10	-1.670-10	-1.370-10	-1.070-10
		-8.630-11	-6.560-11	-4.580-11	-3.180-11	-2.240-11	-1.810-11	-1.400-12	-1.070-11	-1.600-11	-3.730-11
		-1.050-04	-2.220-04	-3.750-07	-4.180-06	-2.940-05	-1.360-04	-4.410-04	-1.070-03	-2.080-03	-3.430-03
12	2.490-01	-1.140-05	-3.920-06	-6.320-07	-8.070-08	-1.090-08	-2.150-09	-7.410-10	-3.970-10	-2.720-10	-2.140-10
		-1.890-10	-1.720-10	-1.560-10	-1.460-10	-1.320-10	-1.170-10	-1.010-10	-8.500-11	-6.900-11	-5.320-11
		-3.470-11	-2.660-11	-1.700-11	-1.050-11	-1.150-12	-1.140-08	-1.100-12	-1.310-11	-1.860-11	-1.280-11
		-2.110-11	-2.660-11	-3.050-10	-1.320-04	-5.140-08	-3.350-07	-1.500-06	-5.470-06	-1.260-05	-1.280-05
11	2.990-01	-5.190-07	-1.430-07	-2.430-08	-4.850-09	-8.450-10	-3.920-10	-2.710-10	-2.200-10	-1.890-10	-1.660-10
		-1.520-10	-1.400-10	-1.240-10	-1.190-10	-1.070-10	-8.550-11	-6.300-11	-7.020-11	-5.720-11	-4.480-11
		-3.260-11	-2.290-11	-3.560-11	-4.110-11	-9.710-12	-1.110-11	-1.220-10	-1.390-04	-1.930-11	-1.970-11
		-2.380-11	-2.820-11	-3.350-11	-4.080-11	-5.070-11	-6.620-11	-2.660-10	-2.020-04	-1.740-08	-1.800-07

Table V sheet 14

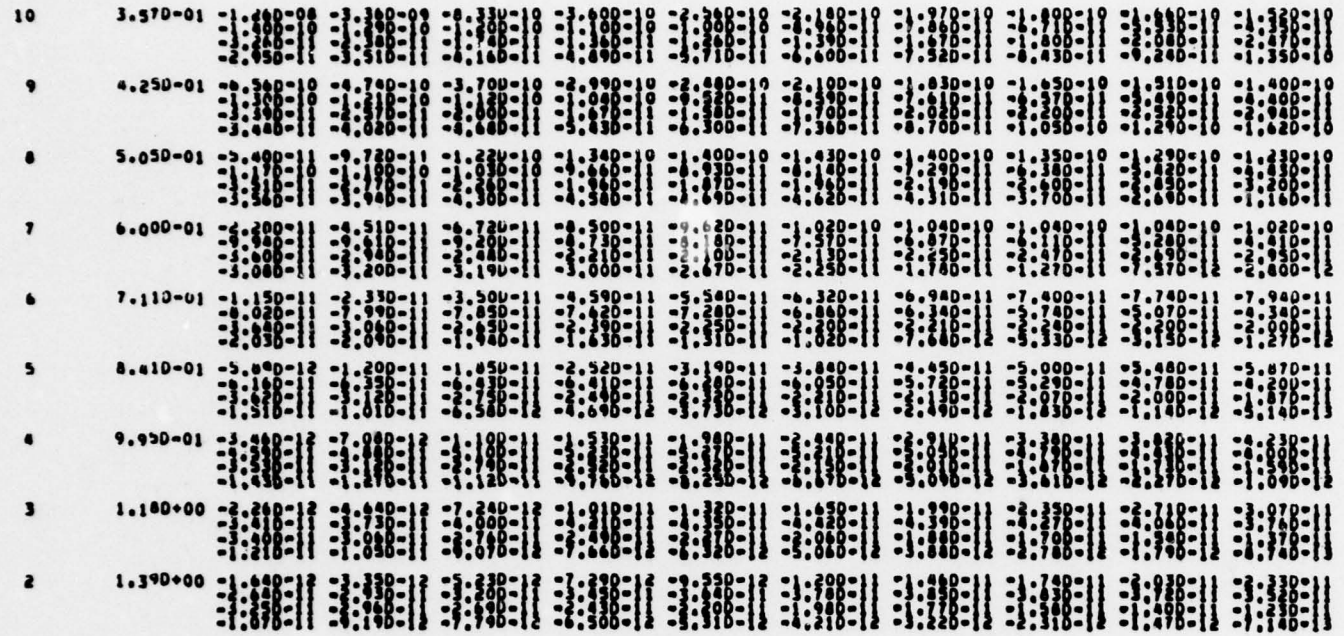


Table V sheet 15

X(I),I=1,IMAX

	0.0	2.510-02	4.670-02	7.720-02	1.090-01	1.440-01	1.830-01	2.260-01	2.750-01	3.280-01
	3.880-01	4.540-01	5.270-01	6.090-01	7.000-01	8.010-01	9.130-01	1.040+00	1.180+00	1.330+00
	1.500+00	1.670+00	1.820+00	1.960+00	2.040+00	2.200+00	2.300+00	2.390+00	2.470+00	2.550+00
	2.610+00	2.670+00	2.730+00	2.770+00	2.820+00	2.860+00	2.890+00	2.920+00	2.950+00	2.980+00
	3.000+00									

J	YTH(J)	JSUBY(I,J),I=2,ITHMAX								
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	6.780-03	0.0	2.750+01	2.100+01	1.280+01	7.470+00	3.080+00	1.300+00	2.090+00	1.260+00	7.130-01
		3.810-01	1.460-01	1.010-01	5.830-02	3.440-02	1.380-02	1.470-02	1.660-02	5.400-02
		4.040-02	-1.490-02	-7.430-03	-6.410-02	-2.100-01	-5.170-01	-1.110+00	-2.170+00	-3.870+00	-5.440+00
		-1.000+01	-1.380+01	-1.600+01	-1.440+01	-1.110+01	-6.440+00	-3.630+00	-1.950+00	-8.340-01	-3.950-01
23	1.330-02	0.0	2.570+01	2.190+01	1.230+01	6.490+00	3.560+00	2.020+00	1.150+00	6.370-01	3.420-01
		1.770-01	8.920-02	4.620-02	2.690-02	1.660-02	1.660-02	1.610-02	1.650-02	1.730-02	1.830-02
		1.910-02	1.720-02	6.960-03	-2.060-02	-1.020-01	-3.000-01	-1.020+00	-2.110+00	-4.160+00	-6.270+00
		-8.200+00	-1.410+01	-1.990+01	-2.220+01	-1.970+01	-1.440+01	-9.200+00	-5.330+00	-2.870+00	-1.310+00
22	2.090-02	0.0	1.910+01	1.710+01	8.550+00	3.700+00	1.650+00	7.830-01	3.870-01	1.950-01	9.770-02
		4.830-02	2.380-02	1.240-02	7.140-03	4.120-03	4.440-03	4.300-03	4.370-03	4.570-03	4.810-03
		4.880-03	4.560-03	3.390-03	1.100-03	-2.440-02	-4.540-02	-1.160-01	-2.890-01	-7.150-01	-1.740+00
		-4.340+00	-9.610+00	-1.710+01	-2.330+01	-2.440+01	-2.160+01	-1.610+01	-1.070+01	-6.500+00	-3.160+00
21	2.990-02	0.0	1.240+01	1.110+01	6.830+00	3.670+00	1.830+00	9.100-01	4.820-01	2.240-01	9.410-02	4.210-02	1.950-02
		9.170-03	4.380-03	2.220-03	1.080-03	5.300-04	2.600-04	1.280-04	7.820-04	7.920-04	8.190-04	8.540-04
		8.140-04	8.150-04	8.080-04	7.880-04	7.600-04	7.200-04	6.680-04	5.980-04	5.060-02	-1.700-01	-5.810-01
		-1.730+00	-4.420+00	-8.110+01	-1.090+01	-1.260+01	-2.540+01	-2.210+01	-1.690+01	-1.140+01	-5.830+00
20	4.040-02	0.0	7.150+00	6.080+00	2.270+00	6.210-01	1.660-01	4.990-02	1.740-02	6.790-03	2.870-03
		1.270-03	5.480-04	2.030-04	1.710-04	1.240-04	1.070-04	1.000-04	1.030-04	1.040-04	1.040-04	1.100-04
		1.120-04	1.050-04	9.030-05	7.160-05	5.330-05	3.600-05	2.360-03	1.800-03	1.840-02	1.140-01
		-3.030-01	-1.910+00	-5.020+00	-1.200+01	-1.900+01	-2.350+01	-2.390+01	-2.080+01	-1.540+01	-8.240+00
19	5.290-02	0.0	3.540+00	2.760+00	8.740-01	1.910-01	3.900-02	8.990-03	2.520-03	8.450-04	3.220-04
		1.340-04	5.440-05	2.910-05	1.700-05	1.240-05	1.080-05	1.030-05	1.030-05	1.040-05	1.060-05
		1.060-05	1.010-05	9.320-06	7.920-06	6.490-06	5.440-06	1.960-04	1.930-04	1.460-03	1.880-02
		-1.080-01	-3.540-01	-2.760+00	-5.060+00	-1.140+01	-1.680+01	-1.980+01	-1.930+01	-1.550+01	-8.820+00
18	6.750-02	0.0	1.440+00	1.010+00	2.720-01	4.850-02	7.670-03	1.350-03	2.990-04	8.410-05	2.860-05
		1.100-05	4.710-06	2.270-06	1.130-06	5.900-07	2.560-07	1.130-07	8.030-07	8.010-07	8.030-07
		7.990-07	7.600-07	6.930-07	5.930-07	4.730-07	3.550-07	2.870-06	1.130-05	5.040-05	2.880-04	2.070-03
		-1.640-02	-1.150-01	-5.960-01	-2.060+00	-5.020+00	-8.840+00	-1.200+01	-1.310+01	-1.130+01	-6.940+00
17	8.470-02	0.0	4.790-01	2.880-01	6.430-02	9.920-03	1.270-03	1.740-04	3.040-05	9.110-06	2.130-06
		3.740-07	1.130-07	4.490-08	1.840-08	8.170-09	4.170-09	2.540-09	1.400-09	1.310-09	1.420-09	1.550-09
		2.110-08	1.470-08	1.110-08	8.990-09	7.340-09	6.120-09	5.240-07	2.330-06	2.420-05	1.420-04	1.130-03
		-1.660-03	-1.610-02	-1.130-01	-5.120-01	-1.540+00	-3.230+00	-5.040+00	-6.050+00	-5.610+00	-3.830+00
16	1.050-01	0.0	1.200-01	6.210-02	1.240-02	1.610-03	1.750-04	1.980-05	2.830-06	5.580-07	1.490-07
		4.950-08	1.950-08	8.370-09	3.770-09	1.840-09	1.020-09	1.770-09	3.620-08	3.490-09	1.360-09
		1.220-09	1.060-09	2.070-09	2.480-09	2.370-09	1.640-08	7.100-08	6.030-07	7.360-06
		-1.060-04	-1.420-03	-1.140-02	-7.970-02	-3.000-01	-7.540-01	-1.340+00	-1.780+00	-1.800+00	-1.390+00
15	1.290-01	0.0	2.230-02	9.890-03	1.750-03	2.040-04	2.000-05	2.040-06	2.600-07	4.620-08	1.140-08
		3.580-09	1.490-09	6.970-10	3.720-10	1.980-10	1.700-10	3.550-10	3.410-10	3.270-10	3.090-10
		2.900-10	2.730-10	2.600-10	2.410-10	2.820-10	7.280-11	-2.010-10	-1.370-09	-1.440-08	-1.070-07
		-3.910-06	-6.970-05	-6.820-04	-6.870-03	-3.260-02	-9.870-02	-2.030-01	-3.020-01	-3.410-01	-3.100-01
14	1.570-01	0.0	2.970-03	1.140-03	1.830-04	2.020-05	1.910-06	1.950-07	2.550-08	4.730-09	1.200-09
		3.810-10	1.550-10	6.300-11	8.020-11	8.110-11	8.470-11	8.770-11	8.920-11	8.870-11	8.600-11
		8.120-11	7.480-11	6.820-11	5.850-11	4.220-11	2.440-11	1.010-11	-1.580-11	-3.100-10	-3.330-09
		-7.380-08	-1.640-06	-2.650-05	-2.650-04	-1.560-03	-5.890-03	-1.450-02	-2.540-02	-3.410-02	-3.810-02
13	1.900-01	0.0	3.800-04	9.390-05	1.470-05	1.590-06	1.580-07	1.810-08	2.890-09	6.700-10	2.040-10
		7.360-11	3.620-11	1.020-11	3.480-11	4.180-11	4.860-11	5.410-11	5.790-11	5.990-11	5.980-11
		2.730-10	1.270-10	4.670-10	1.960-10	2.670-10	1.570-11	-3.310-10	-8.690-10	-1.590-10	-4.680-09
		-2.020-10	-1.850-08	-2.660-07	-3.220-06	-2.360-05	-1.120-04	-3.590-04	-8.170-04	-1.470-03	-2.230-03
12	2.280-01	0.0	1.860-05	5.780-06	8.730-07	1.000-07	1.150-08	1.700-09	3.730-10	1.180-10	4.540-11
		2.000-11	1.330-11	1.630-11	2.310-11	3.090-11	3.830-11	4.450-11	4.910-11	5.200-11	5.270-11
		5.100-11	4.700-11	4.140-11	3.370-11	2.410-11	1.480-11	9.030-12	7.170-12	6.260-12	5.830-12
		3.900-12	-2.880-11	-5.640-10	-6.780-09	-5.610-08	-3.480-07	-1.540-06	-4.020-06	-1.890-05	-2.930-05
11	2.740-01	0.0	8.510-07	2.420-07	3.740-08	4.890-09	7.140-10	1.350-10	3.110-11	6.300-12	5.420-13
		4.910-14	2.830-12	8.320-12	1.410-11	2.420-11	3.140-11	3.840-11	4.360-11	4.700-11	4.830-11
		4.710-11	4.360-11	3.840-11	3.140-11	2.320-11	1.520-11	9.270-12	7.050-12	7.490-12	7.430-12

Table V sheet 16

		7.590-12	7.860-12	8.090-12	7.570-12	2.280-12	-3.470-11	-3.260-10	-3.050-09	-3.050-08	-2.710-07
10	3.280-01	0.0 -1.340-11 4.240-11 1.090-11	2.410-08 -6.410-12 5.040-11 1.220-11	5.860-09 1.510-12 1.510-11 1.370-11	9.240-10 2.460-12 2.960-11 1.910-11	1.280-10 1.790-11 1.240-11 1.620-11	1.590-12 1.580-11 1.580-11 1.660-11	-2.260-11 1.280-11 1.830-11 1.600-11	-2.770-11 3.830-11 9.080-11 1.400-11	-2.610-11 4.220-11 9.550-11 9.950-12	-2.020-11 4.400-11 8.930-11 -9.270-11
9	3.910-01	0.0 -4.350-11 3.980-11 1.970-11	-1.330-10 -1.900-11 1.700-11 1.850-11	-3.970-11 -6.690-12 1.900-11 2.170-11	-6.820-11 6.980-12 2.270-11 2.520-11	-6.900-11 1.130-11 2.200-11 2.870-11	-6.270-11 1.660-11 1.660-11 3.170-11	-5.510-11 2.690-11 1.180-11 3.390-11	-5.070-11 3.680-11 6.980-12 3.300-11	-4.440-11 3.710-11 1.150-11 2.870-11	-3.530-11 3.840-11 1.350-11 1.850-11
8	4.650-01	0.0 -4.720-11 3.500-11 2.160-11	-4.210-10 -5.960-11 3.330-11 2.610-11	-3.250-10 -1.490-11 1.090-11 3.130-11	-2.440-10 -6.850-12 2.900-11 3.740-11	-1.840-10 4.520-12 2.160-11 4.450-11	-1.380-10 1.310-11 2.150-11 5.200-11	-1.030-10 2.070-11 1.930-11 6.330-11	-8.070-11 2.690-11 1.150-11 7.700-11	-6.420-11 3.160-11 1.460-11 6.540-11	-5.000-11 3.440-11 1.800-11 1.180-10
7	5.520-01	0.0 -4.810-11 3.090-11 2.070-11	-2.660-10 -3.370-11 2.930-11 3.100-11	-2.440-10 -2.230-11 2.700-11 3.790-11	-2.090-10 -1.140-11 2.290-11 4.260-11	-1.680-10 -2.140-12 1.900-11 4.780-11	-1.330-10 6.640-12 1.810-11 5.110-11	-1.090-10 1.440-11 1.650-11 5.520-11	-9.010-11 2.090-11 1.560-11 5.920-11	-7.420-11 2.590-11 1.750-11 6.260-11	-5.960-11 2.910-11 2.240-11 6.270-11
6	6.550-01	0.0 -5.030-11 2.780-11 2.290-11	-1.490-10 -3.870-11 2.920-11 3.203-11	-1.490-10 -2.770-11 2.870-11 3.730-11	-1.480-10 -1.740-11 2.180-11 3.850-11	-1.330-10 -9.950-12 1.980-11 3.880-11	-1.180-10 6.810-13 1.840-11 3.940-11	-1.030-10 8.360-12 1.780-11 4.060-11	-8.990-11 1.490-11 1.810-11 4.200-11	-7.580-11 2.000-11 2.050-11 4.300-11	-6.270-11 2.360-11 2.310-11 4.290-11
5	7.760-01	0.0 -4.880-11 2.940-11 2.950-11	-1.100-10 -3.620-11 2.890-11 3.240-11	-1.080-10 -2.980-11 2.830-11 3.350-11	-1.040-10 -2.070-11 2.730-11 3.850-11	-9.900-11 -1.210-11 2.540-11 3.940-11	-9.250-11 -4.150-12 2.290-11 3.800-11	-8.500-11 3.960-12 1.790-11 3.110-11	-7.660-11 9.340-12 1.470-11 3.260-11	-6.770-11 1.450-11 1.940-11 3.320-11	-5.830-11 1.820-11 2.260-11 3.320-11
4	9.180-01	0.0 -4.220-11 1.560-11 2.150-11	-7.460-11 -3.340-11 1.960-11 2.280-11	-7.380-11 -2.830-11 1.870-11 2.360-11	-7.240-11 -2.110-11 1.650-11 2.440-11	-7.030-11 -1.460-11 1.630-11 2.610-11	-6.750-11 -7.230-12 1.640-11 2.760-11	-6.390-11 4.700-13 1.640-11 2.880-11	-5.960-11 8.700-12 1.730-11 2.940-11	-5.440-11 1.430-11 1.860-11 2.960-11	-4.860-11 1.910-11 1.940-11 2.960-11
3	1.090+00	0.0 -3.270-11 1.100-11 1.730-11	-4.270-11 -2.310-11 1.230-11 1.800-11	-4.230-11 -1.960-11 1.240-11 1.870-11	-4.070-11 -1.050-11 1.320-11 1.940-11	-4.780-11 -1.340-11 1.340-11 1.940-11	-4.650-11 -8.140-12 1.370-11 2.050-11	-4.480-11 -3.200-12 1.420-11 2.090-11	-4.240-11 1.380-12 1.490-11 2.120-11	-3.980-11 5.390-12 1.840-11 2.130-11	-3.650-11 8.640-12 1.650-11 2.140-11
2	1.280+00	0.0 -2.200-11 6.590-12 1.220-11	-3.050-11 -1.950-11 6.130-12 1.270-11	-3.040-11 -1.670-11 6.910-12 1.300-11	-3.010-11 -1.030-11 6.360-12 1.330-11	-2.970-11 -1.040-11 6.680-12 1.350-11	-2.910-11 -7.010-12 7.090-12 1.370-11	-2.830-11 -3.620-12 7.040-12 1.390-11	-2.730-11 5.680-13 7.090-12 1.400-11	-2.580-11 3.520-12 7.130-12 1.410-11	-2.400-11 6.960-12 7.180-12 1.410-11
1	1.510+00	0.0 -1.090-11 3.240-12 6.320-12	-1.420-11 -9.690-12 3.470-12 6.440-12	-1.430-11 -8.370-12 3.540-12 6.540-12	-1.420-11 -6.170-12 3.750-12 6.760-12	-1.410-11 -6.390-12 3.810-12 6.850-12	-1.380-11 -4.000-12 3.930-12 6.930-12	-1.350-11 -1.330-12 3.970-12 6.990-12	-1.300-11 1.930-12 3.980-12 6.970-12	-1.250-11 4.340-12 3.980-12 6.930-12	-1.170-11 6.130-12 3.980-12 6.860-12

Table V sheet 17

DELTA(41,3Ø) -- The new residual ($v^{(m+1)} - \psi^{(m)}$) calculated by Stone's method (Gummel's δ). See Sections 4.01 and 4.02 of Chapter I.

XPOS(41) - The x-coordinate corresponding to the ψ-lattice index I.

YPOS(25) -- The y-coordinate corresponding to the ψ-lattice index J.

XTHPOS(41) -- The x-coordinate corresponding to the θ-lattice index I.

YTHPOS(25) -- The y-coordinate corresponding to the θ-lattice index J.

3.02 COMMON/COEFF/

XDLT(42) -- The ψ-lattice spacing $\Delta x(I)$.

YDLT(31) -- The ψ-lattice spacing $\Delta y(J)$.

XTHDLT(41) -- The θ-lattice spacing $\tilde{\Delta}x(I)$.

YTHDLT(25) -- The θ-lattice spacing $\tilde{\Delta}y(J)$.

HNRTH(41,30) -- The "north" coefficient $\tilde{\Delta}x(I)/\Delta y(J+1)$, appearing in eq. (23) as H_p.

HEAST(41,30) -- The "east" coefficient $\tilde{\Delta}y(J)/\Delta x(I+1)$, appearing in eq. (23) as F_p.

HSTH(41,30) -- The "south" coefficient $\tilde{\Delta}x(I)/\Delta y(J)$, appearing in eq. (23) as B_p.

HWEST(41,30) -- The "west" coefficient $\tilde{\Delta}y(J)/\Delta x(I)$, appearing in eq. (23) as D_p.

HCNTR(41,30) -- The "center" coefficient appearing in eq. (23) as E_p.

THNRTH(4Ø,25) -- The "north" coefficient $-\Delta x(I+1)/\tilde{\Delta}y(J+1)$ appearing in eq. (40). (The minus sign is used to make the matrix involved in the block-line iteration for θ positive-definite -- a crucial factor for convergence of that iterative scheme.)

THEAST(4Ø,25) -- The "east" coefficient $-\Delta y(J+1)/\tilde{\Delta}x(I+1)$ appearing in eq. (40). (Same reason for the minus sign.)

THSTH(4Ø,25) -- The "south" coefficient $-\Delta x(I+1)/\tilde{\Delta}y(J)$ appearing in eq. (40). (Same reason for the minus sign.)

THWEST(4Ø,25) -- The "west" coefficient $-\Delta y(J+1)/\tilde{\Delta}x(I)$ appearing in eq. (40). (Same reason for the minus sign.)

QAREA(41,25) -- The area $\tilde{\Delta}x(I)\tilde{\Delta}y(J)$ associated with the ψ-lattice point (I,J), for purposes of calculating the charge contained within that "box."

3.03 COMMON/REAL/

XCHNL -- The channel length (spacing between the source and drain "contacts" in the x-direction).

YSUB -- The substrate thickness in the y-direction.

YOXIDE -- The oxide thickness.

YJNCTN -- The source and drain junction depths in the y-direction.

DONOR -- The donor density in the source and drain.

ACCPTR -- The acceptor density in the substrate region (assumed independent of x and y).

VGATE -- The applied gate voltage.

VSUB -- The applied substrate (back gate) voltage (assumed always to be negative, or zero).

VDRAIN -- The applied drain voltage.

DIELK -- The ratio of dielectric constants (κ_{ox}/κ_{si}).

QSS -- The surface charge density at the oxide-silicon interface. (This can account for the metal-oxide work function difference ϕ_{MS}, as discussed in Section 1.01 of this chapter.)

CURRNT -- The drain current factor J_o discussed in Section 2.05 of Chapter I.

EDENSØ -- The electron density n in the charge-neutral region of the silicon substrate.

PSIEQ -- The electrostatic potential of the charge-neutral region of the substrate (PSIEQ = $-\ell n(N_D N_A) + V_{SUB}$).

3.04 COMMON/INTGR/

IMAX -- Maximum of the index I definiting the x-coordinate.

JMAX -- Maximum of the index J defining the y-coordinate within the silicon substrate.

JMAX5 -- Maximum of the index J defining the y-coordinate within both the silicon substrate and the oxide region (JMAX5 = JMAX + 5).

JUNCTN -- The J index which defines the lower corner of the source and drain "contacts."

ITHMAX -- Maximum of the index I for the θ-lattice (ITHMAX = IMAX - 1).

3.05 COMMON/NRMLZE/

DNSNRM -- Electron and hole density normalization parameter.
PSINRM -- Electrostatic potential normalization parameter.
DEBYE -- Intrinsic Debye length.
QNORM -- Surface charge normalization parameter.
CRTNRM -- Current density normalization parameter.

3.06 COMMON/CNTRL/

OMEGA -- The overrelaxation parameter Ω used for over-relaxation with Gummel's algorithm.

CHAPTER IV

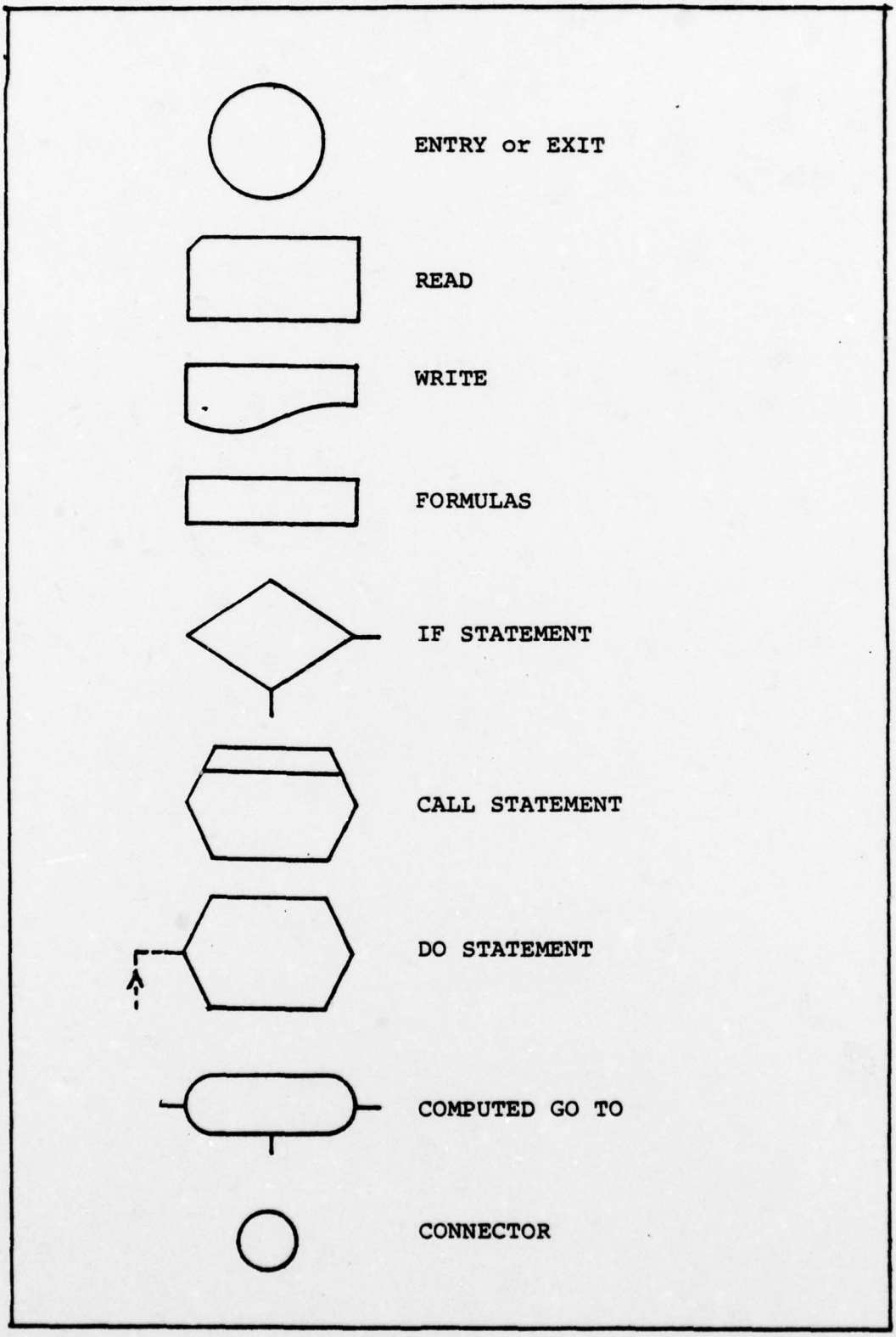
Program Listing, with Flowcharts

With the 41x30 ψ-lattice dimensions specified in the DIMENSION statements incorporated in the program, roughly 336,000 Bytes of core storage are required. The program consists of a main executive program, together with 21 FUNCTION and SUBROUTINE subprograms. (Two of the subprograms, GRELAX, and STONE) are embedded as ENTRY statements in other subprograms, viz. RELAX and STONE.)

Each subprogram is liberally sprinkled with COMMENT statements which, together with the flowcharts provided herein should make the functioning of that subprogram fairly transparent to a reader skilled in FORTRAN IV. We have consistently operated the program using IBM's Level H Extended, Optimization 2, Compiler.

In this chapter, section by section, we provide a brief description of each subprogram, together with a listing, and detailed flowcharts. Regarding the latter, Table VI defines the convention adopted for displaying the various FORTRAN operations in graphic form.

TABLE VI -- SYMBOLS APPEARING IN THE FLOWCHARTS



1.0 MAIN

This is the executive program which controls the functioning of the overall program. It reads the input data in unnormalized form, records them for future reference by the user, normalizes them, then controls the sequence of operations required to trace out characteristic curves of I_D versus V_{DS}, with V_{GS} varied as a parameter. By means of the parameter KRSWCH, it implements either Mock's iterative procedure -- Picard iteration at reduced donor density for coarse convergence, followed by Gummel's algorithm, with full donor density for tight convergence -- or Gummel's algorithm throughout.

```
VEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
 NS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
               SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

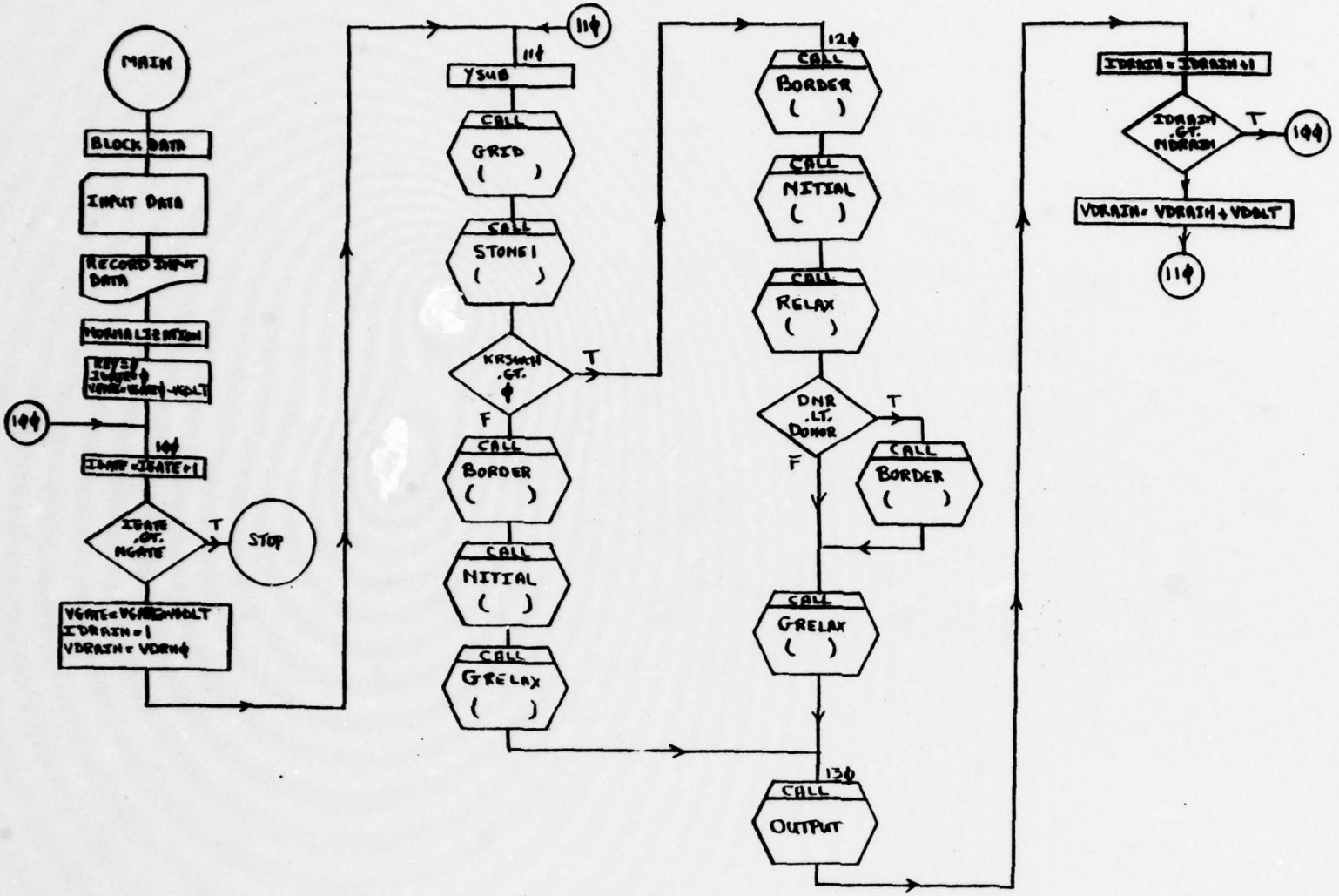
                C      MAIN EXECUTIVE PROGRAM FOR MOSFET ANALYSIS.
                C
                C
                C
                C      INITIALIZE ALL ARRAYS TO ZERO WITH A BLOCK DATA SUBPROGRAM
ISN 0002                BLOCK DATA
ISN 0003                IMPLICIT REAL*8(A-H,O-Z)
ISN 0004                COMMON/ARRAYS/ADUMMY
ISN 0005                COMMON/COEFF/CDUMMY
ISN 0006                REAL*8 ADUMMY(13022)/13022*0.D0/
ISN 0007                REAL*8 CDUMMY(11314)/11314*0.D0/
ISN 0008                END
```

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
ED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

 IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
            SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NC

0002              IMPLICIT REAL*8(A-H,O-Z)
0003              REAL TITLE
          C
0004              COMMON/ARRAYS/PSI(41,30),CDENS(41,25),HDENS(41,25),THETA(40,25),
                 1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),U(41,30),DELTA(41,30
                 2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30),
                 1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                 2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                 1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
0007              COMMON/INTGR/IMAX,JMAX,JMAXS,JUNCTN,ITHMAX
0008              COMMON/NRMLZE/DNSNRM,PSINRM,DEBYE,QNORM,CRTNRM
0009              COMMON/CNTRL/ OMEGA
          C
0010              DIMENSION TITLE(54)
          C
0011              NAMELIST/PRMTRS/  XCHNL,YOXIDE,YJNCTN,DONOR,ACCPTR,QSS,EMOB,
                 1XDELTA,YDELTA,IMAX,JMAX
0012              NAMELIST/VOLTGE/VGDLT,VGATE0,VDDLT,VDRN0,VSUB,NGATE,NDRAIN
0013              NAMELIST/CONTRL/CNVRG1,CNVRG2,SIDLT1,SIDLT2,THDLT1,THDLT2,
                 1ITMAX,ITSIMX,ITTHMX,KRS#CH,OMEGA
          C
          C
0014              READ(5,PRMTRS)
0015              READ(5,VOLTGE)
0016              READ(5,CONTRL)
0017              READ(5,1000)(TITLE(I),I=1,54)
0018         1000 FORMAT(18A4/18A4/18A4)
0019              WRITE(6,1010)(TITLE(I),I=1,54)
0020         1010 FORMAT(1H0,30X,18A4,/,30X,18A4,/,30X,18A4,///,5X,'SUMMARY OF INPU
                 1DATA :',//)
0021              WRITE(6,1020) XCHNL,YOXIDE,YJNCTN,DONOR,ACCPTR,QSS,EMOB,
                 1XDELTA,YDELTA,IMAX,JMAX
0022         1020 FORMAT(1H0,5X,'NAMELIST/PRMTRS/ XCHNL=',1PD11.3,2X,'YOXIDE=',
                 11PD11.3,2X,'YJNCTN=',1PD11.3,2X,'DONOR=',1PD11.3,2X,'ACCPTR=',
                 21PD11.3,/,5X,'QSS=',1PD11.3,2X,'EMOB=',1PD11.3,2X,'XDELTA=',
                 31PD11.3,2X,'YDELTA=',1PD11.3,2X,'IMAX=',I3,2X,'JMAX=',I3)
0023              WRITE(6,1030) VGDLT,VGATE0,VDDLT,VDRN0,VSUB,NGATE,NDRAIN
0024         1030 FORMAT(1H0,5X,'NAMELIST/VOLTGE/ VGDLT=',1PD11.3,2X,'VGATE0=',
                 11PD11.3,2X,'VDDLT=',1PD11.3,2X,'VDRN0=',1PD11.3,2X,'VSUB=',
                 21PD11.3,/,2X,'NGATE=',I3,2X,'NDRAIN=',I3)
0025              WRITE(6,1040)CNVRG1,CNVRG2,SIDLT1,SIDLT2,THDLT1,THDLT2,ITMAX,
                 1ITSIMX,ITTHMX,KRS#CH,OMEGA
0026         1040 FORMAT(1H0,5X,'NAMELIST/CONTRL/ CNVRG1=',1PD11.3,2X,'CNVRG2=',
                 11PD11.3,2X,'SIDLT1=',1PD11.3,2X,'SIDLT2=',1PD11.3,/,5X,'THDLT1=',
                 21PD11.3,2X,'THDLT2=',1PD11.3,2X,'ITMAX=',I3,2X,'ITSIMX=',I3,
                 32X,'ITTHMX=',I3,2X,'KRS#CH=',I3,2X,'OMEGA=',1PD11.3,///)
0027              JMAXS=JMAX+5
          C
          C       CALCULATE NORMALIZATION PARAMETERS & NORMALIZE ALL DATA.
          C       NORMALIZATIONS ASSUME THE FOLLOWING UNITS FOR INPUT & OUTPUT DATA
          C       POTENTIALS-----(VOLTS)
          C       LENGTHS-----(MICRONS)
          C       DONOR & ACCEPTOR DENSITIES-----(CM**-3)
          C       MOBILITY-----(CM**2/VOLT-SEC)
          C       SURFACE CHARGE DENSITY-----(COULOMBS/CM**2)
          C       CURRENT DENSITY-----(AMPERES/CM**2)
          C
0028              DEBYE=(11.7D0*8.854D-14*2.585D-2)/(1.602D-19*1.5D10)
0029              DEBYE=DSQRT(DEBYE)
0030              QNORM=2.585D-2*11.7D0*8.854D-14/DEBYE
0031              CRTNRM=2.585D-2*1.5D10*1.602D-19*EMOB/DEBYE
0032              DEBYE=DEBYE*1.D4
0033              DNSNRM=1.5D10
0034              PSINRM=2.585D-2
          C
```

```
ISN 0035              XCHNL=XCHNL/DEBYE
ISN 0036              YOXIDE=YOXIDE/DEBYE
ISN 0037              YJNCTN=YJNCTN/DEBYE
    0038              DONOR=DONOR/DNSNRM
    0039              ACCPTR=ACCPTR/DNSNRM
ISN 0040              QSS=QSS/QNORM
ISN 0041              VGDLT=VGDLT/PSINRM
ISN 0042              VGATE0=VGATE0/PSINRM
ISN 0043              VDDLT=VDDLT/PSINRM
ISN 0044              VDRN0=VDRN0/PSINRM
ISN 0045              VSUB=VSUB/PSINRM
ISN 0046              DIELK=3.900/11.700
ISN 0047              KEY=0
ISN 0048              IGATE=0
ISN 0049              VGATE=VGATE0-VGDLT
                C
                C     LOOP STARTS HERE. GATE VOLTAGE IS HELD FIXED WHILE DRAIN VOLTAGE
                C     IS INCREMENTED THROUGH THE SPECIFIED RANGE, STARTING AT VD=VDRN0.
                C     THEN GATE VOLTAGE IS INCREMENTED AND THE PROCESS IS REPEATED.
                C
ISN 0050        100   IGATE=IGATE+1
ISN 0051              IF(IGATE.GT.NGATE) STOP
ISN 0053              VGATE=VGATE+VGDLT
ISN 0054              IDRAIN=1
ISN 0055              VDRAIN=VDRN0
ISN 0056        110   YSUB=DSQRT(2.D0*(DLOG(DONOR*ACCPTR)+VDRAIN-VSUB-1.D0)/ACCPTR)
ISN 0057              YSUB=YJNCTN+1.50*YSUB
ISN 0058              CALL GRID(XDELTA,YDELTA,KEY)
ISN 0059              CALL STORE1(IMAX,JMAX,JMAX5)
                C
                C     IF KRSWCH.EQ.0, APPLY ONLY THE GUMMEL ALGORITHM, WITH FULL DONOR
                C     DENSITY AND TIGHT CONVERGENCE CRITERIA. IF KRSWCH.GT.0, APPLY PICAR
                C     ITERATION FIRST, WITH LOOSE CONVERGENCE CRITERIA AND REDUCED DONO
                C     DENSITY (IF APPLICABLE), FOLLOWED BY THE GUMMEL ALGORITHM, WITH
                C     TIGHT CONVERGENCE CRITERIA AND FULL DONOR DENSITY.
                C
ISN 0060              IF(KRSWCH.GT.0) GO TO 120
ISN 0062              CALL BORDER(0.1,DNR)
    0063              CALL NITIAL(DNR)
    0064              CALL GRELAX(ITMAX,ITSIMX,ITTHMX,CNVRG2,SIDLT2,THDLT2,DNR)
ISN 0065              GO TO 130
ISN 0066        120   CALL BORDER(1.0,DNR)
ISN 0067              CALL NITIAL(DNR)
ISN 0068              CALL RELAX(ITMAX,ITSIMX,ITTHMX,CNVRG1,SIDLT1,THDLT1,DNR)
ISN 0069              IF(DNR.LT.DONOR) CALL BORDER(1.1,DNR)
ISN 0071              CALL GRELAX(ITMAX,ITSIMX,ITTHMX,CNVRG2,SIDLT2,THDLT2,DNR)
ISN 0072        130   CALL OUTPUT
                C
                C     STEP DRAIN VOLTAGE & REPEAT UNLESS MAX DRAIN VOLTAGE WAS USED.
                C     IF SO, STEP GATE VOLTAGE & REINITIATE DRAIN VOLTAGE SEQUENCE.
                C
ISN 0073              IDRAIN=IDRAIN+1
ISN 0074              IF(IDRAIN.GT.NDRAIN) GO TO 100
ISN 0076              VDRAIN=VDRAIN+VDDLT
ISN 0077              GO TO 110
ISN 0078              END
```



MAIN SHEET 1 of 1

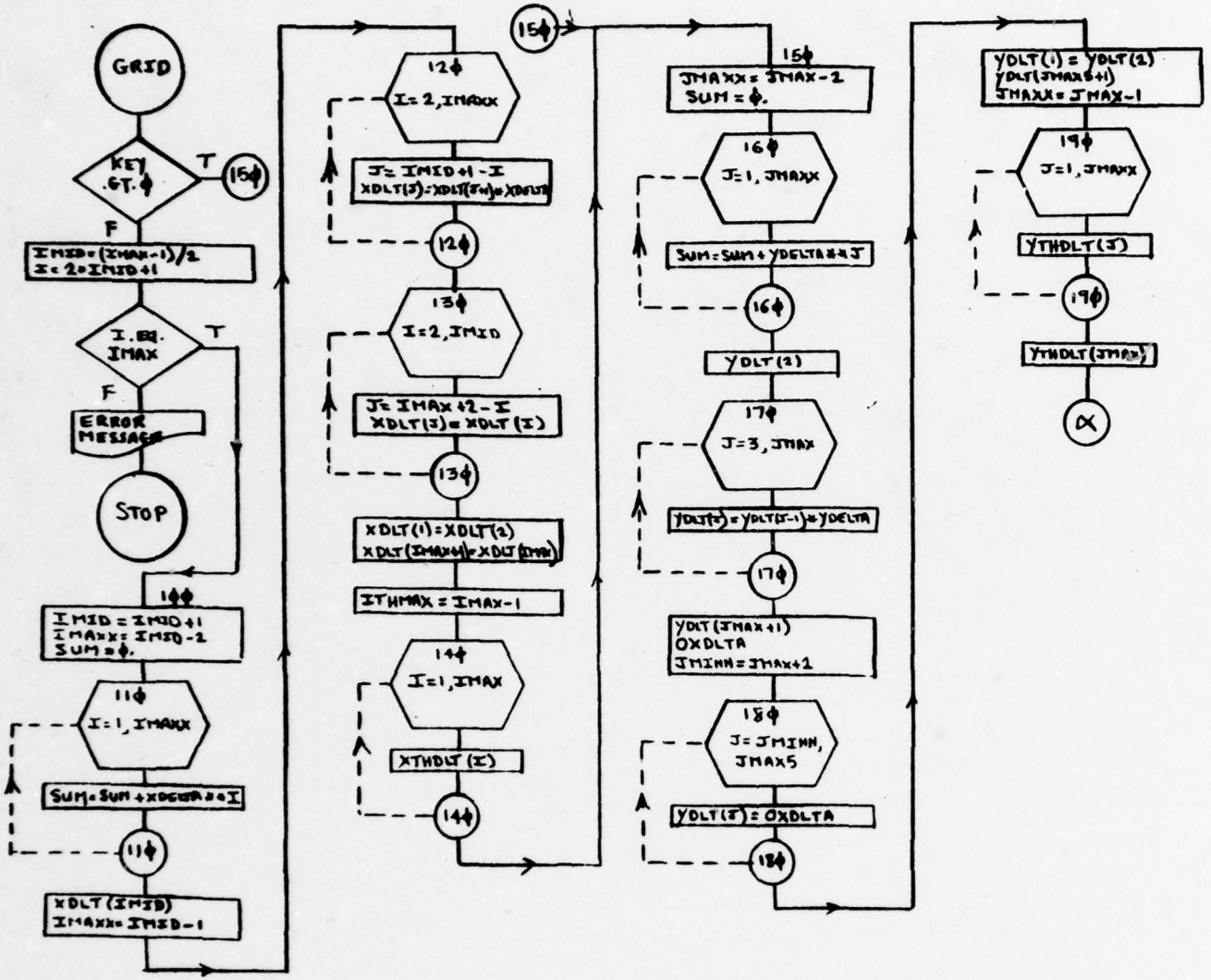
2.0 GRID

This subprogram establishes the ψ and θ-lattice spacings $\Delta x(I)$, $\Delta y(J)$, $\tilde{\Delta} x(I)$, $\tilde{\Delta} y(J)$, following the procedure devised by Mock [1] for generating the graded lattice illustrated in Figure 2 of Chapter I. It then calculates the finite-difference coefficients required in the implementation of Stone's method and of Block-Line iteration. The special procedures for treating the boundary conditions at the oxide-silicon interface, described in Chapter I, Section 3.01.1, are implemented in this subprogram.

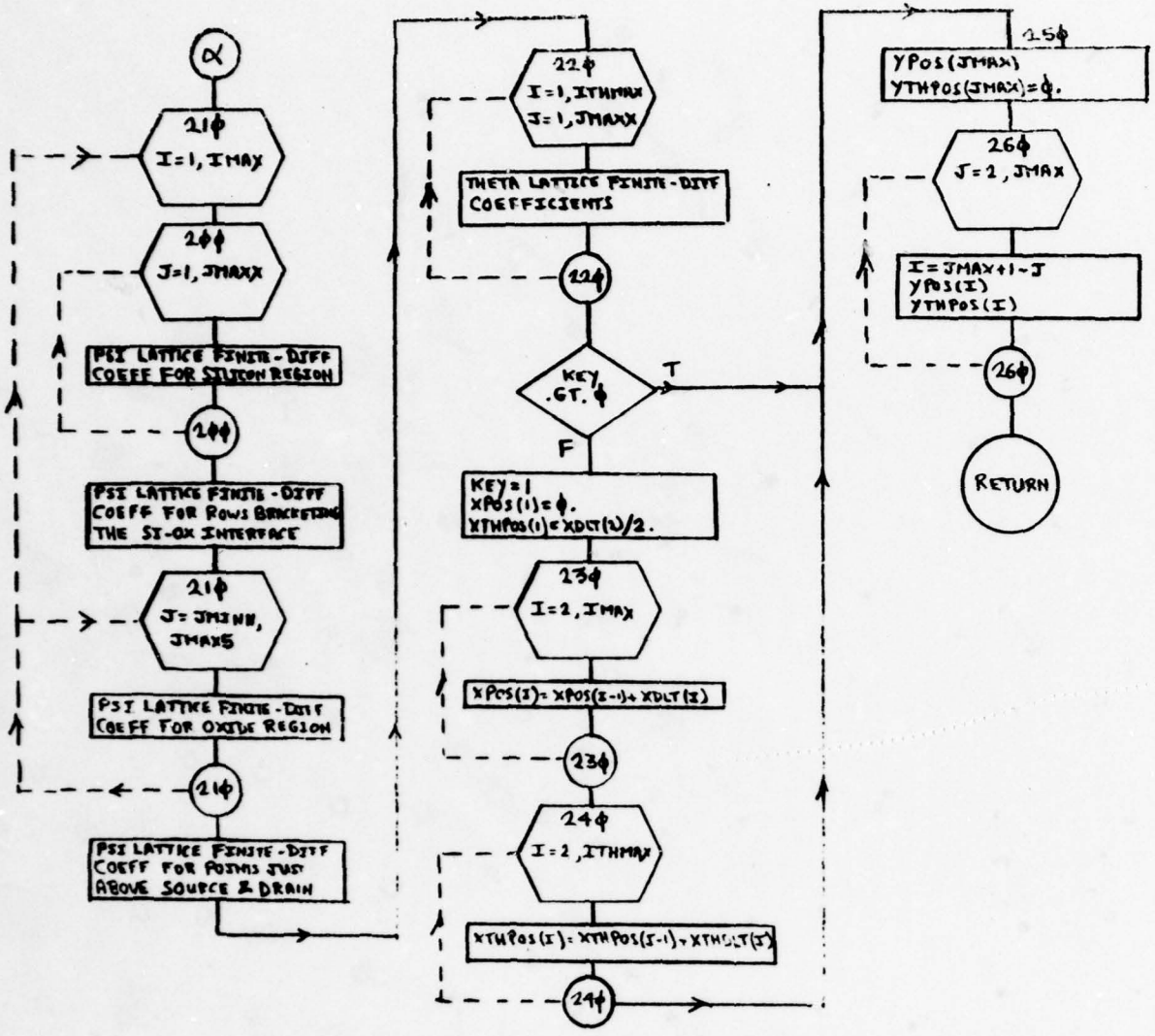
```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO
0002              SUBROUTINE GRID(XDELTA,YDELTA,KEY)
                  C
                  C     SETS UP LATTICE PARAMETERS FOR PSI & THETA. CALCULATES COEFF FOR
                  C     THE FINITE-DIFFERENCE EQUATIONS.
                  C
0003              IMPLICIT REAL*8(A-H,O-Z)
                  C
0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                 1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,3
                 2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30
                 1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25)
                 2THEAST(40,25),THSTH(40,25),THWEST(40,25),QAREA(41,25)
0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                 1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
0007              COMMON/INTGR/IMAX,JMAX,JMAXS,JUNCTN,ITHMAX
                  C
                  C
                  C     BY-PASS XDLT & XTHDLT COMPUTATION IF KEY GT 0.
                  C
0008              IF(KEY.GT.0) GO TO 150
0010              IMID=(IMAX-1)/2
0011              I=2*IMID+1
0012              IF(I.EQ.IMAX) GO TO 100
0014              WRITE(6,1000)
0015         1000 FORMAT(1H1,5X,'ERROR IN INPUT DATA. PARAMETER IMAX MUST BE ODD',
0016              STOP
0017          100 IMID=IMID+1
0018              IMAXX=IMID-2
0019              SUM=0.
                  C
                  C     HORIZONTAL LATTICE PARAMETERS
                  C
0020              DO 110 I=1,IMAXX
0021          110 SUM=SUM+XDELTA**I
0022              XDLT(IMID)=XCHNL/(2.D0*(1.D0+SUM))
0023              IMAXX=IMID-1
0024              DO 120 I=2,IMAXX
0025              J=IMID+1-I
0026          120 XDLT(J)=XDLT(J+1)*XDELTA
0027              DO 130 I=2,IMID
0028              J=IMAX+2-I
0029          130 XDLT(J)=XDLT(I)
0030              XDLT(1)=XDLT(2)
0031              XDLT(IMAX+1)=XDLT(IMAX)
                  C
                  C     THETA LATTICE
                  C
0032              ITHMAX=IMAX-1
0033              DO 140 I=1,IMAX
0034          140 XTHDLT(I)=(XDLT(I)+XDLT(I+1))/2.D0
                  C
                  C     VERTICAL LATTICE PARAMETERS
                  C
0035          150 JMAXX=JMAX-2
0036              SUM=0.
0037              DO 160 J=1,JMAXX
0038          160 SUM=SUM+YDELTA**J
0039              YDLT(2)=YSUB/(1.D0+SUM)
0040              DO 170 J=3,JMAX
0041          170 YDLT(J)=YDLT(J-1)*YDELTA
                  C
                  C     OXIDE REGION
                  C
0042              YDLT(JMAX+1)=YDLT(JMAX)*YDELTA
0043              OXDLTA=(YOXIDE-YDLT(JMAX+1)*DIELK/(1.D0+DIELK))/4.D0
0044              JMINN=JMAX+2
```

```
EL 2.1  ( JAN 75 )            GRID            OS/360  FORTRAN H EXTENDED              DAT
SN 0045                 DO 180 J=JMINN,JMAX5
SN 0046               180 YDLT(J)=DXDLTA
SN 0047                 YDLT(1)=YDLT(2)
   0048                 YDLT(JMAX5+1)=YDLT(JMAX5)
   0049                 JMAXX=JMAX-1
                C
                C       THETA LATTICE
                C
SN 0050                 DO 190 J=1,JMAXX
SN 0051               190 YTHDLT(J)=(YDLT(J)+YDLT(J+1))/2.D0
SN 0052                 YTHDLT(JMAX)=YDLT(JMAX)/2.D0+YDLT(JMAX+1)/(1.D0+DIELK)
                C
                C       FINITE-DIFFERENCE COEFFICIENTS
                C
SN 0053                 DO 210 I=1,IMAX
                C
                C       SILICON REGION
                C
SN 0054                 DO 200 J=1,JMAXX
SN 0055                 GAREA(I,J)=XTHDLT(I)*YTHDLT(J)
SN 0056                 HSTH(I,J)=XTHDLT(I)/YDLT(J)
SN 0057                 HWEST(I,J)=YTHDLT(J)/XDLT(I)
SN 0058                 HEAST(I,J)=YTHDLT(J)/XDLT(I+1)
SN 0059                 HNRTH(I,J)=XTHDLT(I)/YDLT(J+1)
SN 0060               200 HCNTR(I,J)=-(HSTH(I,J)+HWEST(I,J)+HEAST(I,J)+HNRTH(I,J))
                C
                C       THE TWO ROWS BRACKETING THE SI-OX INTERFACE
                C
SN 0061                 J=JMAX
SN 0062                 GAREA(I,J)=XTHDLT(I)*(YDLT(J)/2.D0+YDLT(J+1)/(1.D0+DIELK))
SN 0063                 HSTH(I,J)=XTHDLT(I)/YDLT(J)
SN 0064                 HWEST(I,J)=YTHDLT(J)/XDLT(I)
SN 0065                 HEAST(I,J)=YTHDLT(J)/XDLT(I+1)
SN 0066                 HNRTH(I,J)=XTHDLT(I)/(2.D0*YDLT(J+1)/(1.D0+DIELK))
SN 0067                 HCNTR(I,J)=-(HSTH(I,J)+HWEST(I,J)+HEAST(I,J)+HNRTH(I,J))
SN 0068                 J=JMAX+1
SN 0069                 AVG=YDLT(J+1)/2.D0+YDLT(J)*DIELK/(1.D0+DIELK)
   0070                 HSTH(I,J)=XTHDLT(I)/(2.D0*YDLT(J)*DIELK/(1.D0+DIELK))
   0071                 HWEST(I,J)=AVG/XDLT(I)
SN 0072                 HEAST(I,J)=AVG/XDLT(I+1)
SN 0073                 HNRTH(I,J)=XTHDLT(I)/YDLT(J+1)
SN 0074                 HCNTR(I,J)=-(HSTH(I,J)+HWEST(I,J)+HEAST(I,J)+HNRTH(I,J))
                C
                C       OXIDE REGION
                C
SN 0075                 DO 210 J=JMINN,JMAX5
SN 0076                 AVG=(YDLT(J)+YDLT(J+1))/2.D0
SN 0077                 HSTH(I,J)=XTHDLT(I)/YDLT(J)
SN 0078                 HWEST(I,J)=AVG/XDLT(I)
SN 0079                 HEAST(I,J)=AVG/XDLT(I+1)
SN 0080                 HNRTH(I,J)=XTHDLT(I)/YDLT(J+1)
SN 0081               210 HCNTR(I,J)=-(HSTH(I,J)+HWEST(I,J)+HEAST(I,J)+HNRTH(I,J))
                C
                C       MODIFY THE SOUTH COEFFICIENTS FOR THE TWO POINTS JUST NORTH OF
                C       THE INTERFACE AT THE SOURCE & DRAIN CONTACTS.
                C
SN 0082                 HSTH(1,JMAX+1)=XTHDLT(1)/(YDLT(JMAX+1)*DIELK/(1.D0+DIELK))
SN 0083                 HSTH(IMAX,JMAX+1)=HSTH(1,JMAX+1)
SN 0084                 HCNTR(1,JMAX+1)=-(HSTH(1,JMAX+1)+HWEST(1,JMAX+1)+HNRTH(1,JMAX+1)
                       1+HEAST(1,JMAX+1))
SN 0085                 HCNTR(IMAX,JMAX+1)=HCNTR(1,JMAX+1)
                C
                C       THETA COEFFICIENTS
                C
SN 0086                 DO 220 I=1,ITHMAX
SN 0087                 DO 220 J=1,JMAXX
SN 0088                 THSTH(I,J)=-XDLT(I+1)/YTHDLT(J)
SN 0089                 THWEST(I,J)=-YDLT(J+1)/XTHDLT(I)
SN 0090                 THEAST(I,J)=-YDLT(J+1)/XTHDLT(I+1)
SN 0091               220 THNRTH(I,J)=-XDLT(I+1)/YTHDLT(J+1)
                C
                C       TABULATE MAT & THETA LATTICE COORDINATES.
```

```
      C
0092        IF(KEY.GT.0) GO TO 250
0094        KEY=1
0095        XPOS(1)=0.
0096        XTHPOS(1)=XDLT(2)/2.00
0097        DO 230 I=2,IMAX
0098    230 XPOS(I)=XPOS(I-1)+XDLT(I)
0099        DO 240 I=2,ITHMAX
0100    240 XTHPOS(I)=XTHPOS(I-1)+XTHDLT(I)
0101    250 YPOS(JMAX)=YDLT(JMAX+1)/(1.00+DIELK)
0102        YTHPOS(JMAX)=0.
0103        DO 260 J=2,JMAX
0104        I=JMAX+1-J
0105        YPOS(I)=YPOS(I+1)+YDLT(I+1)
0106    260 YTHPOS(I)=YTHPOS(I+1)+YTHDLT(I+1)
0107        RETURN
0108        END
```



GRID SHEET 1 of 2



GRID SHEET 2 of 2

3.0 BORDER

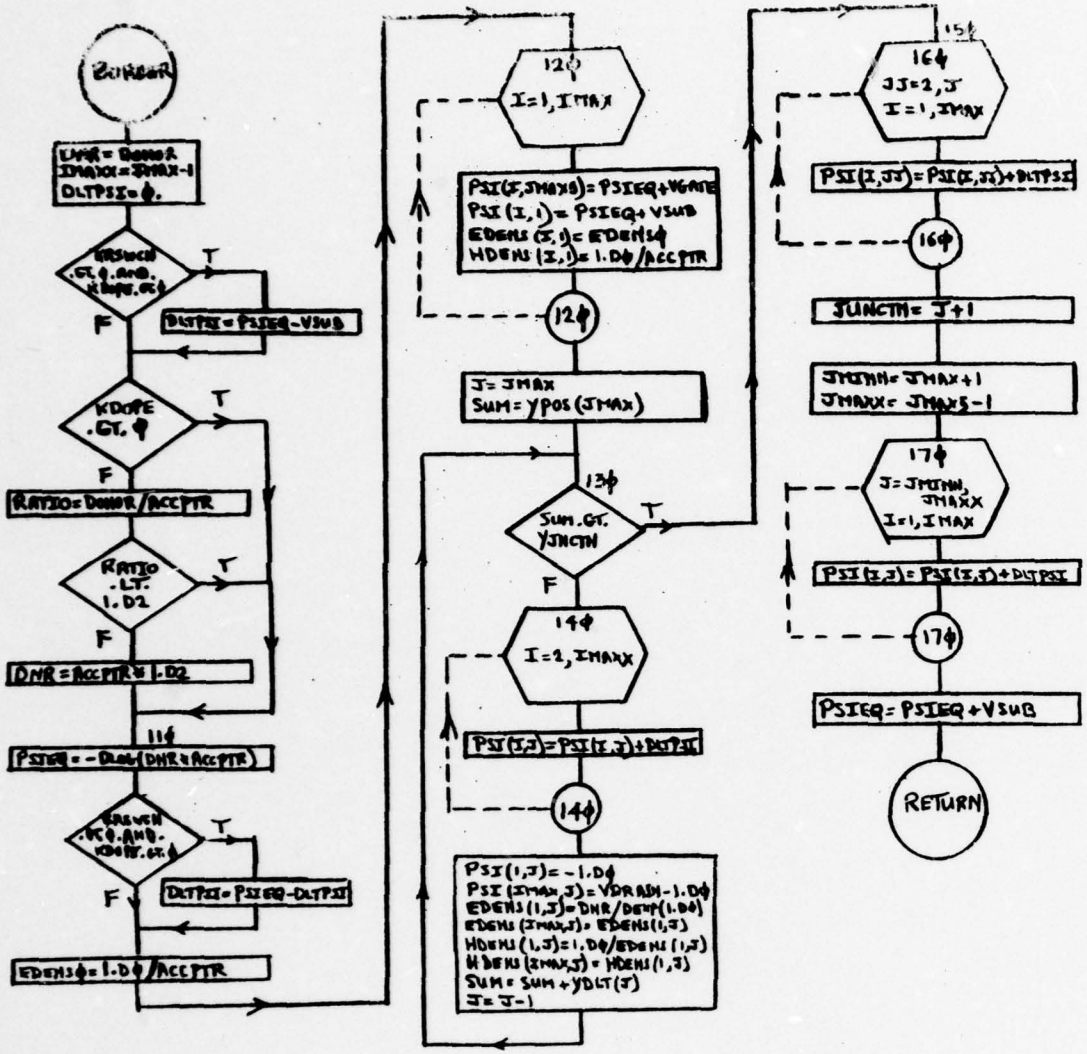
This subprogram assigns the boundary conditions on ψ, n, and p at the borders of the lattice. Since the substrate potential ($\psi_{SUB} = -\ell n(N_D N_A) + V_{SUB}$) depends upon the donor density N_D, and since the latter is generally a reduced value if the Picard iteration method has been used (i.e., if Mock's iterative procedure is being employed) it is necessary to shift the level of the array PSI(I,J), as determined from convergence of the Picard iteration, to account for the increased donor density which will next be used for Gummel's algorithm. This is done by BORDER, the flags KRSWCH and KDOPE determining whether this is necessary.

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
EL OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

 IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
            SOURCE   EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NC

0002              SUBROUTINE BORDER(KRS*CH,KDOPE,DNR)
                  C
                  C     SETS BOUNDARY CONDITIONS ALONG THE FOUR BORDERS. IF REDUCED
                  C     DONOR DENSITY WAS PREVIOUSLY USED (KRS*CH.EQ.1.AND.KDOPE.GT.0),
                  C     MODIFIES THE POTENTIAL PREVIOUSLY OBTAINED BY CONVERGENCE
                  C     OF RELAX TO ACCOUNT FOR THE CHANGE IN SUBSTRATE POTENTIAL DUE
                  C     TO THE INCREASE IN DONOR DENSITY TO ITS CORRECT VALUE.
                  C
0003              IMPLICIT REAL*8(A-H,O-Z)
                  C
0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                 1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,30
                 2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                 1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                 2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                 1VDRAIN,DIELK,GSS,CURRNT,EDENS0,PSIEQ
0007              COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                  C
                  C     KDOPE .EQ.0 SIGNIFIES USE REDUCED DONOR DOPING. KDOPE .EQ.1
                  C     SIGNIFIES USE FULL DOPING.
                  C
0008              DNR=DONOR
0009              IMAXX=IMAX-1
0010              DLTPSI=0.
0011              IF(KRS*CH.GT.0.AND.KDOPE.GT.0) DLTPSI=PSIEQ-VSUB
0013              IF(KDOPE.GT.0) GO TO 110
0015              RATIO=DONOR/ACCPTR
0016              IF(RATIO.LT.1.D2) GO TO 110
0018              DNR=ACCPTR*1.D2
0019           110 PSIEQ=-DLOG(DNR*ACCPTR)
0020              IF(KRS*CH.GT.0.AND.KDOPE.GT.0) DLTPSI=PSIEQ-DLTPSI
0022              EDENS0=1.D0/ACCPTR
                  C
                  C     THE HORIZONTAL BORDERS
                  C
0023              DO 120 I=1,IMAX
0024              PSI(I,JMAX5)=PSIEQ+VGATE
0025              PSI(I,1)=PSIEQ+VSUB
0026              EDENS(I,1)=EDENS0
0027           120 HDENS(I,1)=ACCPTR
                  C
                  C     REGION BORDERED BY THE SOURCE & DRAIN.
                  C
0028              J=JMAX
0029              SUM=YPOS(JMAX)
0030           130 IF(SUM.GT.YJNCTN) GO TO 150
0032              DO 140 I=2,IMAXX
0033           140 PSI(I,J)=PSI(I,J)+DLTPSI
0034              PSI(1,J)=-1.D0
0035              PSI(IMAX,J)=-1.D0+VDRAIN
0036              EDENS(1,J)=DNR/DEXP(1.D0)
0037              EDENS(IMAX,J)=EDENS(1,J)
0038              HDENS(1,J)=1.D0/EDENS(1,J)
0039              HDENS(IMAX,J)=HDENS(1,J)
0040              SUM=SUM+YDLT(J)
0041              J=J-1
0042              GO TO 130
                  C
                  C     THE REGION NOT BORDERED BY THE SOURCE & DRAIN
                  C
0043           150 DO 160 JJ=2,J
0044              DO 160 I=1,IMAX
0045           160 PSI(I,JJ)=PSI(I,JJ)+DLTPSI
0046              JUNCTN=J+1
                  C
```

```
                C       THE OXIDE REGION
                C
     0047               JMINN=JMAX+1
     0048               JMAXX=JMAX5-1
ISN  0049               DO 170 J=JMINN,JMAXX
ISN  0050               DO 170 I=1,IMAX
ISN  0051           170 PSI(I,J)=PSI(I,J)+DLTPSI
ISN  0052               PSIEG=PSIEG+VSUB
ISN  0053               RETURN
ISN  0054               END
```



BORDER SHEET 1 of 1

4.0 NITIAL

This subprogram calculates the initial estimates for $\psi(x,y)$ and $\theta(x,y)$. It first assigns $\psi = -\ell n(N_D N_A) + V_{SUB}$ throughout the silicon substrate, then overwrites this with the potential determined from depletion theory to create space-charge "sheaths" in contact with the source and drain. (An interpolation scheme is applied to provide smooth transitions in the corner regions at the lower edges of the source and drain "contacts.") This procedure establishes $\psi_B(x)$ -- the curve shown dotted in Figure 10(b). The Kennedy-Murley-Motta equations are then used to determine the potential $\psi_s(x)$ along the oxide-silicon interface for $0<x<(x_{channel} - 1.3W_{D_{drain}})$, with a parabolic equation extending $\psi_s(x)$ beyond that point in the manner described in Chapter I, Section 4.04.1. Then, given $\psi_s(x)$ and $\psi_B(x)$, one-dimensional solutions of Poisson's equation for $\psi(y)$ are generated vertically downward from the interface, matching $\psi_B(x)$, with zero $d\psi/dy$, deep within the substrate, and matching $\psi_s(x)$ at the interface. For each such vertical solution, the electron quasi-Fermi level ϕ_n (assumed not to vary with y in these one-dimensional solutions) is adjusted to satisfy the boundary condition on $d\psi/dy$ which must be satisfied at the oxide-silicon interface. The potential within the oxide region is approximated to vary linearly with y, matching $\psi = \psi_s(x)$ at the interface, and $\psi = \psi_{gate}$ at the gate contact.

The initial estimate for $\theta(x,y)$ uses the one-dimensional parabolic estimate discussed in Chapter I, Section 4.04.2, established along the vertical plane corresponding to the edge of the space-charge sheath adjacent to the source.

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
TEL OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N
0002                 SUBROUTINE NITIAL(DNR)
                C
                C    PROVIDES INITIAL ESTIMATES FOR PSI & THETA.
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
                C
0004                 COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                    1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,3
                    2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                 COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30
                    1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25)
                    2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006                 COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                    1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
0007                 COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
0008                 JMAXX=JMAX5-1
0009                 JMINN=JMAX+1
0010                 YY=YDLT(JMINN)*DIELK/(1.D0+DIELK)
0011                 YY=YY-YDLT(JMINN)
0012                 DO 90 J=JMINN,JMAXX
0013                 YY=YY+YDLT(J)
0014                 PSI(1,J)=(PSI(1,JMAX5)-PSI(1,JMAX))*YY/YOXIDE+PSI(1,JMAX)
0015              90 PSI(IMAX,J)=(PSI(IMAX,JMAX5)-PSI(IMAX,JMAX))*YY/YOXIDE+
                   1PSI(IMAX,JMAX)
0016                 JMINN=JUNCTN-1
0017                 IMID=(IMAX-1)/2+1
0018                 DLTA=1.D-2
0019                 DO 100 J=2,JMINN
0020                 DO 100 I=1,IMAX
0021             100 PSI(I,J)=PSIEQ
0022                 DO 110 J=JUNCTN,JMAX
0023                 DO 110 I=2,ITHMAX
0024             110 PSI(I,J)=PSIEQ
                C
                C    PROVIDE SPACE-CHARGE SHEATHS ADJACENT OH THE SOURCE & DRAIN CONT
                C    KEY=1 SIGNIFIES THE SOURCE REGION,KEY=2 THE DRAIN REGION
                C
0025                 KEY=1
0026                 I=1
0027                 II=1
0028                 IDLT=1
0029                 XX=0.
0030                 IBRDER=1
0031                 GO TO 130
0032             120 KEY=2
0033                 I=1
0034                 II=IMAX
0035                 IDLT=-1
0036                 XX=0.
0037                 IBRDER=IMAX
                C
                C    SOURCE & DRAIN SHEATHS USE THE DEPLETION APPROXIMATION.
                C
0038             130 SI=PSI(II,JMAX)-PSIEQ
0039                 WD=DSQRT(2.D0*SI/ACCPTR)
0040                 J=JUNCTN
0041                 YY=0.
                C
                C    SHEATH EDGE BELOW SOURCE (OR DRAIN) CONTACTS.
                C
0042             140 YY=YY+YDLT(J)
0043                 JSTOP=J
0044                 J=J-1
0045                 IF(J.EQ.1) GO TO 150
0047                 IF(YY.GT.WD) GO TO 150
0049                 PSI(II,J)=(YY-WD)*(YY-WD)*ACCPTR/2.D0+PSIEQ
```

```
EL 2.1  ( JAN 75 )          NITIAL          OS/360  FORTRAN H EXTENDED          DATE
SN 0050                GO TO 140
                C
                C      SHEATH HORIZONTALLY ALONG ROW J=JUNCTN.
                C
                150 J=JUNCTN
SN 0052                I=I+1
SN 0053                II=II+IDLT
SN 0054                XX=XA+XDLT(I)
SN 0055                IF(I.EQ.IMAX) GO TO (120,200),KEY
SN 0057                IF(XX.GT.WD) GO TO (120,200),KEY
SN 0059                PSI(II,J)=(XX-WD)*(XX-WD)*ACCPTR/2.D0+PSIEQ
                C
                C      FILL IN PSI VERTICALLY UPWARD ALONG SOURCE (OR DRAIN) REGION.
                C
                160 J=J+1
SN 0061                IF(J.GT.JMAX) GO TO 170
SN 0063                PSI(II,J)=PSI(II,JUNCTN)
SN 0064                GO TO 160
                C
                C      OXIDE REGION ABOVE THE SHEATH.
                C
                170 YY=YDLT(J)*DIELK/(1.D0+DIELK)
SN 0066                YY=YY-YDLT(J)
SN 0067                JMINN=J
SN 0068                DO 180 J=JMINN,JMAXX
SN 0069                YY=YY+YDLT(J)
SN 0070                180 PSI(II,J)=(PSI(II,JMAX5)-PSI(II,JMAX))*YY/YOXIDE+PSI(II,JMAX)
                C
                C      CORNER REGIONS BELOW SOURCE (OR DRAIN), USING AN INTERPOLATION
                C      SCHEME TO PROVIDE A SMOOTH TRANSITION
                C
SN 0071                J=JUNCTN
SN 0072                190 J=J-1
SN 0073                IF(J.LT.JSTOP) GO TO 150
SN 0075                PSI(II,J)=(PSI(IBORDER,J)-PSIEQ)*(PSI(II,JUNCTN)-PSIEQ)/
                      1(PSI(IBORDER,JUNCTN)-PSIEQ)+PSIEQ
SN 0076                GO TO 190
                C
                C      THE KENNEDY-MURLEY 1-D THEORY, AS MODIFIED BY MOTTA, IS NEXT USED
                C      TO ESTIMATE THE POTENTIAL ALONG THE SI-OX INTERFACE FOR VALUES OF
                C      X.LT.(XCHNL-1.5*WD).THIS CUTOFF HAS BEEN EMPIRICALLY SELECTED.
                C
                C      FIRST FIND THE EQUILIBRIUM (MOS CAPACITOR) VALUE UO, USING AN
                C      EQUIVALENT GATE POTENTIAL UG WHICH ACCOUNTS FOR QSS. (ALL "U"
                C      FUNCTIONS USE ZERO REFERENCE POTENTIAL AS THE CHARGE-NEUTRAL
                C      SUBSTRATE REGION.)
                C
                200 UG=PSI(IMID,JMAX5)-PSIEQ+QSS*YOXIDE/DIELK
SN 0078                UO=PSI(IMAX,JMAX)-PSIEQ
SN 0079                ALPHA=DSQRT(2.D0*ACCPTR)*YOXIDE/DIELK
SN 0080                BB=1.D0/(ACCPTR*ACCPTR)
SN 0081                UO=VZERO(UG,ALPHA,BB)
                C
                C      IF THE DRAIN VOLTAGE IS ZERO (MOS CAPACITOR CASE), ONLY A SINGLE
                C      1-D INTEGRATION VERTICALLY DOWNWARD ALONG THE COLUMN I=IMID WILL
                C      SUFFICE FOR FILLING IN PSI IN THE REMAINDER OF THE REGION (KEY=1).
                C      OTHERWISE, 1-D INTEGRATIONS ALONG EACH COLUMN EXTERNAL TO THE
                C      SHEATHS ARE NECESSARY (KEY=2 FOR THE KENNEDY-MURLEY-MOTTA REGION
                C      FOR THE SURFACE POTENTIAL US. KEY=3 FOR THE SUTHERLAND "PARABOLIC
                C      FIT" REGION FOR US).
                C
SN 0082                IF(VDRAIN.GT.0.) GO TO 230
SN 0084                KEY=1
SN 0085                US=UO
SN 0086                I=IMID
SN 0087                GO TO 320
SN 0088                210 IMID=IMID-1
SN 0089                DO 220 I=2,IMID
SN 0090                DO 220 J=2,JMAXX
SN 0091                IF(PSI(I,J).GT.PSI(IMID+1,J)) GO TO 220
SN 0093                PSI(I,J)=PSI(IMID+1,J)
SN 0094                PSI(IMAX+1-I,J)=PSI(I,J)
```

```
0095        220 CONTINUE
0096            GO TO 360
                C
                C       SET UP COEFFICIENTS FOR THE KENNEDY-MURLEY-MOTTA SOLUTION FOR US
                C
0097        230 AA=ALPHA/DSQRT(UO-1.DO)
0098            FACTOR=DEXP(UO)/ACCPTR
0099            FACTOR=FACTOR/(FACTOR+2.DO*ACCPTR)
0100            XLMBDA=(1.DO+AA/2.DO)/(UG-UO+1.DO-(UO-1.5DO)*AA)
0101            AA=(1.DO-XLMBDA)/(UG-UO)
0102            UC=UO+VDRAIN
0103            IF(UG.LE.UC) UC=UG
0105            XC=XCHNL-WD*(1.DO-DSQRT(UC/UD))
0106            EXPNT=(XLMBDA+AA)/AA
0107            E0=FACTOR*(1.DO-(1.DO-AA*(UC-UO))**EXPNT)/XC
0108            EXPNT=1.DO/EXPNT
                C
                C       THE KENNEDY-MURLEY -MOTTA SOLUTION FOR US IS APPLIED FOR XX.LT.
                C       (XCHNL-1.3*WD). BEYOND THAT A PARABOLIC SOLUTION FOR US IS
                C       APPLIED,DETERMINED EMPIRICALLY BY SUTHERLAND.
                C
0109            XC=XCHNL-1.3DO*WD
0110            IF(XC.GE.1.D-1*XCHNL) GO TO 235
0112            WRITE(6,1000)
0113        1000 FORMAT(1H0,5X,'RUN TERMINATED IN SUBROUTINE NITIAL. ',/,
                   1' THE DRAIN VOLTAGE EXCEEDS THAT FOR WHICH THE PROGRAM ',/,
                   2' WAS DESIGNED. (THE LOGIC FAILS FOR PARABOLIC EXTENSION',/,
                   3' OF THE KENNEDY-MURLEY-MOTTA SOLUTION FOR PSI ALONG THE',/,
                   4' SI-OX INTERFACE',///)
0114            STOP
0115        235 KEY=2
0116            I=1
0117            XX=0.
0118        240 J=JMAX
0119            I=I+1
0120            XX=XX+XDLT(I)
0121            IF(XX.GT.XC) GO TO 250
0123            US=UO+(1.DO-(1.DO-E0*XX)**EXPNT)/AA
0124            GO TO 310
                C
                C       SET UP COEFFICIENTS FOR PARABOLIC SOLUTION FOR US.
                C
0125        250 KEY=3
0126            UST=US
0127            XX=XX-XDLT(I)
0128            I=I+1
0129            IMINN=I
0130            FACTOR=XLMBDA+AA
0131            EXPNT=XLMBDA/FACTOR
0132            BB=(E0/FACTOR)/((1.DO-E0*XX)**EXPNT)
                C
                C       TERMINATION POINT FOR THE PARABOLIC SOLUTION DEPENDS ON WHETHER
                C       UG.GT.UD. IF SO,TERMINATION IS AT XCHNL. IF NOT, TERMINATION
                C       OCCURS WHEN US.GT.UG.
                C
0133            IF(UG.GE.UD) GO TO 280
0135            DD=0.
0136        260 I=I+1
0137            IF(I.GT.IMAX) GO TO 270
0139            DD=DD+XDLT(I)
0140            US=PSI(I,J)-PSIEQ
0141            IF(US.LT.UG) GO TO 260
0143        270 AA=(US-UST-BB*DD)/(DD*DD)
0144            IMAXX=I-1
0145            GO TO 290
0146        280 DD=XCHNL-XX
0147            AA=(UD-UST-BB*DD)/(DD*DD)
0148            IMAXX=IMAX-1
0149        290 I=IMINN
0150            XX=0.
                C
                C       PARABOLIC EQUATION FOR US IN THE INTERVAL WHERE IT APPLIES.
```

NOT SHOWN IN THE FLOWCHART

```
VEL 2.1  ( JAN 75 )          NITIAL          OS/360  FORTRAN H EXTENDED          D
                C
ISN 0151          300 J=JMAX
ISN 0152              I=I+1
ISN 0153              IF(I.GT.IMAXX) GO TO 360
ISN 0155              XX=AA+XDLT(I)
ISN 0156              US=AA*XX*XX+BB*XX+UST
                C
                C     ASSIGN US TO THE SURFACE POINT ONLY IF IT EXCEEDS THE POTENTIAL
                C     ALREADY THERE DUE TO THE SHEATHS.
                C
ISN 0157          310 IF(US.GT.(PSI(I,J)-PSIEQ)) GO TO 320
ISN 0159              IF(KEY.EQ.2) GO TO 240
ISN 0161              GO TO 300
                C
                C     FIRST FILL IN THE OXIDE POTENTIALS ABOVE THE INTERFACE.
                C
ISN 0162          320 J=JMAX+1
ISN 0163              YY=YDLT(J)*DIELK/(1.D0+DIELK)
ISN 0164              YY=YY-YDLT(J)
ISN 0165              JMINN=J
ISN 0166              DO 330 J=JMINN,JMAXX
ISN 0167              YY=YY+YDLT(J)
ISN 0168          330 PSI(I,J)=((PSI(I,JMAXS)-PSIEQ)-US)*YY/YOXIDE+US+PSIEQ
                C
                C     SET UP TO INTEGRATE A 1-D POISSON EQ FROM THE SURFACE DOWNWARD,
                C     WHICH MATCHES THE POTENTIAL UB WITH ZERO DERIVATIVE THERE.
                C
ISN 0169              UB=PSI(I,JUNCTN)-PSIEQ
ISN 0170              UDLT=US-UB
ISN 0171              IF(UDLT.GT.0) GO TO 335
ISN 0173              WRITE(6,1010)
ISN 0174         1010 FORMAT(1H0,5X,'RUN TERMINATED IN SUBROUTINE NITIAL.',/,
                     1' THE GATE VOLTAGE IS NOT HIGH ENOUGH TO CAUSE THE',/,
                     2' ELECTRIC FIELD AT THE SI-OX INTERFACE TO BE DIRECTED',/,
                     3' VERTICALLY DOWNWARD-- A CONDITION WHICH MUST BE SAT-',/,
                     4' ISFIED FOR THE 1-D SOLUTIONS USED IN NITIAL TO APPLY.',//)
ISN 0175              STOP
ISN 0176          335 FACTOR=0.
ISN 0177              IF(UB.LT.1.D1)FACTOR=DEXP(-UB)
ISN 0179              QF=-(1.D0-FACTOR)
ISN 0180              UGTERM=((UG-US)/ALPHA)*((UG-US)/ALPHA)
ISN 0181              IF(UDLT.GT.1.D-4) QF=(UGTERM-UDLT-FACTOR*(DEXP(-UDLT)-1.D0))/
                     1(DEXP(UDLT)-1.D0)
ISN 0183              J=JMAX
ISN 0184              Y=0.
ISN 0185              YY=YDLT(J+1)/(1.D0+DIELK)
ISN 0186              DY=YY/3.D0
ISN 0187              SI=US
ISN 0188              YZERO=0.
ISN 0189              SIZERO=SI
ISN 0190              KNT=0
ISN 0191              KFLAG=0
                C
                C     USE PREDICTOR-CORRECTOR INTEGRATION UNTIL Y & YZERO BRACKET YY.
                C
ISN 0192          340 KNT=KNT+1
ISN 0193              YNEG=YZERO
ISN 0194              SINEG=SIZERO
ISN 0195              YZERO=Y
ISN 0196              SIZERO=SI
ISN 0197              CALL ONEDIM(SI,UB,FACTOR,QF,DY,DLTA,KNT,KFLAG)
                C
                C     TERMINATE IF KFLAG.GT.0. ( THE 1-D SOLUTIONREACHED A MINIMUM VAL
                C
ISN 0198              IF(KFLAG.GT.0) GO TO(210,240,300),KEY
                C
                C     TEST FOR BRACKETING. CONTINUE INTEGRATION IF NOT.
                C
ISN 0200              Y=Y+DY
ISN 0201          350 IF(Y.LT.YY) GO TO 340
                C
                C     SINCE BRACKETING OCCURRED, INTERPOLATE FOR PSI AT YY.
```

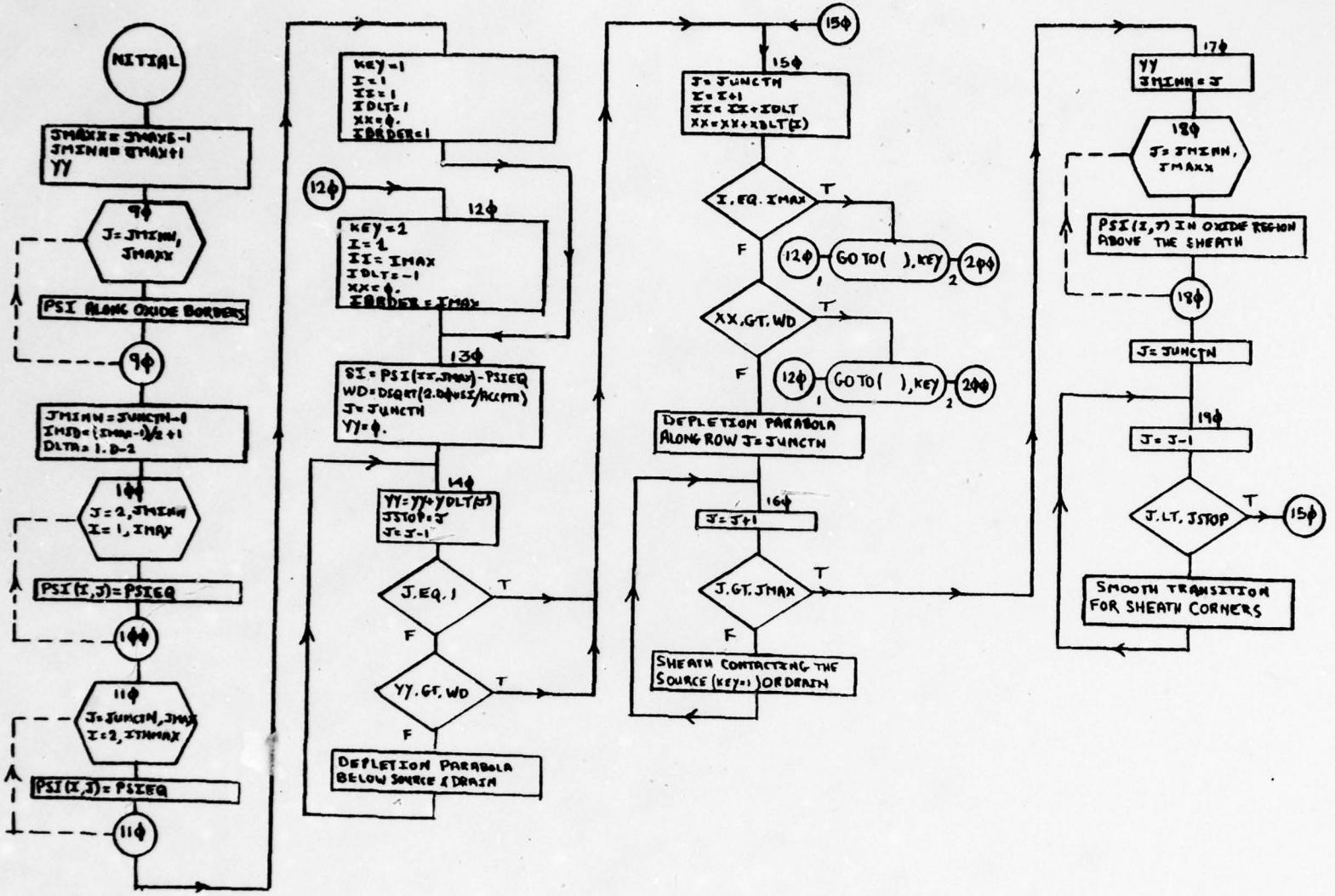
NOT SHOWN IN THE FLOWCHART

```
      C
0203        SITEMP=GTERP(Y,YZERO,YNEG,SI,SIZERO,SINEG,YY)
0204        DLT=DABS(SITEMP-UB)
0205        IF(DLT.LT.1.D-2) GO TO (210,240,300),KEY
0207        SITEMP=SITEMP+PSIEG
0208        IF(SITEMP.LT.PSI(I,J)) GO TO (210,240,300),KEY
0210        PSI(I,J)=SITEMP
0211        YY=YY+YDLT(J)
0212        J=J-1
0213        IF(J.EQ.1) GO TO (210,240,300),KEY
0215        GO TO 350
      C
      C     SUPPLY AN INITIAL ESTIMATE FOR THE STREAM FUNCTION.
      C
0216  360   I=1
0217        J=JUNCTN
0218        SI=PSI(I,J)
0219  370   I=I+1
0220        IF(PSI(I,J).GE.SI) GO TO 380
0222        SI=PSI(I,J)
0223        GO TO 370
0224  380   J=JMAX
0225        US=(PSI(I,J)+PSI(I,J+1))/2.D0+USS*YPOS(J)/2.D0-PSIEG
0226        UGTERM=((UG-US)/ALPHA)*((UG-US)/ALPHA)
0227        QF=(UGTERM-US-DEXP(-US)+1.D0)/(DEXP(US)-1.D0)
0228        SI=-DLOG(ACCPTR*QF)+PSIEG
0229        YY=0.
0230  390   IF(SI.GT.PSI(I,J)) GO TO 400
0232        YY=YTHPOS(J)
0233        J=J-1
0234        IF(J.EQ.1) GO TO 400
0236        GO TO 390
0237  400   Y=YY
0238        JMAXX=J
0239        DO 410 J=1,JMAXX
0240        DO 410 I=1,ITHMAX
0241  410   THETA(I,J)=1.D0
0242        JMINN=JMAXX+1
0243        DO 420 J=JMINN,JMAX
0244        DO 420 I=1,ITHMAX
0245        YY=YTHPOS(J)/Y
0246  420   THETA(I,J)=YY*(2.D0-YY)
0247        RETURN
0248        END
```

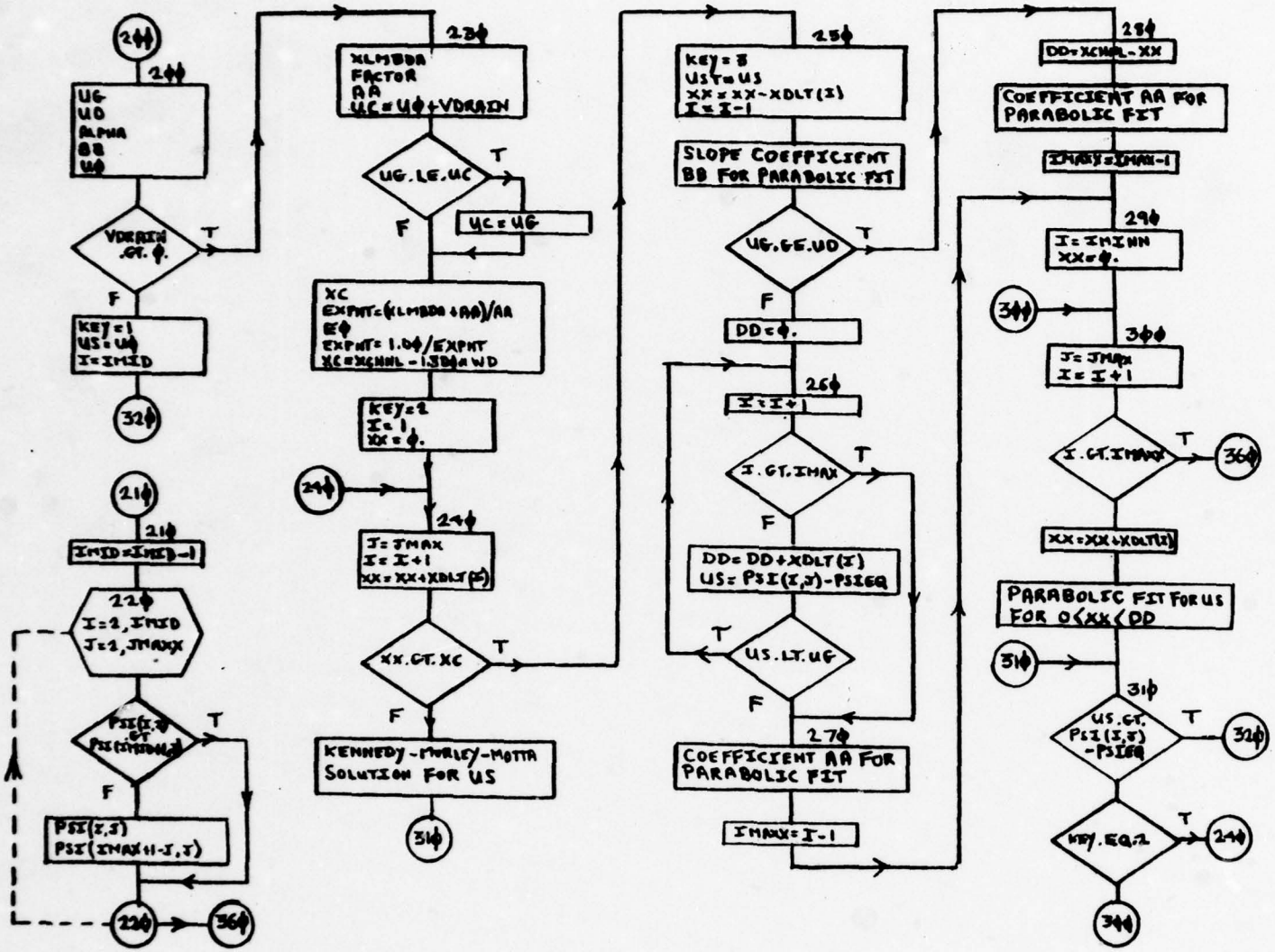
```
VEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                  DA
UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

IONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

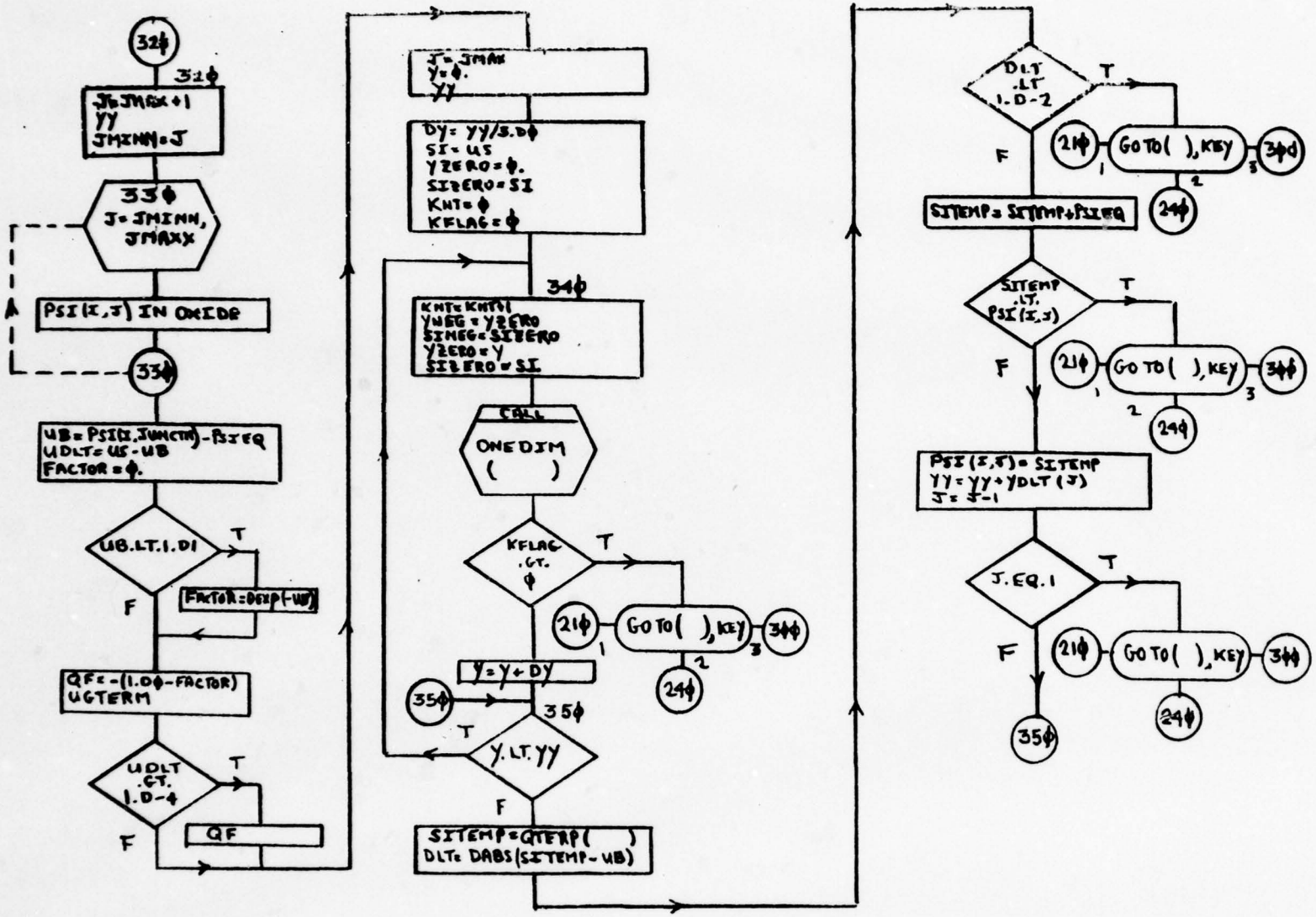
ISN 0002            FUNCTION QTERP(YP,Y0,YN,FP,F0,FN,YY)
                C
                C   PARABOLIC INTERPOLATION WITH UNEQUAL INTERVALS FOR SUBROUTINE
                C   NITIAL
                C
ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
                C
ISN 0004            DP=YP-Y0
ISN 0005            DN=Y0-YN
ISN 0006            DY=YY-Y0
ISN 0007            DENOM=DP*DN*(DP+DN)
ISN 0008            A=(DN*FP+DP*FN-(DP+DN)*F0)/DENOM
ISN 0009            DP=DP*DP
ISN 0010            DN=DN*DN
ISN 0011            B=(DN*FP-DP*FN+(DP-DN)*F0)/DENOM
ISN 0012            QTERP=DY*(A*DY+B)+F0
ISN 0013            RETURN
ISN 0014            END
```



NITIAL SHEET 1 of 4



NITIAL SHEET 2 of 4



NITIAL SHEET 3 of 4

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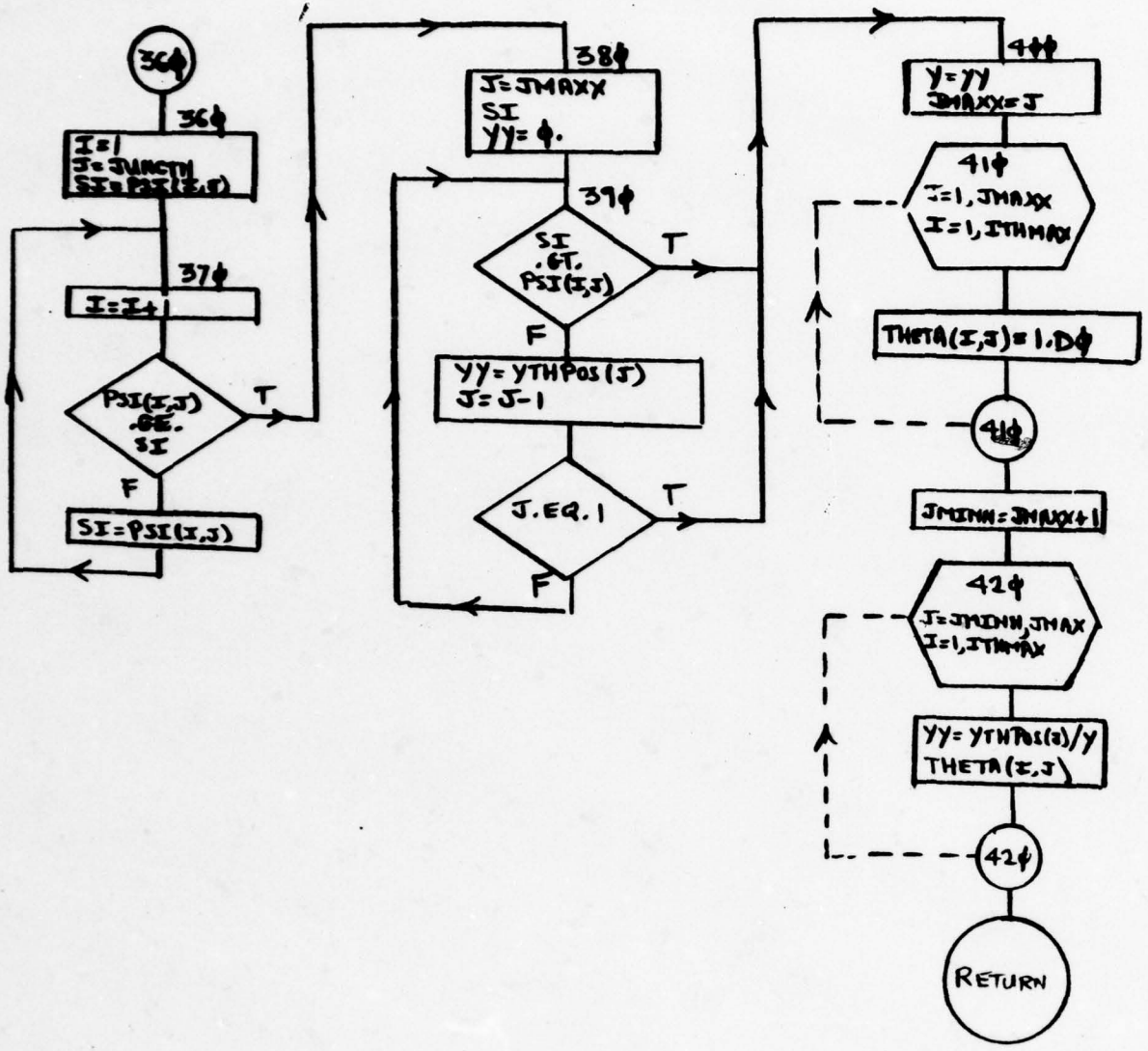


NITIAL SHEET 4 of 4

5.0 VZERO & DERIV

FUNCTION VZERO solves a transcendental equation for the surface potential V_o of the Kennedy-Motta-Murley theory. (This is the result of a one-dimensional solution of Poisson's equation in y which assumes that equilibrium conditions, viz. $\phi_n = \phi_p$, apply close to the source, as in MOS capacitor theory.) The subprogram uses the method of secants [14] to achieve this, and implicitly assumes that the energy band bending will always be in the direction which causes inversion at the silicon surface. Thus, when attempting to model p-channel structures by using this n-channel model (as was described in Chapter III) the user must assure that the above condition is not violated. Otherwise, an error message will be printed and the run will terminate.

FUNCTION DERIV provides the derivatives $d\psi/dy$ required by ONEDIM, which generates the one-dimensional solutions for $\psi(y)$ needed in implementing NITIAL, as discussed in the preceeding section.

---

[14] F.S. Acton, NUMERICAL METHODS THAT WORK, Harper & Row, New York, 1970, p. 52.

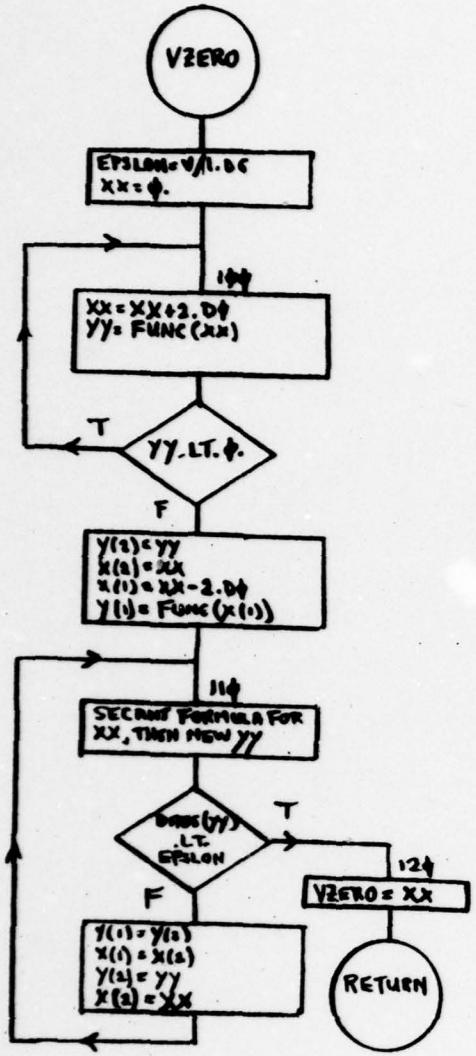
```
( JAN 75 )                    OS/360  FORTRAN H EXTENDED                DATE 77.183
PTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
 ECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
      SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NOANSF NOT

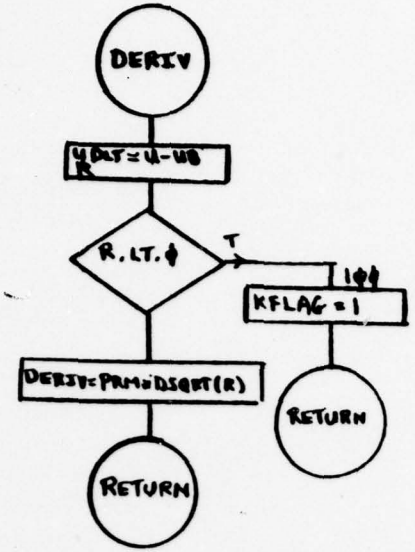
          FUNCTION DERIV(PRM,U,UB,FACTOR,QF,KFLAG)
      C
      CC      PROVIDES DPSIDY/DY FOR SUBROUTINE ONEDIM.
      C
          IMPLICIT REAL*8(A-H,O-Z)
          UDLT=U-UB
          R=UDLT+FACTOR*(DEXP(-UDLT)-1.D0)+QF*(DEXP(UDLT)-1.D0)
          IF(R.LT.0) GO TO 100
          DERIV=PRM*DSQRT(R)
          RETURN
        100 KFLAG=1
          RETURN
          END
```

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

0002                 FUNCTION VZERO(V,A,B)
                C
                C    CALCULATES V0 OF THE ONE DIMENSIONAL KENNEDY MODEL
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
0004                 DIMENSION X(2),Y(2)
0005                 FUNC(X)=X+A*DSQRT(DEXP(-X)+X-1.D0+B*(DEXP(X)-1.D0))-V
0006                 EPSLON=V/1.D6
0007                 XX=0.
                C
                C    INCREMENT XX UNTIL YY CHANGES SIGN FROM MINUS TO PLUS.
                C
0008              100 XX=XX+2.D0
0009                 YY=FUNC(XX)
0010                 IF(YY.LT.0.) GO TO 100
                C
                C    USE METHOD OF SECANTS TO REDUCE YY BELOW EPSLON.
                C
0012                 Y(2)=YY
0013                 X(2)=XX
0014                 X(1)=XX-2.D0
0015                 Y(1)=FUNC(X(1))
0016              110 XX=(X(2)*Y(1)-X(1)*Y(2))/(Y(1)-Y(2))
0017                 YY=FUNC(XX)
0018                 IF(DABS(YY).LT.EPSLON) GO TO 120
0020                 Y(1)=Y(2)
0021                 X(1)=X(2)
0022                 Y(2)=YY
0023                 X(2)=XX
0024                 GO TO 110
0025              120 VZERO=XX
0026                 RETURN
0027                 END
```





FUNCTIONS VZERO & DERIV SHEET 1 Of 1

6.0 ONEDIM

This subroutine implements the Adams-Bashforth-Moulton predictor-corrector algorithm [15], with a simple modification to improve its accuracy [16], to obtain the one-dimensional solutions of Poisson's equation required for NITIAL. It utilizes a 4th order Runge-Kutta starting procedure [17] to initialize the solution. Interval halving and doubling are invoked when tests show that these measures are acceptable.

---

[15] F.S. Acton, loc. cit., p. 146.

[16] ENCYCLOPAEDIC DICTIONARY OF MATHEMATICS FOR ENGINEERS AND APPLIED SCIENTISTS, Pergamon Press, 1976, (I.N. Sneddon, Editor), p. 182.

[17] J.B. Scarborough, NUMERICAL MATHEMATICAL ANALYSIS, Fifth Edition, Johns Hopkins Press, 1962, p. 357.

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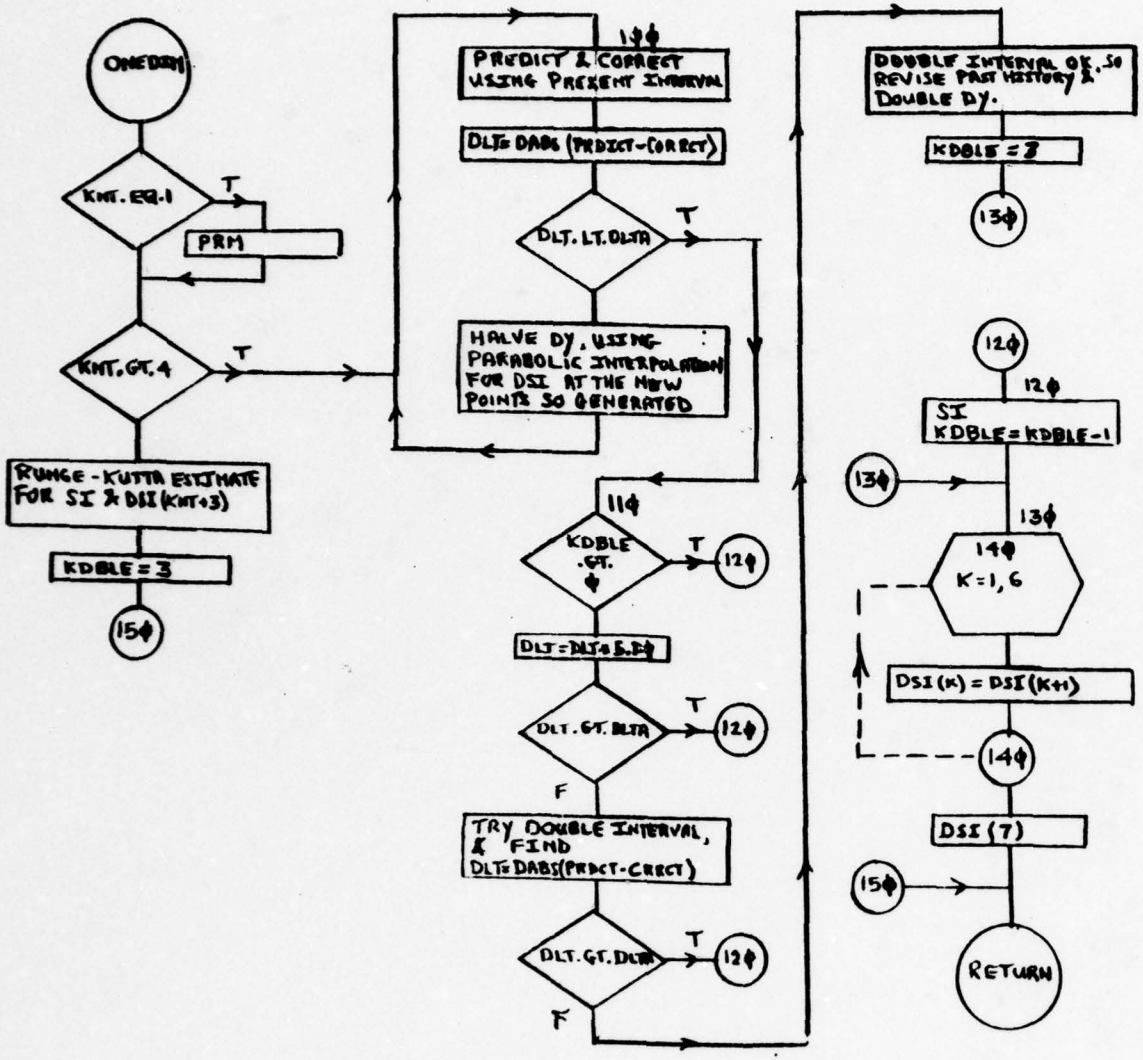
/EE 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
ESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

IONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

ISN 0002              SUBROUTINE ONEDIM(SI,DB,FACTOR,QF,DY,DLTA,KNT,KFLAG)
                C
                C     USES ADAMS-BASHFORD-MOULTON PREDICTOR-CORRECTOR (WITH RUNGE-KUTTA
                C     STARTING PROCEDURE) TO INTEGRATE 1-D SOLUTION OF POISSON'S EQ FOR
                C     SUBROUTINE NITIAL.
                C
ISN 0003              IMPLICIT REAL*8(A-H,O-Z)
                C
ISN 0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                     1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,30
                     2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
ISN 0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                     1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                     2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
ISN 0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                     1VDRAIN,DIELK,GSS,CURRNT,EDENS0,PSIEQ
ISN 0007              COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
ISN 0008              DIMENSION DSI(7)
                C
                C     USE RUNGE-KUTTA FOR THE FIRST FOUR INTEGRATION STEPS.
                C
ISN 0009              IF(KNT.EQ.1) PRM=-DSQRT(2.D0*ACCPTR)
ISN 0011              IF(KNT.GT.4) GO TO 100
ISN 0013              DK1=DERIV(PRM,SI,DB,FACTOR,QF,KFLAG)
ISN 0014              ETA=SI+DK1*DY/2.D0
ISN 0015              DK2=DERIV(PRM,ETA,DB,FACTOR,QF,KFLAG)
ISN 0016              ETA=SI+DK2*DY/2.D0
ISN 0017              DK3=DERIV(PRM,ETA,DB,FACTOR,QF,KFLAG)
ISN 0018              ETA=SI+DK3*DY
ISN 0019              DK4=DERIV(PRM,ETA,DB,FACTOR,QF,KFLAG)
ISN 0020              DSI(KNT+3)=(DK1+2.D0*(DK2+DK3)+DK4)/6.D0
ISN 0021              SI=SI+DSI(KNT+3)*DY
    0022              KDBLE=3
    0023              DSI(KNT+3)=DERIV(PRM,SI,DB,FACTOR,QF,KFLAG)
ISN 0024              GO TO 150
                C
                C     PREDICTOR-CORRECTOR USING PRESENT INTERVAL.
                C
ISN 0025            100 PRDICT=(55.D0*DSI(7)-59.D0*DSI(6)+37.D0*DSI(5)-9.D0*DSI(4))*
                     1DY/24.D0
ISN 0026              ETA=SI+PRDICT
ISN 0027              DSINEW=DERIV(PRM,ETA,DB,FACTOR,QF,KFLAG)
ISN 0028              CORRCT=(9.D0*DSINEW+19.D0*DSI(7)-5.D0*DSI(6)+DSI(5))*DY/24.D0
ISN 0029              DLT=DABS(PRDICT-CORRCT)
                C
                C     HALVE THE INTERVAL IF DLT EXCEEDS DLTA.
                C
ISN 0030              IF(DLT.LT.DLTA) GO TO 110
                C
                C     PARABOLIC INTERPOLATION COEFFICIENTS FOR THE HALF INTERVAL VALUE
                C
ISN 0032              AA=(DSI(7)-2.D0*DSI(6)+DSI(5))/(2.D0*DY*DY)
ISN 0033              BB=(DSI(7)-DSI(5))/(2.D0*DY)
ISN 0034              AAA=(DSI(6)-2.D0*DSI(5)+DSI(4))/(2.D0*DY*DY)
ISN 0035              BBB=(DSI(6)-DSI(4))/(2.D0*DY)
ISN 0036              DY=DY/2.D0
ISN 0037              DSI(1)=DSI(4)
ISN 0038              DSI(3)=DSI(5)
ISN 0039              DSI(5)=DSI(6)
ISN 0040              DSI(2)=DY*(AAA*DY-BBB)+DSI(3)
ISN 0041              DSI(4)=DY*(AA*DY-BB)+DSI(5)
ISN 0042              DSI(6)=DY*(AA*DY+BB)+DSI(5)
ISN 0043              GO TO 100
                C
                C     TENTATIVELY DOUBLE THE INTERVAL, BUT ONLY IF 3 STEPS HAVE BEEN
                C     TAKEN SINCE THE LAST DOUBLING, AND ONLY IF DLT IS LESS THAN
```

```
2.1  ( JAN 75 )          ONEDIM          OS/360  FORTRAN H EXTENDED          JA

      C       ONE FIFTH OF DELTA.
      C
0044     110  IF(KDBLE.GT.0) GO TO 120
0046          DLT=DLT*5.D0
0047          IF(DLT.GT.DLTA) GO TO 120
0049          PRDCT=(55.D0*DSI(7)-59.D0*DSI(5)+37.D0*DSI(3)-9.D0*DSI(1))*
             1DY/12.D0
0050          ETA=SI+PRDCT
0051          DSINE=DERIV(PRM,ETA,DE,FACTOR,QF,KFLAG)
0052          CRRCT=(9.D0*DSINE+19.D0*DSI(7)-5.D0*DSI(5)+DSI(3))*DY/12.D0
0053          DLT=DABS(PRDCT-CRRCT)
0054          IF(DLT.GT.DLTA) GO TO 120
      C
      C       DOUBLE INTERVAL IS OK, SO DO IT.
      C
0056          DSI(6)=DSI(5)
0057          DSI(5)=DSI(3)
0058          DSI(4)=DSI(1)
0059          DY=2.D0*DY
0060          SI=SI+(19.D0*PRDCT+251.D0*CRRCT)/270.D0
0061          KDBLE=3
0062          GO TO 130
      C
      C       NO DOUBLING OCCURRED. COMPUTE NEW SI & SHIFT DERIVATIVES LEFT
      C       FOR THE NEXT INTEGRATION STEP, UPDATING THE PRESENT DERIVATIVE.
      C
0063     120  SI=SI+(19.D0*PRDICT+251.D0*CORRCT)/270.D0
0064          KDBLE=KDBLE-1
0065     130  DO 140 K=1,6
0066     140  DSI(K)=DSI(K+1)
0067          DSI(7)=DERIV(PRM,SI,DE,FACTOR,QF,KFLAG)
0068     150  RETURN
0069          END
```



ONEDIM SHEET 1 of 1

7.0 RELAX & GRELAX

This subprogram implements the Picard iteration method described in Chapter I, Section 4.01. It calculates and applies the Chebyscheff sequence of relaxation parameters, reinitiating said sequence whenever the maximum residual at step (m+1) is detected to exceed that encountered at step (m), as suggested by Mock.

7.01 GRELAX

GRELAX, which appears as an ENTRY statement in RELAX (in order to reduce storage requirements because it utilizes some arrays also used by RELAX), implements the overrelaxed Gummel's algorithm, described in Chapter I, Section 4.02.

```
EVEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                DA
UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

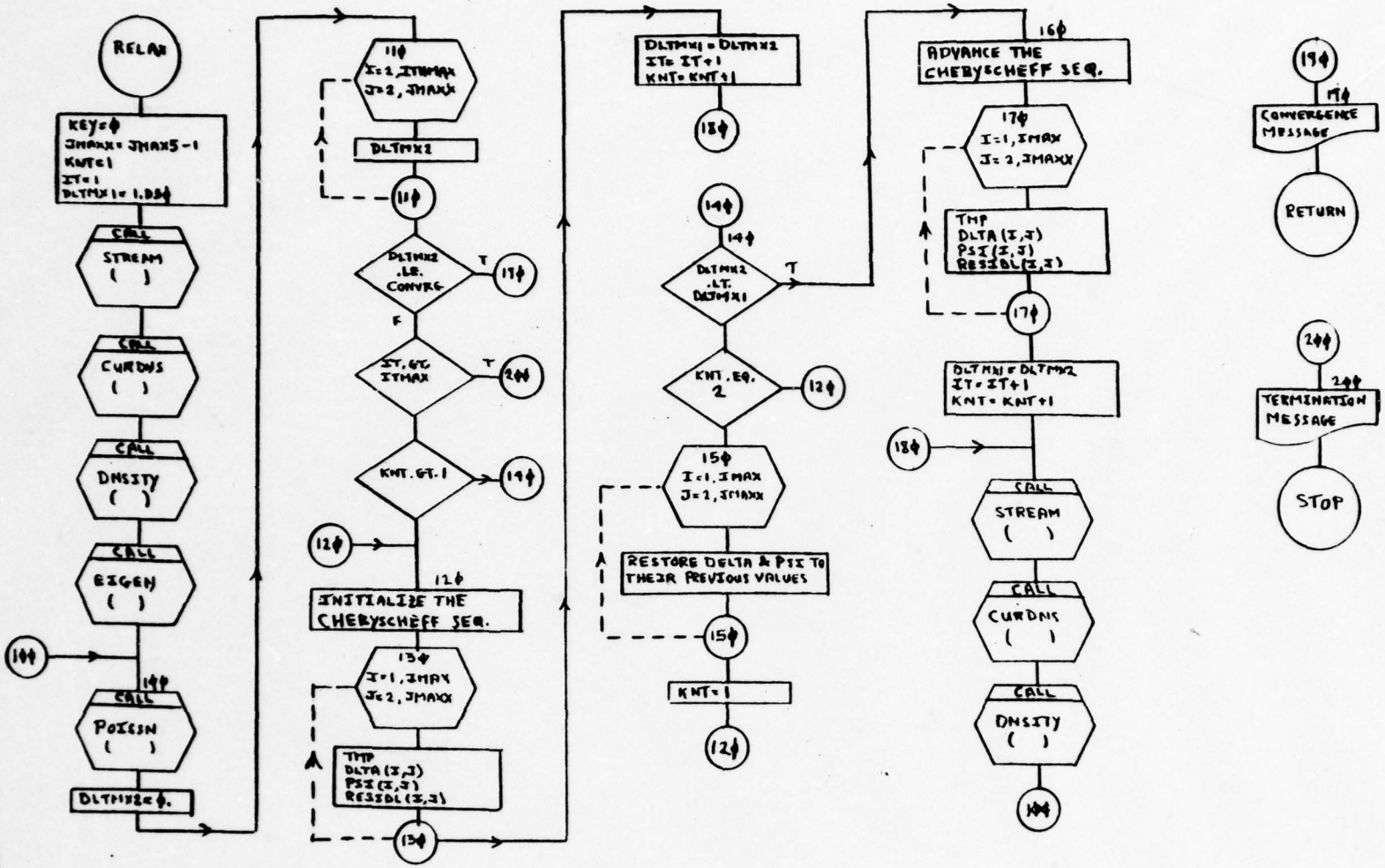
IONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

ISN 0002              SUBROUTINE RELAX(ITMAX,ITSIMX,ITTHMX,CONVRG,SIDLT,THDLT,DNR)
                C
                C     IMPLEMENTS CHEBYSCHEFF RELAXATION SEQUENCE FOR THE PICARD
                C     ITERATION (KEY=0) OR GUMMEL'S ALGORITHM (KEY=1) FOR A SELF-
                C     CONSISTENT SOLUTION FOR PSI,EDENS, & THETA. SEE FORSYTHE & WASO*
                C     "FINITE-DIFFERENCE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS",
                C     *ILEY,1960, P255.
                C
ISN 0003              IMPLICIT REAL*8(A-H,O-Z)
                C
ISN 0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                     1D(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,3
                     2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
ISN 0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30
                     1THEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25)
                     2THEAST(40,25),THSTH(40,25),THWEST(40,25),CAREA(41,25)
ISN 0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                     1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
ISN 0007              COMMON/INTGR/IMAX,JMAX,JMAXS,JUNCTN,ITHMAX
ISN 0008              COMMON/CNTRL/ OMEGA
                C
ISN 0009              DIMENSION DELTA(41,30), RESIDL(41,30)
                C
ISN 0010              KEY=0
ISN 0011              JMAXX=JMAXS-1
ISN 0012              KNT=1
ISN 0013              IT=1
ISN 0014              DLTMX1=1.D50
                C
                C     GIVEN THE INITIAL ESTIMATE FOR PSI FROM SUBROUTINE NITIAL, FIND
                C     THE CORRESPONDING INITIAL ESTIMATES FOR THETA,EDENS, &HDENS.
                C
ISN 0015              CALL STREAM(ITTHMX,THDLT)
ISN 0016              CALL CURDNS(DNR)
ISN 0017              CALL DNSITY(DNR)
                C
                C     ESTIMATE THE RANGE(AA,BB) OF THE MATRIX EIGENVALUES.
                C
ISN 0018              CALL EIGEN(AA,BB)
                C
                C     ITERATION LOOP STARTS AT 100, ENDS JUST BEFORE 190.
                C
ISN 0019          100 CALL POISSN(ITSIMX,SIDLT,KEY)
ISN 0020              DLTMX2=0.
                C
                C     FIND MAXIMUM RESIDUAL & TERMINATE IF IT IS LESS THAN THE
                C     SPECIFIED MAXIMUM VALUE.
                C
ISN 0021              DO 110 I=2,ITHMAX
ISN 0022              DO 110 J=2,JMAXX
ISN 0023              DLTMX2=DMAX1(DABS(DELTA(I,J)),DLTMX2)
ISN 0024          110 CONTINUE
ISN 0025              IF(DLTMX2.LE.CONVRG) GO TO 190
ISN 0027              IF(IT.GT.ITMAX) GO TO 200
                C
                C     INITIALIZE CHEBYSCHEFF SEQUENCE IF KNT=1
                C
ISN 0029              IF(KNT.GT.1) GO TO 140
ISN 0031          120 COSH=(BB+AA)/(BB-AA)
ISN 0032              COSHH=COSH
ISN 0033              COSH0=1.D0
ISN 0034              ALPHA=2.D0/(BB+AA)
ISN 0035              DO 130 I=1,IMAX
ISN 0036              DO 130 J=2,JMAXX
ISN 0037              TMP=PSI(I,J)
ISN 0038              DELTA(I,J)=DELTA(I,J)
ISN 0039              PSI(I,J)=PSI(I,J)+ALPHA*DELTA(I,J)
```

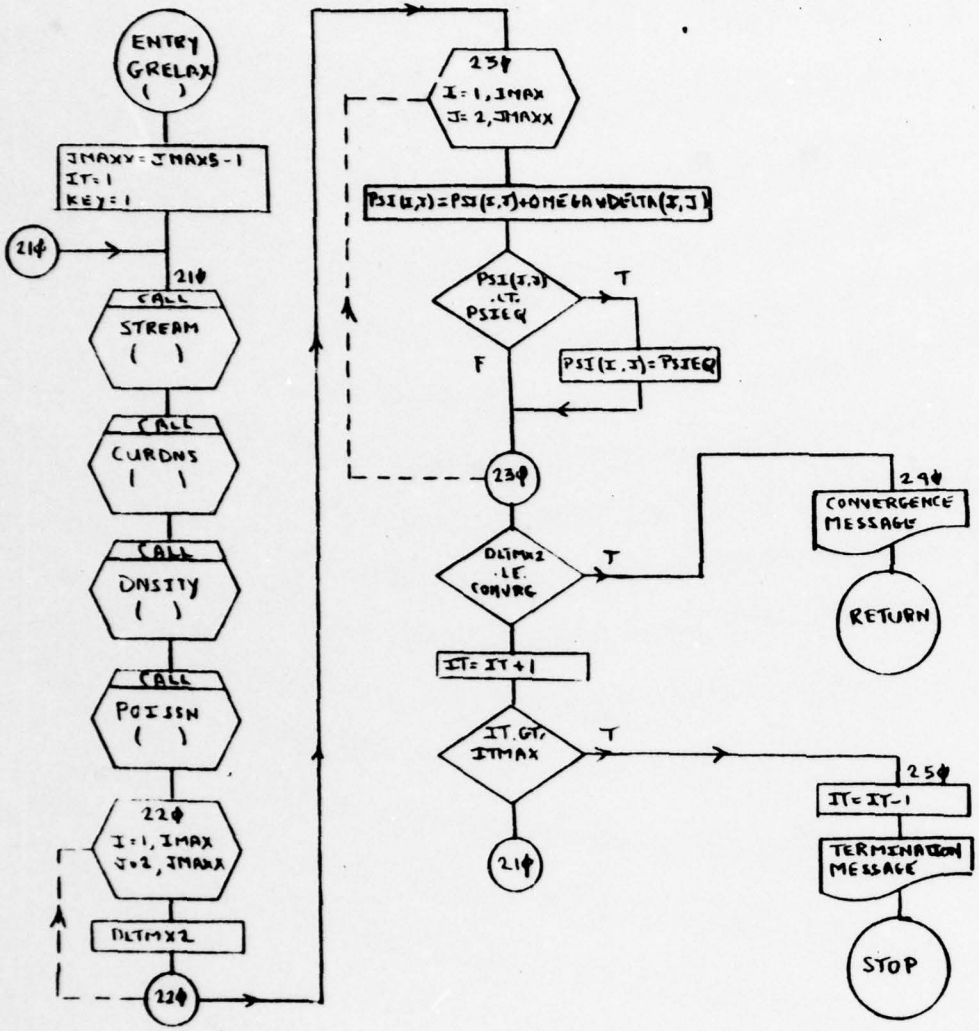
```
0040          130 RESIDL(I,J)=PSI(I,J)-TMP
0041              DLTMX1=DLTMX2
0042              IT=IT+1
0043              KNT=KNT+1
0044              GO TO 180
                C
                C     IF THE MAXIMUM DELTA HAS INCREASED, RATHER THAN DECREASED,
                C     REINITIALIZE THE CHEBYSCHEFF SEQUENCE. USE THE PREVIOUS VALUES O
                C     DELTA & PSI, UNLESS THE SEQUENCE WAS JUST PREVIOUSLY RESET.
                C     OTHERWISE USE PRESENT VALUES TO AVOID AN ENDLESS LOOP.
                C
0045          140 IF(DLTMX2.LT.DLTMX1) GO TO 160
0047              IF(KNT.EQ.2) GO TO 120
0049              DO 150 I=1,IMAX
0050              DO 150 J=2,JMAXX
0051              DELTA(I,J)=DLTA(I,J)
0052          150 PSI(I,J)=PSI(I,J)-RESIDL(I,J)
0053              KNT=1
0054              GO TO 120
                C
                C     CHEBYSCHEFF SEQUENCE OF ALPHAS & BETAS, USING A RECURSIVE
                C     FORMULA FOR THE REQUIRED HYPERBOLIC FUNCTIONS.
                C
0055          160 COSHN=COSH0
0056              COSH0=COSHP
0057              COSHP=2.D0*COSH*COSH0-COSHN
0058              ALPHA=4.D0*COSH0/(COSHP*(BB-AA))
0059              BETA=COSHN/COSHP
0060              DO 170 I=1,IMAX
0061              DO 170 J=2,JMAXX
0062              TMP=PSI(I,J)
0063              DLTA(I,J)=DELTA(I,J)
0064              PSI(I,J)=PSI(I,J)+ALPHA*DELTA(I,J)+BETA*RESIDL(I,J)
0065          170 RESIDL(I,J)=PSI(I,J)-TMP
0066              DLTMX1=DLTMX2
0067              IT=IT+1
0068              KNT=KNT+1
                C
                C     UPDATE THETA,EDENS,&HDENS THEN CONTINUE ITERATION.
                C
0069          180 CALL STREAM(ITTHMX,THDLT)
0070              CALL CURDNS(DNR)
0071              CALL DNSITY(DNR)
0072              GO TO 100
                C
                C     TERMINATION MESSAGES.
                C
0073          190 WRITE(6,1000) IT
0074         1000 FORMAT(1H0,5X,I5,'  ITERATIONS WERE REQUIRED IN RELAX')
0075              RETURN
0076          200 IT=IT-1
0077              WRITE(6,1010) IT,DLTMX2,DLTMX1,CONVRG
0078         1010 FORMAT(1HJ,5X,'MAXIMUM ITERATIONS (ITMAX=',I5,'  ) WERE EXCEEDED
                1IN RELAX.',3X,'DLTMX2=',1PD10.2,'  DLTMX1=',1PD10.2,'  CONVRG=',
                21PD10.2,//)
0079              STOP
                C
                C     ENTRY GRELAX IMPLEMENTS GUMMEL'S ALGORITHM
                C
0080              ENTRY GRELAX(ITMAX,ITSIMX,ITTHMX,CONVRG,SIDLT,THDLT,DNR)
0081              JMAXX=JMAX5-1
0082              IT=1
0083              KEY=1
0084          210 DLTMX2=0.
0085              CALL STREAM(ITTHMX,THDLT)
0086              CALL CURDNS(DNR)
0087              CALL DNSITY(DNR)
0088              CALL POISSN(ITSIMX,SIDLT,KEY)
0089              DO 220 I=1,IMAX
0090              DO 220 J=2,JMAXX
0091              DLTMX2=DMAX1(DABS(DELTA(I,J)),DLTMX2)
0092              DLT=DABS(DELTA(I,J))
```

```
:VEL 2.1  ( JAN 75 )          RELAX          OS/360  FORTRAN H EXTENDED
 ISN 0093          IF(DLT.LT.DLTMX2) GO TO 220
 ISN 0095          IMAX=I
 ISN 0096          JMAX=J
 ISN 0097        220 CONTINUE
 ISN 0098          DO 230 I=1,IMAX
 ISN 0099          DO 230 J=2,JMAX
 ISN 0100          PSI(I,J)=PSI(I,J)+OMEGA*DELTA(I,J)
 ISN 0101          IF(PSI(I,J).LT.PSIEQ) PSI(I,J)=PSIEQ
 ISN 0103        230 CONTINUE
 ISN 0104          IF(DLTMX2.LE.CONVRG) GO TO 240
 ISN 0106          IT=IT+1
 ISN 0107          IF(IT.GT.ITMAX) GO TO 250
 ISN 0109          GO TO 210
                C
                C     TERMINATION MESSAGES
                C
 ISN 0110        240 WRITE(6,1030) IT
 ISN 0111       1030 FORMAT(1H0,5X,I5,'  ITERATIONS WERE REQUIRED IN GRELAX',///)
 ISN 0112          RETURN
 ISN 0113        250 IT=IT-1
 ISN 0114          WRITE(6,1040) IT,DLTMX2,CONVRG
 ISN 0115       1040 FORMAT(1H0,5X,'MAXIMUM ITERATIONS( ITMAX=',I5,'  ) WERE
                    1EXCEEDED IN GRELAX.',3X,'DLTMX2=',1PD10.2,'  CONVRG=',
                    21PD10.2,///)
 ISN 0116          STOP
 ISN 0117          END
```

THESE INSTRUCTIONS ARE N
NEEDED. THEY WERE
INSERTED FOR DEBUGGING
& WERE INADVERTENTLY
INCLUDED WHEN RUNNING
THIS LISTING.



RELAX SHEET 1 of 2



RELAX SHEET 2 of 2

8.0 STREAM

This subprogram solves for the stream function $\theta(x,y)$, and controls the Block-Line iteration algorithm until convergence is achieved. Given the potentials at the four ψ-lattice points NE, SE, SW, and NW of the θ-lattice point under consideration, it selects the one of 16 possible cases which assures that only negative exponents will be encountered in the operation DEXP(x), and that only differences in potential between adjacent ψ-lattice points will appear as such exponents. It calls subroutine BLOCK, each iteration, to implement the algorithm. It calls upon function programs SIFUNC and EXPFNC to evaluate exponential functions which supply, in turn, appropriate approximations when exponential underflow, or division by zero operations are imminent.

```
.VEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                  07
.UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

 ONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC
ISN 0002            SUBROUTINE STREAM(ITHMAX,RDELT)
                C
                C     SOLVES FOR THE STREAM FUNCTION THETA.
                C
ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
                C
ISN 0004            COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25)
                   1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,
                   2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
ISN 0005            COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,3
                   1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25
                   2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
ISN 0006            COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                   1VDRAIN,DIELK,GSS,CURRNT,EDENS0,PSIEQ
ISN 0007            COMMON/INTGR/IMAX,JMAX,JMAXS,JUNCTN,ITHMAX
                C
                C
                C     SET UP THE ITERATION. LOOP STARTS AT 100, ENDS JUST ABOVE 300
                C
ISN 0008            ITH=1
ISN 0009            J=JMAX
ISN 0010        100 J=J-1
ISN 0011            IF(J.LT.1) GO TO 300
ISN 0013            I=0
ISN 0014        110 I=I+1
ISN 0015            IF(I.GT.ITHMAX) GO TO 100
                C
                C     TO AVOID EXPONENT OVERFLOW OR UNDERFLOW, THE FINITE-DIFFERENCE
                C     EQUATION IS FACTORED BY DEXP(PSIMIN), WHERE PSIMIN IS THE MINIM
                C     VALUE OF PSI AT THE FOUR POINTS OF THE PSI LATTICE SURROUNDING
                C     THE POINT (I,J). THE COEFFICIENTS ARE FURTHER MANIPULATED SO TH
                C     ONLY DIFFERENCES IN PSI APPEAR AS EXPONENTS WHICH ARE NEGATIVE
                C     SO THAT SUITABLE APPROXIMATIONS CAN BE MADE WHEN LARGE NEGATIVE
                C     EXPONENTS ARE ENCOUNTERED. FUNCTION SUBPROGRAMS SIFUNC(DSI) AND
                C     EXPFNC(DSI) HANDLE THESE APPROXIMATIONS.
                C
                C     FIRST FIND PSIMIN AND ITS LOCATION. THE POINTS SW,SE,NE,&NW OF
                C     THE POINT (I,J) ARE DESIGNATED 1,2,3,4,RESPECTIVELEY.
                C
ISN 0017            SI1=PSI(I,J)
ISN 0018            SI2=PSI(I+1,J)
ISN 0019            SI3=PSI(I+1,J+1)
ISN 0020            SI4=PSI(I,J+1)
ISN 0021            SIMIN=DMIN1(SI1,SI2,SI3,SI4)
ISN 0022            IF(SI1.EQ.SIMIN) GO TO 120
ISN 0024            IF(SI2.EQ.SIMIN) GO TO 160
ISN 0026            IF(SI3.EQ.SIMIN) GO TO 190
ISN 0028            GO TO 220
                C
                C     POINT #1 (SW) IS THE LOCATION OF PSIMIN.
                C
ISN 0029        120 DSI=SI2-SI1
ISN 0030            FS=SIFUNC(DSI)
ISN 0031            DSI=SI4-SI1
ISN 0032            FW=SIFUNC(DSI)
ISN 0033            IF(SI3.GT.SI2) GO TO 130
ISN 0035            DSI=SI2-SI3
ISN 0036            FE=SIFUNC(DSI)
ISN 0037            DSI=SI3-SI1
ISN 0038            FE=FE*EXPFNC(DSI)
ISN 0039            GO TO 140
ISN 0040        130 DSI=SI3-SI2
ISN 0041            FE=SIFUNC(DSI)
ISN 0042            DSI=SI2-SI1
ISN 0043            FE=FE*EXPFNC(DSI)
ISN 0044        140 IF(SI3.GT.SI4) GO TO 150
ISN 0046            DSI=SI4-SI3
```

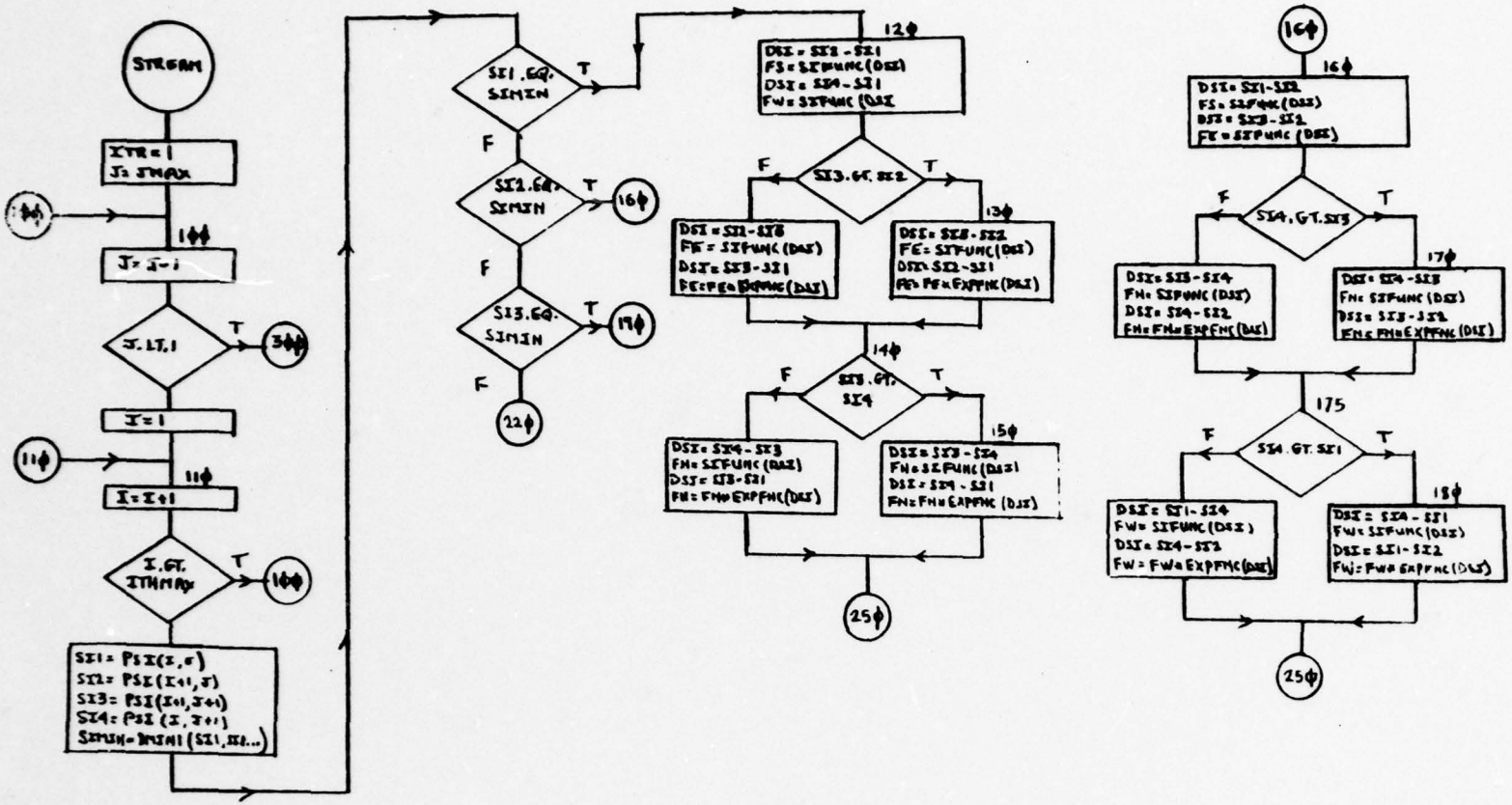
```
0047              FN=SIFUNC(DSI)
0048              DSI=SI3-SI1
0049              FN=FN*EXPFNC(DSI)
0050              GO TO 250
0051          150 DSI=SI3-SI4
0052              FN=SIFUNC(DSI)
0053              DSI=SI4-SI1
0054              FN=FN*EXPFNC(DSI)
0055              GO TO 250
                  C
                  C     POINT #2 (SE) IS THE LOCATION OF PSIMIN.
                  C
0056          160 DSI=SI1-SI2
0057              FS=SIFUNC(DSI)
0058              DSI=SI3-SI2
0059              FE=SIFUNC(DSI)
0060              IF(SI4.GT.SI3) GO TO 170
0062              DSI=SI3-SI4
0063              FN=SIFUNC(DSI)
0064              DSI=SI4-SI2
0065              FN=FN*EXPFNC(DSI)
0066              GO TO 175
0067          170 DSI=SI4-SI3
0068              FN=SIFUNC(DSI)
0069              DSI=SI3-SI2
0070              FN=FN*EXPFNC(DSI)
0071          175 IF(SI4.GT.SI1) GO TO 180
0073              DSI=SI1-SI4
0074              FW=SIFUNC(DSI)
0075              DSI=SI4-SI2
0076              FW=FW*EXPFNC(DSI)
0077              GO TO 250
0078          180 DSI=SI4-SI1
0079              FW=SIFUNC(DSI)
0080              DSI=SI1-SI2
0081              FW=FW*EXPFNC(DSI)
0082              GO TO 250
                  C
                  C     POINT #3 (NE) IS THE LOCATION OF PSIMIN.
                  C
0083          190 DSI=SI2-SI3
0084              FE=SIFUNC(DSI)
0085              DSI=SI4-SI3
0086              FN=SIFUNC(DSI)
0087              IF(SI1.GT.SI2) GO TO 200
0089              DSI=SI2-SI1
0090              FS=SIFUNC(DSI)
0091              DSI=SI1-SI3
0092              FS=FS*EXPFNC(DSI)
0093              GO TO 205
0094          200 DSI=SI1-SI2
0095              FS=SIFUNC(DSI)
0096              DSI=SI2-SI3
0097              FS=FS*EXPFNC(DSI)
0098          205 IF(SI1.GT.SI4) GO TO 210
0100              DSI=SI4-SI1
0101              FW=SIFUNC(DSI)
0102              DSI=SI1-SI3
0103              FW=FW*EXPFNC(DSI)
0104              GO TO 250
0105          210 DSI=SI1-SI4
0106              FW=SIFUNC(DSI)
0107              DSI=SI4-SI3
0108              FW=FW*EXPFNC(DSI)
0109              GO TO 250
                  C
                  C     POINT #4 (NW) IS THE LOCATION OF PSIMIN.
                  C
0110          220 DSI=SI1-SI4
0111              FW=SIFUNC(DSI)
0112              DSI=SI3-SI4
0113              FN=SIFUNC(DSI)
```

```
ISN 0114              IF(SI2.GT.SI1) GO TO 230
ISN 0116              DSI=SI1-SI2
ISN 0117              FS=SIFUNC(DSI)
      N 0118          DSI=SI2-SI4
      N 0119          FS=FS*EXPFNC(DSI)
ISN 0120              GO TO 235
ISN 0121          230 DSI=SI2-SI1
ISN 0122              FS=SIFUNC(DSI)
ISN 0123              DSI=SI1-SI4
ISN 0124              FS=FS*EXPFNC(DSI)
ISN 0125          235 IF(SI2.GT.SI3) GO TO 240
ISN 0127              DSI=SI3-SI2
ISN 0128              FE=SIFUNC(DSI)
ISN 0129              DSI=SI2-SI4
ISN 0130              FE=FE*EXPFNC(DSI)
ISN 0131              GO TO 250
ISN 0132          240 DSI=SI2-SI3
ISN 0133              FE=SIFUNC(DSI)
ISN 0134              DSI=SI3-SI4
ISN 0135              FE=FE*EXPFNC(DSI)
                  C
                  C     HAVING CALCULATED THE AVERAGE EXP(-PSI) FUNCTIONS FOR THE
                  C     MIDPOINTS E,W,N,&S OF THE POINT (I,J) (MUCK'S "A" FUNCTIONS)
                  C     NOW CALCULATE THE MATRIX COEFFICIENTS B,D,E,F,H,AND Q FOR THE
                  C     POINT (I,J).
                  C
ISN 0136          250 B(I,J)=THSTH(I,J)*FS
ISN 0137              D(I,J)=THWEST(I,J)*FW
ISN 0138              F(I,J)=THEAST(I,J)*FE
ISN 0139              H(I,J)=THNRTH(I,J)*FN
ISN 0140              E(I,J)=-(B(I,J)+D(I,J)+F(I,J)+H(I,J))
ISN 0141              Q(I,J)=0.
                  C
                  C     MODIFY THE COEFFICIENTS ALONG THE BOTTOM ROW.
                  C
ISN 0142              IF(J.GT.1) GO TO 200
ISN 0144              Q(I,1)=-B(I,1)
                  C
                  C     SIMILARLY ALONG THE LEFT BORDER
                  C
ISN 0145          200 IF(I.GT.1) GO TO 280
ISN 0147              IF(J.LT.JUNCTN) GO TO 270
ISN 0149              E(I,J)=E(I,J)+D(I,J)
ISN 0150              GO TO 110
ISN 0151          270 Q(I,J)=Q(I,J)-D(I,J)
ISN 0152              GO TO 110
                  C
                  C     SIMILARLY ALONG THE RIGHT BORDER
                  C
ISN 0153          280 IF(I.LT.ITHMAX) GO TO 110
ISN 0155              IF(J.LT.JUNCTN) GO TO 290
ISN 0157              E(I,J)=E(I,J)+F(I,J)
ISN 0158              GO TO 110
ISN 0159          290 Q(I,J)=Q(I,J)-F(I,J)
ISN 0160              GO TO 110
                  C
                  C     FIND THETA USING BLOCK-LINE ITERATION (SLOR).
                  C
ISN 0161          300 CALL BLOCK(RESID)
ISN 0162              IF(RESID.LE.RDELT) RETURN
ISN 0164              ITR=ITR+1
ISN 0165              IF(ITR.GT.ITRMAX) GO TO 310
ISN 0167              GO TO 300
ISN 0168          310 WRITE(6,1000)
ISN 0169         1000 FORMAT(1H0,5X,'MAXIMUM ITERATIONS EXCEEDED IN STREAM',//)
ISN 0170              STOP
ISN 0171              END
```

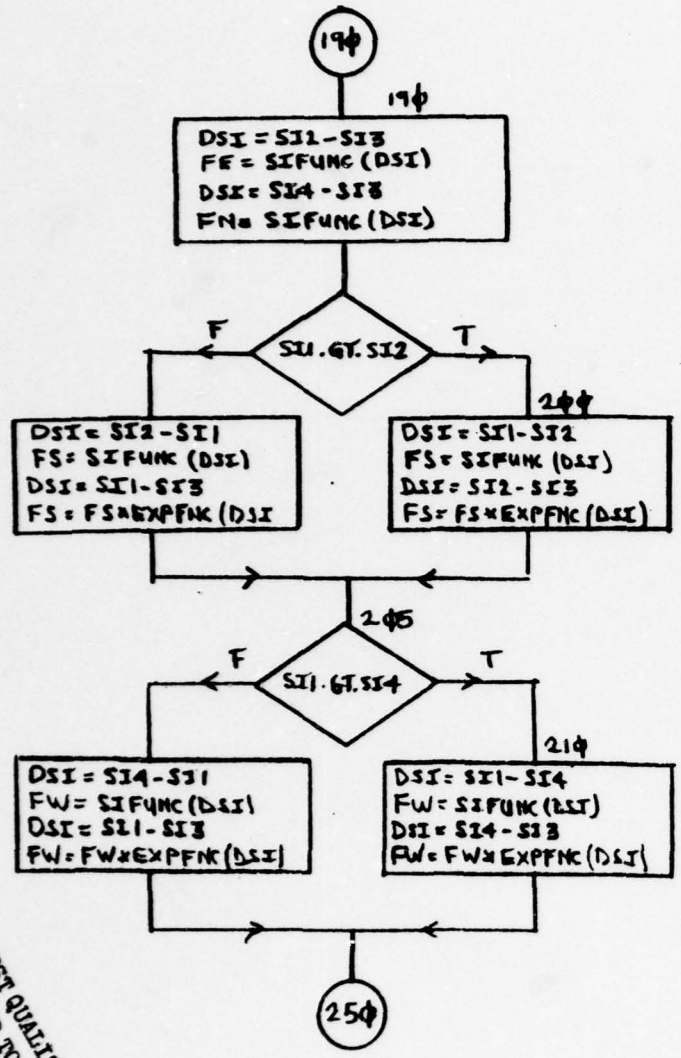
```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
ED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
 IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
            SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N
0002                FUNCTION SIFUNC(DSI)
                C
                C       CALCULATES MUCK'S "A" FUNCTIONS OF PSI FOR SUBROUTINE STREAM.
                C
0003                IMPLICIT REAL*8(A-H,O-Z)
                C
0004                IF(DSI.GT.1.7D2) GO TO 100
0006                IF(DSI.LT.1.D-9) GO TO 110
0008                SIFUNC=(1.D0-DEXP(-DSI))/DSI
0009                RETURN
0010            100 SIFUNC=1.D0/DSI
0011                RETURN
0012            110 SIFUNC=1.D0
0013                RETURN
0014                END
```

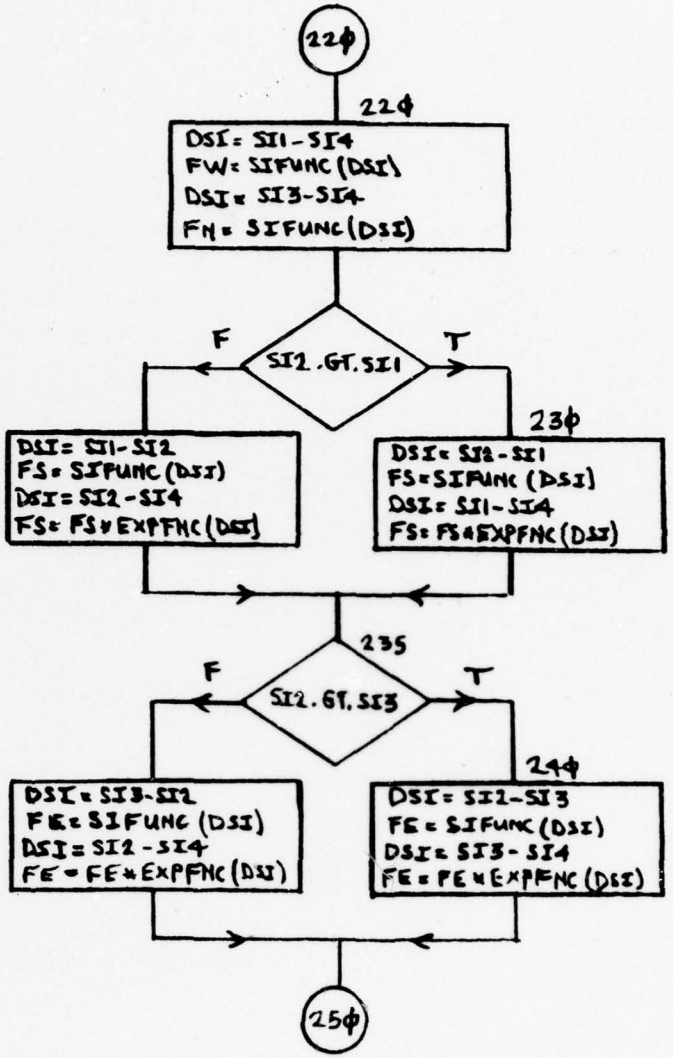
```
VEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
IONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

  1 0002                FUNCTION EXPFNC(DSI)
                C
                C       CALCULATES DEXP(-DSI) FOR SUBROUTINE STREAM.
                C
ISN 0003                IMPLICIT REAL*8(A-H,O-Z)
                C
ISN 0004                IF(DSI.GT.1.7D2) GO TO 100
ISN 0006                EXPFNC=DEXP(-DSI)
ISN 0007                RETURN
ISN 0008            100 EXPFNC=0.
ISN 0009                RETURN
ISN 0010                END
```

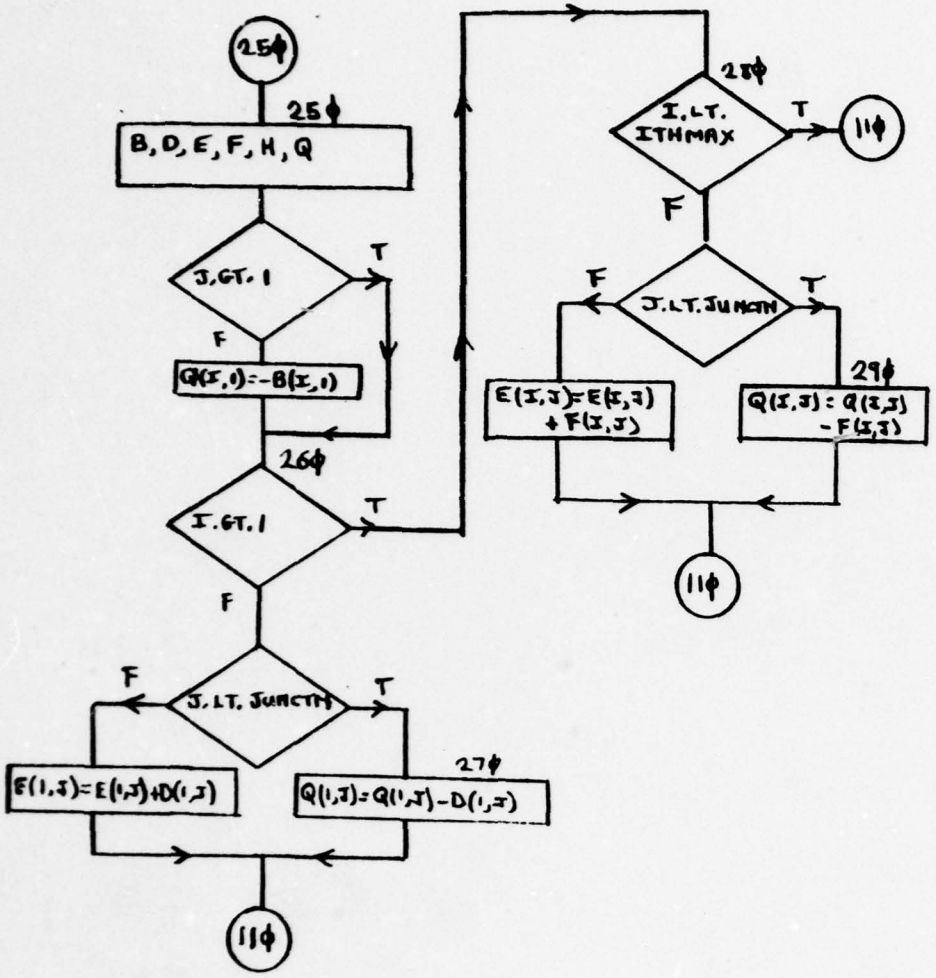


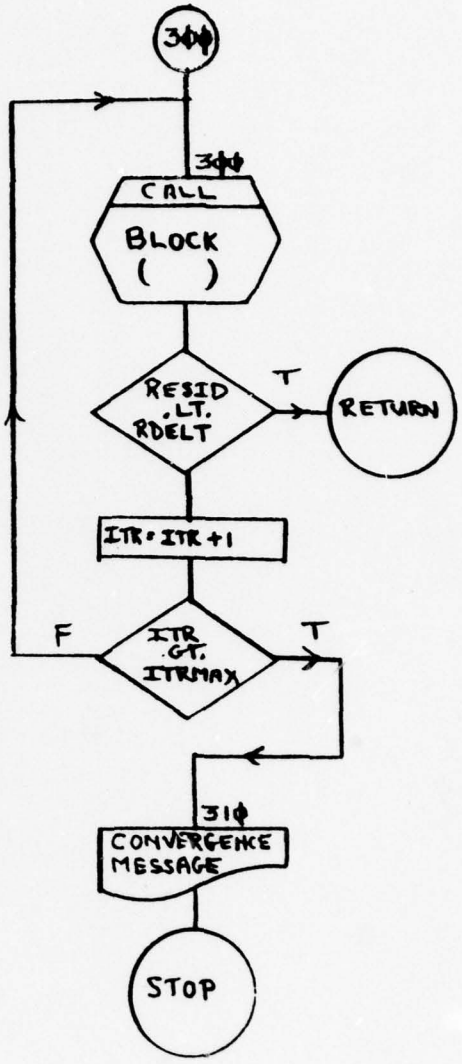
STREAM SHEET 1 of 3





STREAM SHEET 2 of 3





STREAM SHEET 3 of 3

9.0 BLOCK

This subprogram implements the Block-Line iteration method for matrix inversion described by Varga [18], the iteration procedure itself being controlled by subroutine STREAM.

---

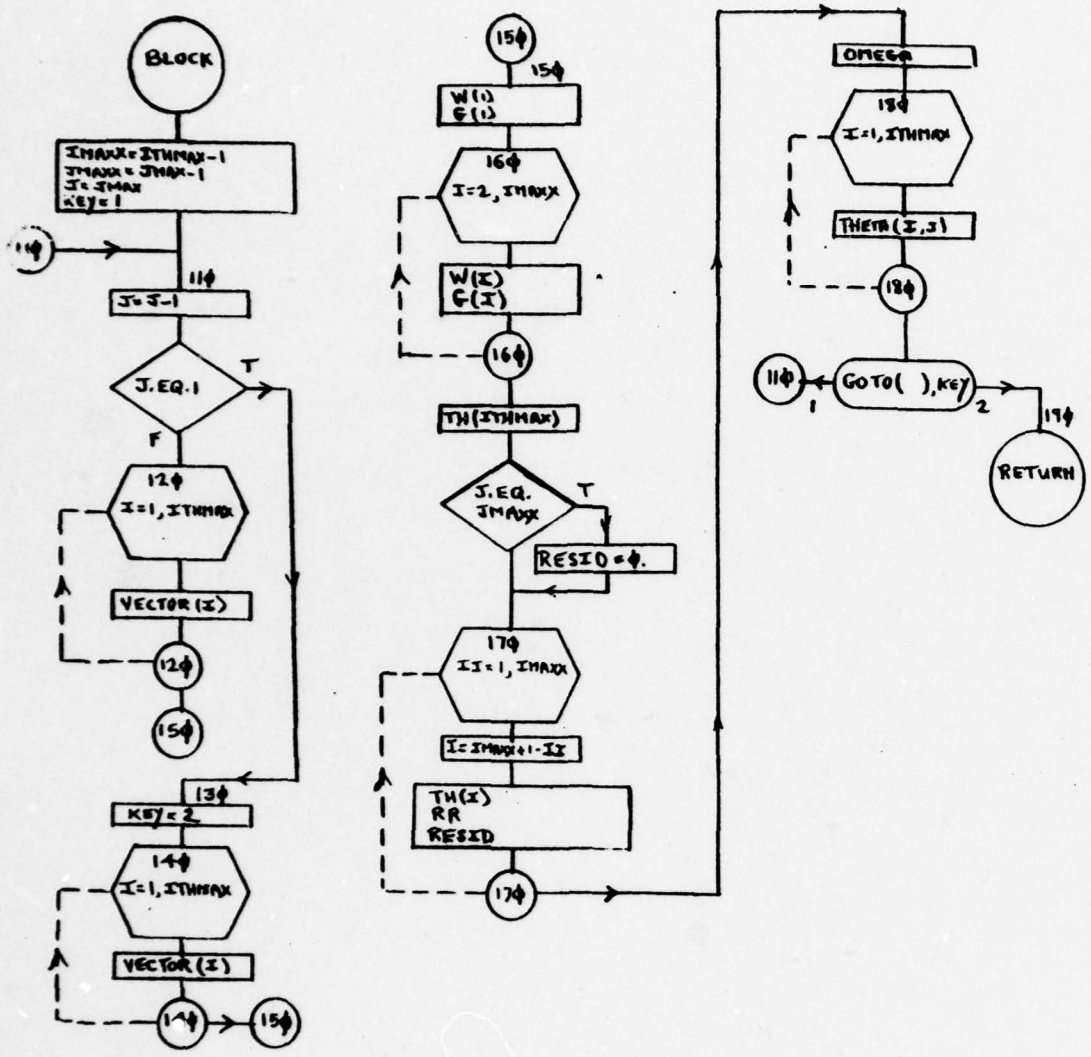
[18] R.S. Varga, MATRIX ITERATIVE ANALYSIS, Prentice-Hall, 1962, pp. 195-199.

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
EC OPTIONS:  NODECK,LOAD,OPT=2,SOURCE,MAP

, IN EFFECT:  NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
              SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NE

CC02                SUBROUTINE BLOCK(RESID)
                C
                C     IMPLEMENTS BLOCK-LINE ITERATION FOR SOLVING FOR THE STREAM
                C     FUNCTION THETA.  SEE VARGA "ITERATIVE MATRIX ANALYSIS", PRENTICE-
                C     HALL, 1962,  PP. 194-199.
                C
0003                IMPLICIT REAL*8(A-H,O-Z)
                C
0004                COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                   1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,30
                   2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                   1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                   2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006                COMMON/REAL/XCHNL,YSUB,YCAIDE,YJNCTN,DUNOR,ACCPTR,VGATE,VSUB,
                   1VDRAIN,DIELK,USS,CURRNT,EDENS0,PSIEQ
0007                COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
0008                DIMENSION W(40),G(40),TH(40),VECTOR(40)
                C
0009                IMAXX=ITHMAX-1
0010                JMAXX=JMAX-1
0011                J=JMAX
                C
                C     OMIT THE ROW J=JMAX, WHERE THETA IS FIXED AT ZERO.
                C     INDEX DOWNWARD BY ROWS. KEY IS USED AS A SWITCH FOR THE
                C     COMPUTED GO TO STATEMENT FOLLOWING STATEMENT # 180 BELOW.
                C
0012                KEY=1
0013              110 J=J-1
0014                IF(J.EQ.1) GO TO 130
                C
                C     ALL INTERIOR ROWS.
                C
0016                DO 120 I=1,ITHMAX
0017              120 VECTOR(I)=-B(I,J)*THETA(I,J-1)-H(I,J)*THETA(I,J+1)+Q(I,J)
0018                GO TO 150
                C
                C     THE ROW J=1. (SET KEY =2 TO FLAG THIS AS THE LAST ROW TREATED.)
                C
0019              130 KEY=2
0020                DO 140 I=1,ITHMAX
0021              140 VECTOR(I)=-H(I,J)*THETA(I,J+1)+Q(I,J)
                C
                C     RECURSIVE DETERMINATION OF THETA(I,J). FIRST SET UP THE VECTORS
                C     W AND G DEFINED BY VARGA.
                C
0022              150 W(1)=F(1,J)/E(1,J)
0023                G(1)=VECTOR(1)/E(1,J)
0024                DO 160 I=2,IMAXX
0025                DENOM=E(I,J)-D(I,J)*W(I-1)
0026                W(I)=F(I,J)/DENOM
0027              160 G(I)=(VECTOR(I)-D(I,J)*G(I-1))/DENOM
                C
                C     CALCULATE THETA(I,J) USING VARGA'S EQ (6.46')
                C
0028                TH(ITHMAX)=(VECTOR(ITHMAX)-D(ITHMAX,J)*G(IMAXX))/
                   1(E(ITHMAX,J)-D(ITHMAX,J)*W(IMAXX))
0029                IF(J.EQ.JMAXX) RESID=0.
0031                DO 170 II=1,IMAXX
0032                I=IMAXX+1-II
0033                TH(I)=G(I)-W(I)*TH(I+1)
0034                RR=DABS(THETA(I,J)-TH(I))
0035              170 RESID=DMAX1(DABS(RR),RESID)
                C
                C     FILL IN NEW THETA (USING OMEGA=1.0) FOR ROW J. REPEAT PROCEDURE
                C     FOR ROW J-1 UNLESS J=1 (FLAGGED BY KEY=2). OTHERWISE TERMINATE.
```

```
      C
ISN 0036            OMEGA=1.000
ISN 0037            DO 180 I=1,ITHMAX
ISN 0038        180 THETA(I,J)=OMEGA*(TH(I)-THETA(I,J))+THETA(I,J)
ISN 0039            GO TO (110,190),KEY
ISN 0040        190 RETURN
ISN 0041            END
```



BLOCK SHEET 1 of 1

10.0 DNSITY

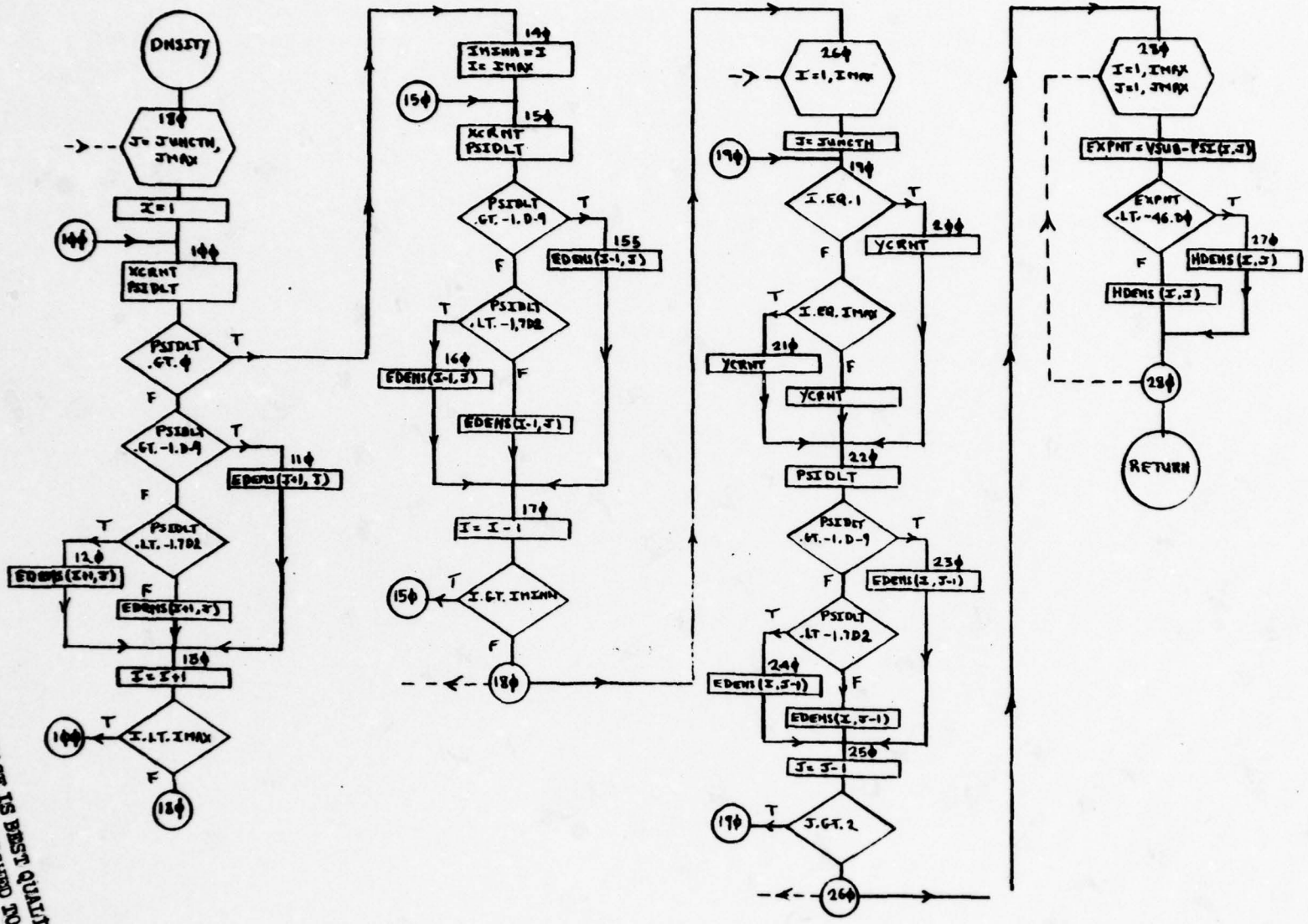
This subprogram implements Mock's "marching method," described in Chapter I, Section 3.04, for determining the electron density n(x,y). It applies appropriate approximations for DEXP(-x) when x is sufficiently large.

```
2.1  ( JAN 75 )                          05/360  FORTRAN H EXTENDED                    DAT
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

0002                 SUBROUTINE DNSITY(DNR)
                C
                C    USES NUCKS INTEGRAL METHOD FOR FINDING ELECTRON DENSITY. USES
                C    THE BOLTZMANN RELATIONSHIP FOR HOLE DENSITY.
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
                C
0004                 COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                    1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,30
                    2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                 COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30),
                    1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                    2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006                 COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                    1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
0007                 COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
                C    ELECTRON DENSITY IN THE REGION "IN CONTACT WITH" THE SOURCE &
                C    DRAIN.
                C
                C    INTEGRATE AWAY FROM SOURCE UNTIL THE POTENTIAL MINIMUM IS
                C    ENCOUNTERED.
                C
0008                 DO 180 J=JUNCTN,JMAX
0009                 I=1
0010            100  XCRNT=CURRNT*(THETA(I,J-1)-THETA(I,J))/YTHDLT(J)
0011                 PSIDLT=PSI(I+1,J)-PSI(I,J)
0012                 IF(PSIDLT.GT.0.) GO TO 140
0014                 IF(PSIDLT.GT.-1.D-9) GO TO 110
0016                 IF(PSIDLT.LT.-2.D1) GO TO 120
0018                 EDENS(I+1,J)=EDENS(I,J)*DEXP(PSIDLT)+XCRNT*XDLT(I+1)*
                    1(DEXP(PSIDLT)-1.D0)/PSIDLT
0019                 GO TO 130
0020            110  EDENS(I+1,J)=EDENS(I,J)+XCRNT*XDLT(I+1)
0021                 GO TO 130
0022            120  EDENS(I+1,J)=-XCRNT*XDLT(I+1)/PSIDLT
0025            130  CONTINUE
0024                 I=I+1
0025                 IF(I.LT.IMAX) GO TO 100
0027                 GO TO 180
                C
                C    INTEGRATE AWAY FROM DRAIN FOR X BEYOND POTENTIAL MINIMUM.
                C
0028            140  IMINN=1
0029                 IF(IMINN.EQ.1) IMINN=2
0031                 I=IMAX
0032            150  XCRNT=CURRNT*(THETA(I-1,J-1)-THETA(I-1,J))/YTHDLT(J)
0033                 PSIDLT=PSI(I-1,J)-PSI(I,J)
0034                 IF(PSIDLT.GT.-1.D-9) GO TO 155
0036                 IF(PSIDLT.LT.-2.D1) GO TO 160
0038                 EDENS(I-1,J)=EDENS(I,J)*DEXP(PSIDLT)-XCRNT*XDLT(I)*
                    1(DEXP(PSIDLT)-1.D0)/PSIDLT
0039                 GO TO 170
0040            155  EDENS(I-1,J)=EDENS(I,J)-XCRNT*XDLT(I)
0041                 GO TO 170
0042            160  EDENS(I-1,J)=XCRNT*XDLT(I)/PSIDLT
0043            170  CONTINUE
0044                 I=I-1
0045                 IF(I.GT.IMINN) GO TO 150
0047            180  CONTINUE
                C
                C    INTEGRATE DOWNWARD FROM THE ROW CONTACTING THE LOWER BOUNDARY OF
                C    THE SOURCE & DRAIN TO FILL IN THE SUBSTRATE REGION BELOW THAT ROW
                C
0048                 DO 260 I=1,IMAX
0049                 J=JUNCTN
0050            190  IF(I.EQ.1) GO TO 200
```

```
ISN 0052              IF(I.EQ.IMAX) GO TO 210
ISN 0054              YCRNT=-CURRNT*(THETA(I,J-1)-THETA(I-1,J-1))/XTHDLT(I)
ISN 0055              GO TO 220
ISN 0056          200 YCRNT=-CURRNT*(THETA(1,J-1)-1.D0)/XTHDLT(1)
ISN 0057              GO TO 220
ISN 0058          210 YCRNT=-CURRNT*(1.D0-THETA(ITHMAX,J-1))/XTHDLT(IMAX)
ISN 0059          220 PSIDLT=PSI(I,J-1)-PSI(I,J)
ISN 0060              IF(PSIDLT.GT.-1.D-9) GO TO 230
ISN 0062              IF(PSIDLT.LT.-2.D1) GO TO 240
ISN 0064              EDENS(I,J-1)=EDENS(I,J)*DEXP(PSIDLT)+YCRNT*YDLT(J)*
                     1(DEXP(PSIDLT)-1.D0)/PSIDLT
ISN 0065              GO TO 250
ISN 0066          230 EDENS(I,J-1)=EDENS(I,J)+YCRNT*YDLT(J)
ISN 0067              GO TO 250
ISN 0068          240 EDENS(I,J-1)=-YCRNT*YDLT(J)/PSIDLT
ISN 0069          250 CONTINUE
ISN 0070              J=J-1
ISN 0071              IF(J.GT.2) GO TO 190
ISN 0073          260 CONTINUE
                  C
                  C     CALCULATE THE HOLE DENSITY
                  C
ISN 0074              DO 280 I=1,IMAX
ISN 0075              DO 280 J=1,JMAX
ISN 0076              EXPNT=VSUB-PSI(I,J)
ISN 0077              IF(EXPNT.LT.-2.D1) GO TO 270
ISN 0079              HDENS(I,J)=DEXP(EXPNT)/DNR
ISN 0080              GO TO 280
ISN 0081          270 HDENS(I,J)=2.0D-9
ISN 0082          280 CONTINUE
ISN 0083              RETURN
ISN 0084              END
```



DNSITY SHEET 1 of 1

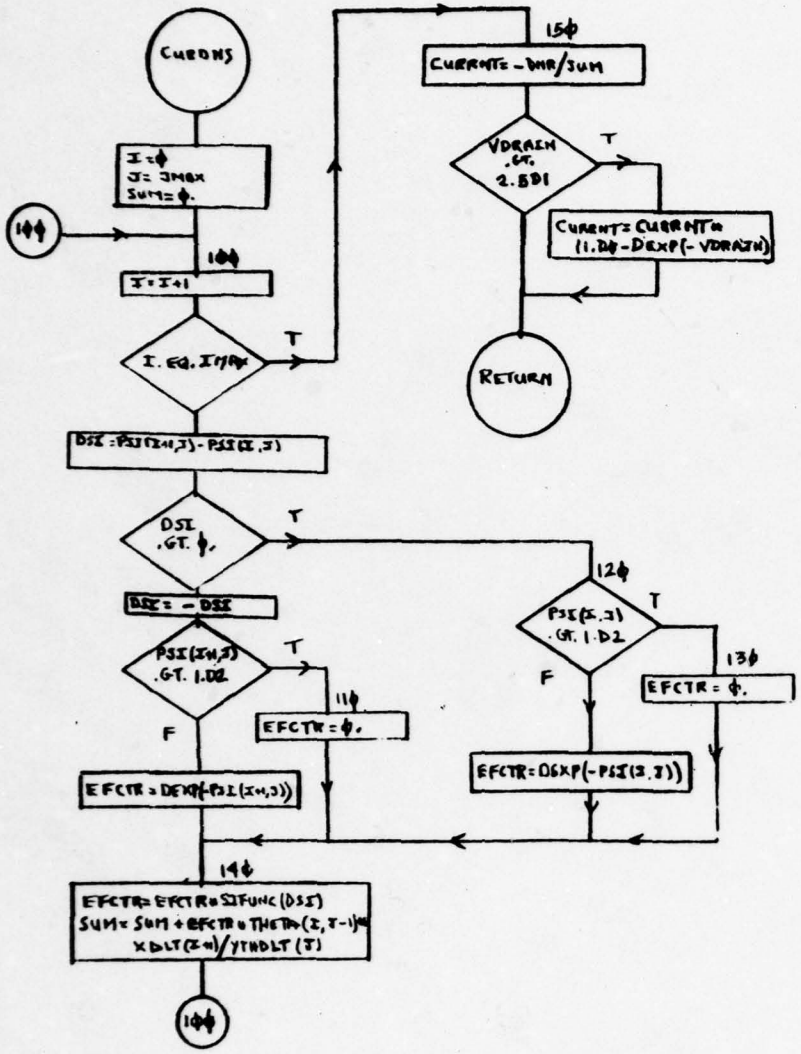
11.0 CURDNS

This subprogram calculates the drain current factor J_o, as described in Chapter I, Section 3.03. The trapezoidal rule is invoked in evaluating the integral discussed there along the row of the ψ-lattice just below the oxide-silicon interface.

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

0002                 SUBROUTINE CURDNS(DNR)
                C
                C    CALCULATES THE CURRENT DENSITY SCALE FACTOR JO  (CURRNT) BY
                C    INTEGRATING FROM SOURCE TO DRAIN ALONG THE ROW JUST BELOW THE
                C    SI-OX INTERFACE.
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
                C
                C
0004                 COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,30
                2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                 COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006                 COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                1VDRAIN,DIELK,GSS,CURRNT,EDENS0,PSIEQ
0007                 COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
0008                 IF(VDRAIN.GT.0.) GO TO 90
0010                 CURRNT=0.
0011                 RETURN
                C
                C    EVALUATE THE LINE INTEGRAL OF EXP(-PSI)*(DTHETA/DY) ALONG THE ROW
                C    JUST BELOW THE INTERFACE. THE IF STATEMENTS BELOW ARE TO AVOID
                C    OVERFLOW OR UNDERFLOW DUE TO LARGE POSITIVE OR NEGATIVE EXPONENTS
                C    AS WELL AS OVERFLOW DUE TO DIVISION BY A NUMBER CLOSE TO ZERO.
                C
0012              90 I=0
0013                 J=JMAX
0014                 SUM=0.
0015             100 I=I+1
0016                 IF(I.EQ.IMAX) GO TO 150
0018                 DSI=PSI(I+1,J)-PSI(I,J)
0019                 IF(DSI.GT.0) GO TO 120
0021                 DSI=-DSI
0022                 IF(PSI(I+1,J).GT.1.7D2) GO TO 110
0024                 EFCTR=DEXP(-PSI(I+1,J))
0025                 GO TO 140
0026             110 EFCTR=0.
0027                 GO TO 140
0028             120 IF(PSI(I,J).GT.1.7D2) GO TO 130
0030                 EFCTR=DEXP(-PSI(I,J))
0031                 GO TO 140
0032             130 EFCTR=0.
0033             140 EFCTR=EFCTR*SIFUNC(DSI)
0034                 SUM=SUM+EFCTR*THETA(I,J-1)*XDLT(I+1)/YTHDLT(J)
0035                 GO TO 100
0036             150 CURRNT=-DNR/SUM
0037                 IF(VDRAIN.LT.1.7D2) CURRNT=CURRNT*(1.D0-DEXP(-VDRAIN))
0039                 RETURN
0040                 END
```



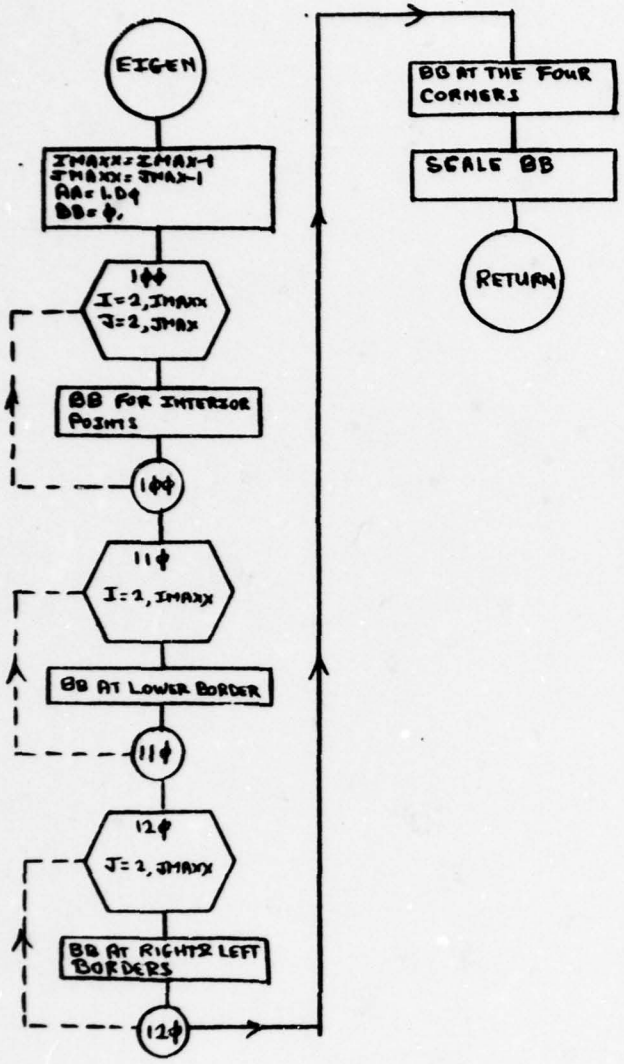
CURDNS SHEET 1 of 1

12.0 EIGEN

This subprogram evaluates the upper and lower bounds, b and a, of the eigenvealues of equation (24), the finite-difference equation for ψ described in Chapter I, Section 4.01. The lower bound, a, is set to unity, since its value is not critical for the Picard iteration. The upper bound, b, is evaluated from eq. (54), using a two-dimensional trapezoidal rule to approximate the integral contained therein.

```
:VEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
:UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP
TIONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                 SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

ISN 0002              SUBROUTINE EIGEN(AA,BB)
                    C
                    C     ESTIMATES THE UPPER & LOWER BOUNDS BB AND AA OF THE EIGENVALUES
                    C     THE MATRIX USED WITH THE PICARD ALGORITHM.INTEGRATES (EDENS+HDEN
                    C     OVER THE SUBSTRATE REGION. USING THE TRAPEZOIDAL RULE.
                    C
ISN 0003              IMPLICIT REAL*8(A-H,O-Z)
                    C
                    C
ISN 0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                     1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,3
                     2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
ISN 0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30
                     1THEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25)
                     2THEAST(40,25),THSTH(40,25),THWEST(40,25),DAREA(41,25)
ISN 0006              COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                     1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
ISN 0007              COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                    C
                    C     THE REGION EXCLUDING THE BORDERS
                    C
ISN 0008              IMAXX=IMAX-1
ISN 0009              JMAXX=JMAX-1
ISN 0010              AA=1.D0
ISN 0011              BB=0.
ISN 0012              DO 100 I=2,IMAXX
ISN 0013              DO 100 J=2,JMAX
ISN 0014          100 BB=BB+(EDENS(I,J)+HDENS(I,J))*XTHDLT(I)*YTHDLT(J)
                    C
                    C     THE LOWER BORDER, EXCLUDING THE CORNERS.
                    C
ISN 0015              DO 110 I=2,IMAXX
ISN 0016          110 BB=BB+(EDENS(I,1)+HDENS(I,1))*XTHDLT(I)*YDLT(2)/2.D0
                    C
                    C     THE RIGHT AND LEFT BORDERS EXCLUDING THE CORNERS.
                    C
ISN 0017              DO 120 J=2,JMAXX
ISN 0018              BB=BB+(EDENS(1,J)+HDENS(1,J))*XDLT(2)*YTHDLT(J)/2.D0
ISN 0019          120 BB=BB+(EDENS(IMAX,J)+HDENS(IMAX,J))*XDLT(IMAX)*YTHDLT(J)/2.D0
                    C
                    C     THE FOUR CORNERS
                    C
ISN 0020              BB=BB+(EDENS(1,1)+HDENS(1,1))*XDLT(2)*YDLT(2)/4.D0
ISN 0021              BB=BB+(EDENS(IMAX,1)+HDENS(IMAX,1))*XDLT(IMAX)*YDLT(2)/4.D0
ISN 0022              BB=BB+(EDENS(1,JMAX)+HDENS(1,JMAX))*XDLT(2)*YTHDLT(JMAX)/2.D0
ISN 0023              BB=BB+(EDENS(IMAX,JMAX)+HDENS(IMAX,JMAX))*XDLT(IMAX)*YTHDLT(JMAX
                     1/2.D0
ISN 0024              BB=BB*YSUB/(4.D0*XCHNL)
ISN 0025              BB=BB+1.D0
ISN 0026              RETURN
ISN 0027              END
```



EIGEN SHEET 1 of 1

13.0 POISSN

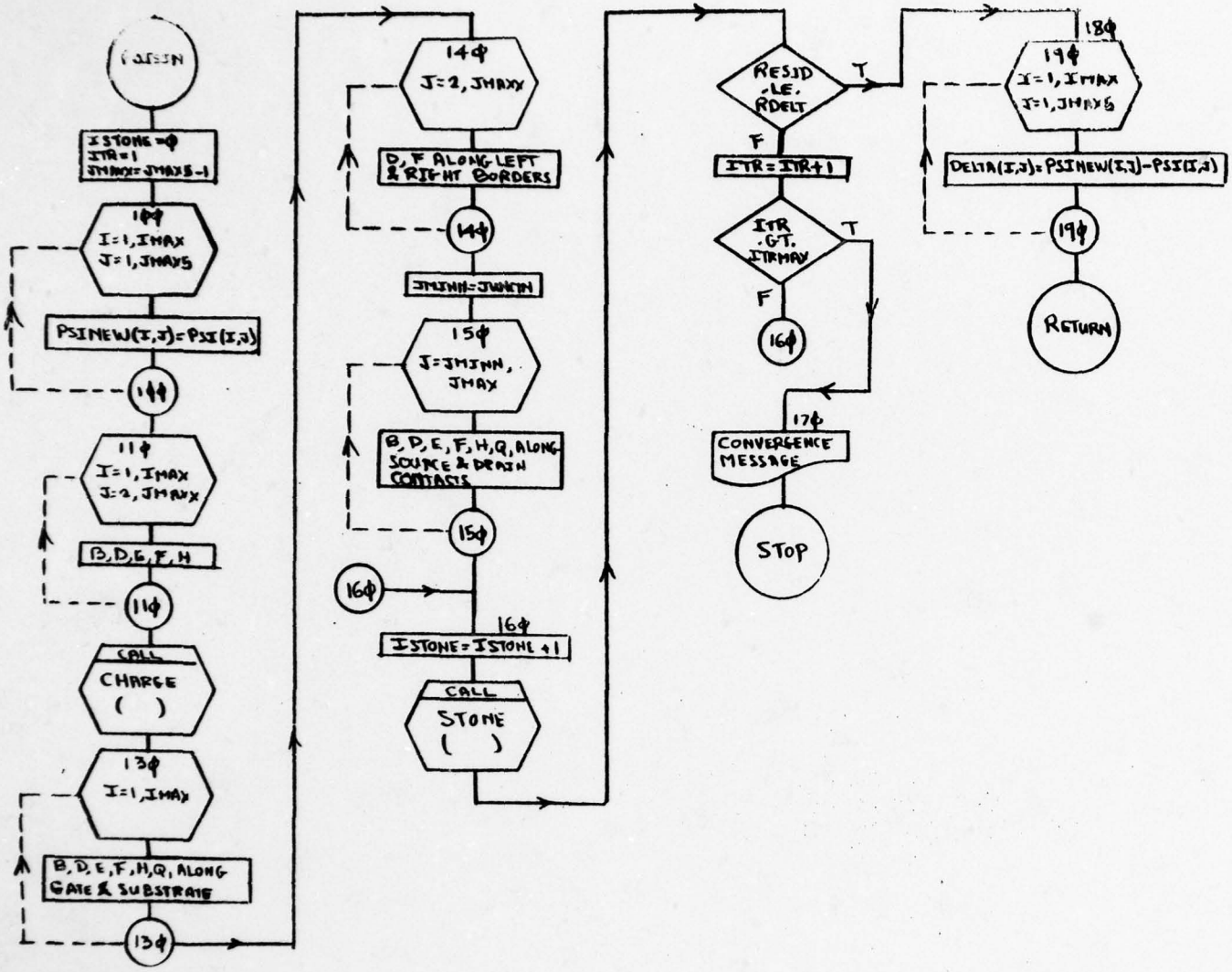
This subprogram controls Stone's iterative method for finding $\psi(x,y)$, given the charge densities n, p, and N_A. It calls upon subroutines CHARGE (to calculate the charge term Q_p) and STONE (to apply Stone's method), during each iteration.

```
2.1  ( JAN 75 )                    05/360  FORTRAN H EXTENDED                    DAT
ED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

 IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
            SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

0002                  SUBROUTINE POISSN(ITRMAX,RDELT,KEY)
                C
                C     SOLVES POISSONS EQUATION FOR POTENTIAL USING STONES METHOD.
                C
0003                  IMPLICIT REAL*8(A-H,O-Z)
                C
0004                  COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                     1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,30
                     2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                  COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                     1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                     2THEAST(40,25),THSTH(40,25),THWEST(40,25),QAREA(41,25)
0006                  COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                     1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEG
0007                  COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
                C
0008                  DIMENSION PSINEW(41,30)
                C
                C     SET UP COEFFICIENTS FOR SUBROUTINE STONE
                C
0009                  ISTONE=0
0010                  ITR=1
0011                  JMAXX=JMAX5-1
0012                  DO 100 I=1,IMAX
0013                  DO 100 J=1,JMAX5
0014             100  PSINEW(I,J)=PSI(I,J)
                C
                C     GENERAL EXPRESSIONS FOR STONE'S COEFFICIENTS. SPECIAL VALUES FOR
                C     THE BOUNDARIES ARE SUBSTITUTED BELOW.
                C
0015                  DO 110 I=1,IMAX
0016                  DO 110 J=2,JMAXX
0017                  B(I,J)=HSTH(I,J)
0018                  D(I,J)=HWEST(I,J)
0019                  E(I,J)=HCNTR(I,J)
0020                  F(I,J)=HEAST(I,J)
0021             110  H(I,J)=HNRTH(I,J)
                C
                C     CHARGE DENSITY THROUGH SUBSTRATE.
                C
0022                  CALL CHARGE(KEY)
                C
                C     SUPPLY COEFFICIENTS ALONG THE GATE & SUBSTRATE CONTACTS.
                C
0023                  DO 130 I=1,IMAX
0024                  B(I,1)=0.
0025                  D(I,1)=0.
0026                  E(I,1)=1.D0
0027                  F(I,1)=0.
0028                  H(I,1)=0.
0029                  G(I,1)=PSIEG
0030                  B(I,JMAX5)=0.
0031                  D(I,JMAX5)=0.
0032                  E(I,JMAX5)=1.D0
0033                  F(I,JMAX5)=0.
0034                  H(I,JMAX5)=0.
0035             130  G(I,JMAX5)=PSI(2,JMAX5)
                C
                C     MODIFY COEFFICIENTS ALONG LEFT & RIGHT BORDERS
                C
0036                  DO 140 J=2,JMAXX
0037                  D(1,J)=0.
0038                  F(1,J)=2.D0*F(1,J)
0039                  D(IMAX,J)=2.D0*D(IMAX,J)
0040             140  F(IMAX,J)=0.
                C
                C     MODIFY WHAT WAS JUST DONE ALONG SOURCE & DRAIN CONTACTS.
```

```
                C
ISN 0041            JMINN=JUNCTN
ISN 0042            DO 150 J=JMINN,JMAX
ISN 0043            B(1,J)=0.
ISN 0044            E(1,J)=1.00
ISN 0045            F(1,J)=0.
ISN 0046            H(1,J)=0.
ISN 0047            G(1,J)=PSI(1,J)
ISN 0048            B(IMAX,J)=0.
ISN 0049            D(IMAX,J)=0.
ISN 0050            E(IMAX,J)=1.00
ISN 0051            F(IMAX,J)=0.
ISN 0052            H(IMAX,J)=0.
ISN 0053          150 G(IMAX,J)=PSI(IMAX,J)
                C
                C       ITERATIVE STONES METHOD
                C
ISN 0054          160 ISTONE=ISTONE+1
ISN 0055            CALL STONE(ISTONE,IMAX,JMAX5,RESID,PSINEW)
ISN 0056            IF(RESID.LE.RDELT) GO TO 180
ISN 0058            ITR=ITR+1
ISN 0059            IF(ITR.GT.ITRMAX) GO TO 170
ISN 0061            GO TO 160
ISN 0062          170 WRITE(6,1000)
ISN 0063         1000 FORMAT(1H0,5X,'MAXIMUM ITERATIONS EXCEEDED IN POISSN',//)
ISN 0064            STOP
ISN 0065          180 DO 190 I=1,IMAX
ISN 0066            DO 190 J=1,JMAX5
ISN 0067          190 DELTA(I,J)=PSINEW(I,J)-PSI(I,J)
ISN 0068            RETURN
ISN 0069            END
```



POISSN SHEET 1 of 1

14.0 STONE & STONE1

This subprogram implements Stone's method for determining $\psi(x,y)$, given the charge term $\rho(x,y)$ in Poisson's equation. The iterative scheme necessary is controlled by subroutine POISSN. This particular subprogram uses a notation which deviates from the notation utilized in other subprograms to make it more transparent to those who study Stone's paper [7]. For example, the indices (I,J) used elsewhere become (J,K), as defined by Stone. Similarly, the potential $\psi(x,y)$ used elsewhere becomes T(x,y), since Stone discusses solving Poisson's equation in the context of a heat flow problem. The variables AA, BB, etc., represent Stone's a, b, etc. The intention in doing this was to make STONE a "utility" program which can be extracted and used in other situations by those who wish to implement Stone's method for their purposes.

14.01 STONE1

This subprogram, which appears as an ENTRY statement in STONE, implements Stone's suggested sequence of his relaxation parameters, α, and initializes the arrays which are internal to STONE. Some of the notation utilized here (e.g., XCHHL, (YSUB+YOXIDE), JMAX5) would require revision were the reader desirous of extracting STONE for use as a "utility" program.

```
2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DAT
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

0002                 SUBROUTINE STONE(ISTONE,JMAX,KMAX,RESID,T)
                C
                C    IMPLEMENTS STONE'S METHOD FOR MATRIX FACTORIZATION.
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
                C
                C
0004                 COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,30
                2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                 COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                2THEAST(40,25),THSTH(40,25),THWEST(40,25),AREA(41,25)
0006                 COMMON/REAL/ACHNL,YSUB,YOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
                C
0007                 DIMENSION T(JMAX,KMAX)
0008                 DIMENSION EE(41,30),FF(41,30),V(41,30),ALPHA(18)
                C
                C    DEFINE FREQUENTLY USED FUNCTIONS.(N FOR FORWARD K,R FOR REVERSE).
                C
0009                 BN(B,E)=B/(1.D0+ALPH*E)
0010                 CN(D,F)=D/(1.D0+ALPH*F)
0011                 DN1(E,F)=BB*(ALPH*E-F)
0012                 DN2(F,E)=CC*(ALPH*F-E)
0013                 EN(F,E)=(F-ALPH*BB*E)/DD
0014                 FN(H,F)=(H-ALPH*CC*F)/DD
0015                 BR(H,E)=H/(1.D0+ALPH*E)
0016                 CR(D,F)=D/(1.D0+ALPH*F)
0017                 DR1(E,F)=BB*(ALPH*E-F)
0018                 DR2(F,E)=CC*(ALPH*F-E)
0019                 ER(F,E)=(F-ALPH*BB*E)/DD
0020                 FR(B,F)=(B-ALPH*CC*F)/DD
                C
                C
                C    ISTONE FLAGS WHICH ALPHA TO USE AND WHETHER FORWARD (ISTONE ODD)
                C    OR REVERSE (ISTONE EVEN) INDEXING OF K IS TO BE IMPLEMENTED.
                C
0021                 IF(ISTONE.GT.18) ISTONE=1
0023                 ALPH=ALPHA(ISTONE)
0024                 JMAXX=JMAX-1
0025                 KMAXX=KMAX-1
0026                 J=2*(ISTONE/2)
0027                 IF(J.EQ.ISTONE) GO TO 170
                C
                C    ISTONE IS ODD. INDEX K FORWARD
                C
                C
                C    THE CORNER POINT (1,1)
                C
0029                 DD=E(1,1)
0030                 EE(1,1)=F(1,1)/DD
0031                 FF(1,1)=H(1,1)/DD
0032                 R=G(1,1)-(E(1,1)*T(1,1)+F(1,1)*T(2,1)+H(1,1)*T(1,2))
0033                 RR=DABS(R)
0034                 RESID=RR
0035                 V(1,1)=R/DD
                C
                C    THE ROW K=1, EXCLUDING THE RIGHT CORNER POINT.
                C
0036                 DO 100 J=2,JMAXX
0037                 JP=J+1
0038                 JN=J-1
0039                 CC=CN(D(J,1),FF(JN,1))
0040                 DD=E(J,1)+DN2(FF(JN,1),EE(JN,1))
0041                 EE(J,1)=F(J,1)/DD
0042                 FF(J,1)=FN(H(J,1),FF(JN,1))
```

```
VCL 2.1  ( JAN 75 )          STONE          05/360  FORTRAN H EXTENDED          DA1

ISN 0043              R=G(J,1)-(B(J,1)*T(JN,1)+E(J,1)*T(J,1)+F(J,1)*T(JP,1)+H(J,1)*
                     1T(J,2))
ISN 0044              RR=DABS(R)
ISN 0045              IF(RR.GT.RESID) RESID=RR
ISN 0047          100 V(J,1)=(R-CC*V(JN,1))/DD
                  C
                  C       THE CORNER POINT (JMAX,1)
                  C
ISN 0048              J=JMAX
ISN 0049              JN=JMAXX
ISN 0050              CC=CN(D(J,1),FF(JN,1))
ISN 0051              DD=E(J,1)+DN2(FF(JN,1),EE(JN,1))
ISN 0052              EE(J,1)=0.
ISN 0053              FF(J,1)=FN(H(J,1),FF(JN,1))
ISN 0054              R=G(J,1)-(D(J,1)*T(JN,1)+E(J,1)*T(J,1)+H(J,1)*T(J,2))
ISN 0055              RR=DABS(R)
ISN 0056              IF(RR.GT.RESID) RESID=RR
ISN 0058              V(J,1)=(R-CC*V(JN,1))/DD
                  C
                  C       ALL ROWS EXCEPT ROW KMAX
                  C
ISN 0059              DO 120 K=2,KMAXX
ISN 0060              KP=K+1
ISN 0061              KN=K-1
                  C
                  C       THE LEFT BORDER POINT
                  C
ISN 0062              BB=BN(B(1,K),EE(1,KN))
ISN 0063              DD=E(1,K)+DN1(EE(1,KN),FF(1,KN))
ISN 0064              EE(1,K)=EN(F(1,K),EE(1,KN))
ISN 0065              FF(1,K)=H(1,K)/DD
ISN 0066              R=G(1,K)-(B(1,K)*T(1,KN)+E(1,K)*T(1,K)+F(1,K)*T(2,K)+H(1,K)*
                     1T(1,KP))
ISN 0067              RR=DABS(R)
ISN 0068              IF(RR.GT.RESID) RESID=RR
ISN 0070              V(1,K)=(R-BB*V(1,KN))/DD
                  C
                  C       THE ROW K EXCLUDING THE RIGHT BORDER POINT.
                  C
ISN 0071              DO 110 J=2,JMAXX
ISN 0072              JP=J+1
ISN 0073              JN=J-1
ISN 0074              BB=BN(B(J,K),EE(J,KN))
ISN 0075              CC=CN(D(J,K),FF(JN,K))
ISN 0076              DD=E(J,K)+DN1(EE(J,KN),FF(J,KN))+DN2(FF(JN,K),EE(JN,K))
ISN 0077              EE(J,K)=EN(F(J,K),EE(J,KN))
ISN 0078              FF(J,K)=FN(H(J,K),FF(JN,K))
ISN 0079              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+
                     1F(J,K)*T(JP,K)+H(J,K)*T(J,KP))
ISN 0080              RR=DABS(R)
ISN 0081              IF(RR.GT.RESID) RESID=RR
ISN 0083          110 V(J,K)=(R-BB*V(J,KN)-CC*V(JN,K))/DD
                  C
                  C       THE RIGHT BORDER POINT
                  C
ISN 0084              J=JMAX
ISN 0085              JN=JMAXX
ISN 0086              BB=BN(B(J,K),EE(J,KN))
ISN 0087              CC=CN(D(J,K),FF(JN,K))
ISN 0088              DD=E(J,K)+DN1(EE(J,KN),FF(J,KN))+DN2(FF(JN,K),EE(JN,K))
ISN 0089              EE(J,K)=0.
ISN 0090              FF(J,K)=FN(H(J,K),FF(JN,K))
ISN 0091              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+H(J,K)*
                     1T(J,KP))
ISN 0092              RR=DABS(R)
ISN 0093              IF(RR.GT.RESID) RESID=RR
ISN 0095          120 V(J,K)=(R-BB*V(J,KN)-CC*V(JN,K))/DD
                  C
                  C       THE CORNER POINT (1,KMAX)
                  C
ISN 0096              K=KMAX
ISN 0097              KN=KMAXX
```

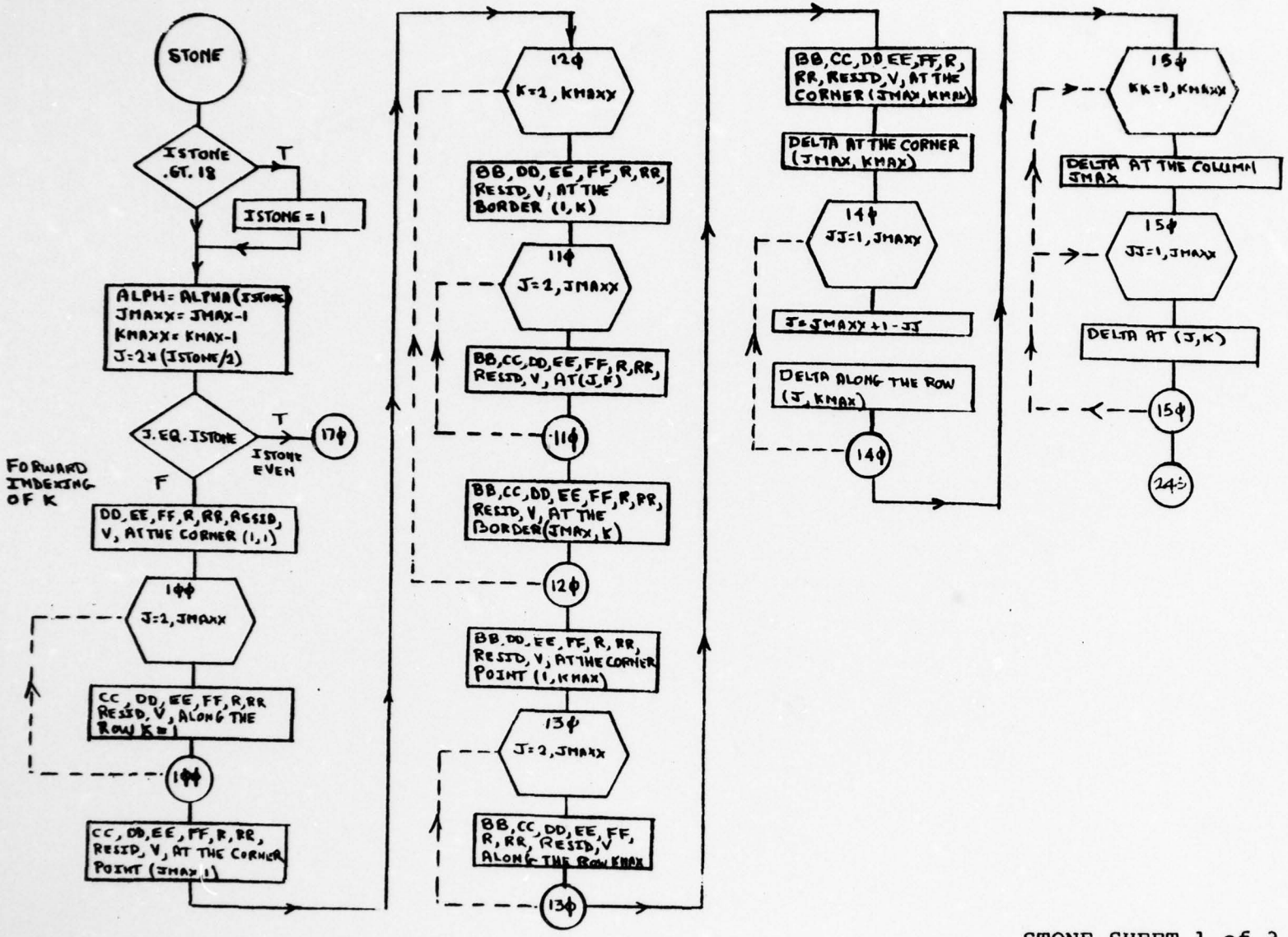
```
0098              BB=BN(B(1,K),EE(1,KN))
0099              DD=E(1,K)+DN1(EE(1,KN),FF(1,KN))
0100              EE(1,K)=EN(F(1,K),EE(1,KN))
0101              FF(1,K)=0.
0102              R=G(1,K)-(B(1,K)*T(1,KN)+E(1,K)*T(1,K)+F(1,K)*T(2,K))
0103              RR=DABS(R)
0104              IF(RR.GT.RESID) RESID=RR
0106              V(1,K)=(R-BB*V(1,KN))/DD
                C
                C     THE ROW KMAX. EXCLUDING THE CORNER POINT (JMAX,KMAX)
                C
0107              DO 130 J=2,JMAXX
0108              JP=J+1
0109              JN=J-1
0110              BB=BN(B(J,K),EE(J,KN))
0111              CC=CN(D(J,K),FF(JN,K))
0112              DD=E(J,K)+DN1(EE(J,KN),FF(J,KN))+DN2(FF(JN,K),EE(JN,K))
0113              EE(J,K)=EN(F(J,K),EE(J,KN))
0114              FF(J,K)=0.
0115              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+F(J,K)*
                 1T(JP,K))
0116              RR=DABS(R)
0117              IF(RR.GT.RESID) RESID=RR
0119          130 V(J,K)=(R-BB*V(J,KN)-CC*V(JN,K))/DD
                C
                C     THE CORNER POINT (JMAX,KMAX)
                C
0120              J=JMAX
0121              JN=JMAXX
0122              BB=BN(B(J,K),EE(J,KN))
0123              CC=CN(D(J,K),FF(JN,K))
0124              DD=E(J,K)+DN1(EE(J,KN),FF(J,KN))+DN2(FF(JN,K),EE(JN,K))
0125              EE(J,K)=0.
0126              FF(J,K)=0.
0127              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K))
0128              RR=DABS(R)
0129              IF(RR.GT.RESID) RESID=RR
0131              V(J,K)=(R-BB*V(J,KN)-CC*V(JN,K))/DD
                C
                C     INDEX K AND J IN REVERSE TO FIND DELTA
                C
                C     THE CORNER POINT (JMAX,KMAX)
                C
0132              K=KMAX
0133              DELTA(JMAX,K)=V(JMAX,K)
                C
                C     THE ROW KMAX
                C
0134              DO 140 JJ=1,JMAXX
0135              J=JMAXX+1-JJ
0136              JP=J+1
0137          140 DELTA(J,K)=V(J,K)-EE(J,K)*DELTA(JP,K)
                C
                C     THE ROW K, TREATING THE RIGHT BORDER FIRST.
                C
0138              DO 150 KK=1,KMAXX
0139              K=KMAXX+1-KK
0140              KP=K+1
0141              DELTA(JMAX,K)=V(JMAX,K)-FF(JMAX,K)*DELTA(JMAX,KP)
0142              DO 150 JJ=1,JMAXX
0143              J=JMAXX+1-JJ
0144              JP=J+1
0145          150 DELTA(J,K)=V(J,K)-EE(J,K)*DELTA(JP,K)-FF(J,K)*DELTA(J,KP)
0146              GO TO 240
                C
                C     ISTONE IS EVEN. INDEX K IN REVERSE ORDER.
                C
                C     THE CORNER POINT (1,KMAX)
                C
0147          170 K=KMAX
0148              KN=KMAXX
0149              DD=E(1,K)
```

```
ISN 0150              EE(1,K)=F(1,K)/DD
ISN 0151              FF(1,K)=B(1,K)/DD
ISN 0152              R=G(1,K)-(B(1,K)*T(1,KN)+E(1,K)*T(1,K)+F(1,K)*T(2,K))
ISN 0153              RR=DABS(R)
ISN 0154              RESID=RR
ISN 0155              V(1,K)=R/DD
                C
                C     THE ROW KMAX
                C
ISN 0156              DO 180 J=2,JMAXX
ISN 0157              JP=J+1
ISN 0158              JN=J-1
ISN 0159              CC=CR(D(J,K),FF(JN,K))
ISN 0160              DD=E(J,K)+DR2(FF(JN,K),EE(JN,K))
ISN 0161              EE(J,K)=F(J,K)/DD
ISN 0162              FF(J,K)=FR(B(J,K),FF(JN,K))/DD
ISN 0163              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+F(J,K)*
                     1T(JP,K))
ISN 0164              RR=DABS(R)
ISN 0165              IF(RR.GT.RESID) RESID=RR
ISN 0167        180   V(J,K)=(R-CC*V(JN,K))/DD
                C
                C     THE CORNER POINT (JMAX,KMAX)
                C
ISN 0168              J=JMAX
ISN 0169              JN=JMAXX
ISN 0170              CC=CR(D(J,K),FF(JN,K))
ISN 0171              DD=E(J,K)+DR2(FF(JN,K),EE(JN,K))
ISN 0172              EE(J,K)=0.
ISN 0173              FF(J,K)=FR(B(J,K),FF(JN,K))
ISN 0174              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K))
ISN 0175              RR=DABS(R)
ISN 0176              IF(RR.GT.RESID) RESID=RR
ISN 0178              V(J,K)=(R-CC*V(JN,K))/DD
                C
                C     ALL ROWS EXCEPT THE ROW K=1
                C
ISN 0179              DO 200 KK=2,KMAXX
ISN 0180              K=KMAXX+2-KK
ISN 0181              KP=K+1
ISN 0182              KN=K-1
                C
                C     THE LEFT BORDER POINT (1,K)
                C
ISN 0183              BB=BR(H(1,K),EE(1,KP))
ISN 0184              DD=E(1,K)+DR1(EE(1,KP),FF(1,KP))
ISN 0185              EE(1,K)=ER(F(1,K),EE(1,KP))
ISN 0186              FF(1,K)=B(1,K)/DD
ISN 0187              R=G(1,K)-(B(1,K)*T(1,KN)+E(1,K)*T(1,K)+F(1,K)*T(2,K)+H(1,K)*
                     1T(1,KP))
ISN 0188              RR=DABS(R)
ISN 0189              IF(RR.GT.RESID) RESID=RR
ISN 0191              V(1,K)=(R-BB*V(1,KP))/DD
                C
                C     THE ROW K EXCLUDING THE RIGHT BORDER POINT
                C
ISN 0192              DO 190 J=2,JMAXX
ISN 0193              JP=J+1
ISN 0194              JN=J-1
ISN 0195              BB=BR(H(J,K),EE(J,KP))
ISN 0196              CC=CR(D(J,K),FF(JN,K))
ISN 0197              DD=E(J,K)+DR1(EE(J,KP),FF(J,KP))+DR2(FF(JN,K),EE(JN,K))
ISN 0198              EE(J,K)=ER(F(J,K),EE(J,KP))
ISN 0199              FF(J,K)=FR(B(J,K),FF(JN,K))
ISN 0200              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+F(J,K)*
                     1T(JP,K)+H(J,K)*T(J,KP))
ISN 0201              RR=DABS(R)
ISN 0202              IF(RR.GT.RESID) RESID=RR
ISN 0204        190   V(J,K)=(R-BB*V(J,KP)-CC*V(JN,K))/DD
                C
                C     THE RIGHT BORDER POINT
                C
```

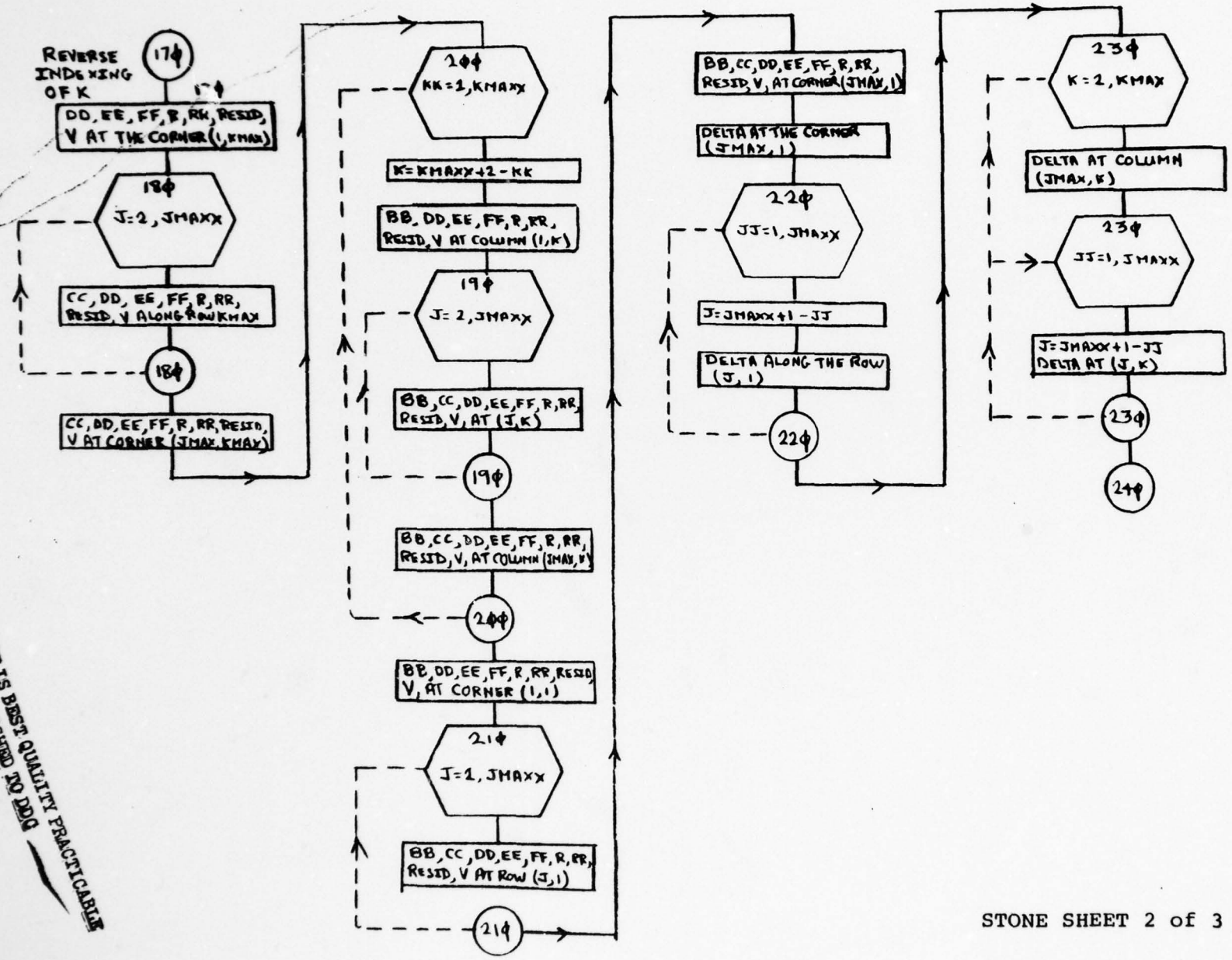
THIS IS AN ERROR & SHOULD BE DELETED!

```
0205              J=JMAX
0206              JN=JMAXX
0207              BB=BR(H(J,K),EE(J,KP))
0208              CC=CR(D(J,K),FF(JN,K))
0209              DD=E(J,K)+DR1(EE(J,KP),FF(J,KP))+DR2(FF(JN,K),EE(JN,K))
0210              EE(J,K)=0.
0211              FF(J,K)=FR(D(J,K),FF(JN,K))
0212              R=G(J,K)-(B(J,K)*T(J,KN)+D(J,K)*T(JN,K)+E(J,K)*T(J,K)+H(J,K)*
                 1T(J,KP))
0213              RR=DABS(R)
0214              IF(RR.GT.RESID) RESID=RR
0216          200 V(J,K)=(R-BB*V(J,KP)-CC*V(JN,K))/DD
                C
                C     THE CORNER POINT (1,1)
                C
0217              BB=BR(H(1,1),EE(1,2))
0218              DD=E(1,1)+DR1(EE(1,2),FF(1,2))
0219              EE(1,1)=ER(F(1,1),EE(1,2))
0220              FF(1,1)=0.
0221              R=G(1,1)-(E(1,1)*T(1,1)+F(1,1)*T(2,1)+H(1,1)*T(1,2))
0222              RR=DABS(R)
0223              IF(RR.GT.RESID) RESID=RR
0225              V(1,1)=(R-BB*V(1,2))/DD
                C
                C     THE ROW K=1, EXCLUDING THE RIGHT CORNER POINT.
                C
0226              DO 210 J=2,JMAXX
0227              JP=J+1
0228              JN=J-1
0229              BB=BR(H(J,1),EE(J,2))
0230              CC=CR(D(J,1),FF(JN,1))
0231              DD=E(J,1)+DR1(EE(J,2),FF(J,2))+DR2(FF(JN,1),EE(JN,1))
0232              EE(J,1)=ER(F(J,1),EE(J,2))
0233              FF(J,1)=0.
0234              R=G(J,1)-(D(J,1)*T(JN,1)+E(J,1)*T(J,1)+F(J,1)*T(JP,1)+H(J,1)*
                 1T(J,2))
0235              RR=DABS(R)
0236              IF(RR.GT.RESID) RESID=RR
0238          210 V(J,1)=(R-BB*V(J,2)-CC*V(JN,1))/DD
                C
                C     THE CORNER POINT (JMAX,1)
                C
0239              J=JMAX
0240              JN=JMAXX
0241              BB=BR(H(J,1),EE(J,2))
0242              CC=CR(D(J,1),FF(JN,1))
0243              DD=E(J,1)+DR1(EE(J,2),FF(J,2))+DR2(FF(JN,1),EE(JN,1))
0244              EE(J,1)=0.
0245              FF(J,1)=0.
0246              R=G(J,1)-(D(J,1)*T(JN,1)+E(J,1)*T(J,1)+H(J,1)*T(J,2))
0247              RR=DABS(R)
0248              IF(RR.GT.RESID) RESID=RR
0250              V(J,1)=(R-BB*V(J,2)-CC*V(JN,1))/DD
                C
                C     INDEX K FORWARD & J REVERSE TO FIND DELTA
                C
                C     THE POINT (JMAX,1)
                C
0251              DELTA(JMAX,1)=V(JMAX,1)
                C
                C     THE ROW K=1
                C
0252              DO 220 JJ=1,JMAXX
0253              J=JMAXX+1-JJ
0254              JP=J+1
0255          220 DELTA(J,1)=V(J,1)-EE(J,1)*DELTA(JP,1)
                C
                C     ALL REMAINING ROWS
                C
0256              DO 230 K=2,KMAX
0257              KN=K-1
                C
```

```
EL 2.1  ( JAN 75 )          STONE          05/360  FORTRAN H EXTENDED          DATE
                C       THE RIGHT BORDER POINT
                C
SN 0258                 DELTA(JMAX,K)=V(JMAX,K)-FF(JMAX,K)*DELTA(JMAX,KN)
                C
                C       ALL REMAINING POINTS OF THE RO#
                C
SN 0259                 DO 230 JJ=1,JMAXX
SN 0260                 J=JMAXX+1-JJ
SN 0261                 JP=J+1
SN 0262           230   DELTA(J,K)=V(J,K)-EE(J,K)*DELTA(JP,K)-FF(J,K)*DELTA(J,KN)
                C
                C       CALCULATE NEW T(J,K)
                C
SN 0263           240   DO 250 J=1,JMAX
SN 0264                 DO 250 K=1,KMAX
SN 0265           250   T(J,K)=T(J,K)+DELTA(J,K)
SN 0266                 RETURN
                C
                C       ENTRY STONE1 SETS UP ALPHA VALUES & INITIALIZES ALL ARRAYS
                C
SN 0267                 ENTRY STONE1(IMAX,JMAX,JMAX5)
                C
SN 0268                 DO 320 J=2,IMAX
SN 0269                 DO 320 K=2,JMAX5
SN 0270                 DX=ADLT(J)/XCHNL
SN 0271                 DY=ADLT(K)/(YSUB+YUXIDE)
SN 0272           320   EE(J,K)=2.D0/(1.D0/(DX*DX)+1.D0/(DY*DY))
                C       FIND AVERAGE
                C
SN 0273                 ALPHX=0.
SN 0274                 DO 330 J=2,IMAX
SN 0275                 DO 330 K=2,JMAX5
SN 0276           330   ALPHX=ALPHX+EE(J,K)
SN 0277                 ALPHX=ALPHX/(IMAX*JMAX5)
                C
                C       IMPLEMENT STONE'S EQ(27)
                C
SN 0278                 DO 340 J=1,9
SN 0279                 ALPHY=J-1
   0280                 ALPHY=ALPHY/8.D0
   0281           340   EE(J,1)=ALPHX**ALPHY
                C
                C       IMPLEMENT THE SEQUENCE OF 18 ALPHAS SUGGESTED BY STONE
                C
SN 0282                 ALPHA(1)=EE(9,1)
SN 0283                 ALPHA(3)=EE(8,1)
SN 0284                 ALPHA(5)=EE(3,1)
SN 0285                 ALPHA(7)=EE(8,1)
SN 0286                 ALPHA(9)=EE(5,1)
SN 0287                 ALPHA(11)=EE(2,1)
SN 0288                 ALPHA(13)=EE(7,1)
SN 0289                 ALPHA(15)=EE(4,1)
SN 0290                 ALPHA(17)=EE(1,1)
SN 0291                 DO 350 J=2,18,2
SN 0292           350   ALPHA(J)=ALPHA(J-1)
SN 0293                 DO 360 J=1,18
SN 0294           360   ALPHA(J)=1.D0-ALPHA(J)
                C
                C
                C       INITIALIZE ALL ARRAYS INTERNAL TO STONE TO ZERO.
                C
SN 0295                 DO 370 J=1,IMAX
SN 0296                 DO 370 K=1,JMAX5
SN 0297                 EE(J,K)=0.
SN 0298                 FF(J,K)=0.
SN 0299                 V(J,K)=0.
SN 0300           370   DELTA(J,K)=0.
SN 0301                 RETURN
SN 0302                 END
```



STONE SHEET 1 of 3



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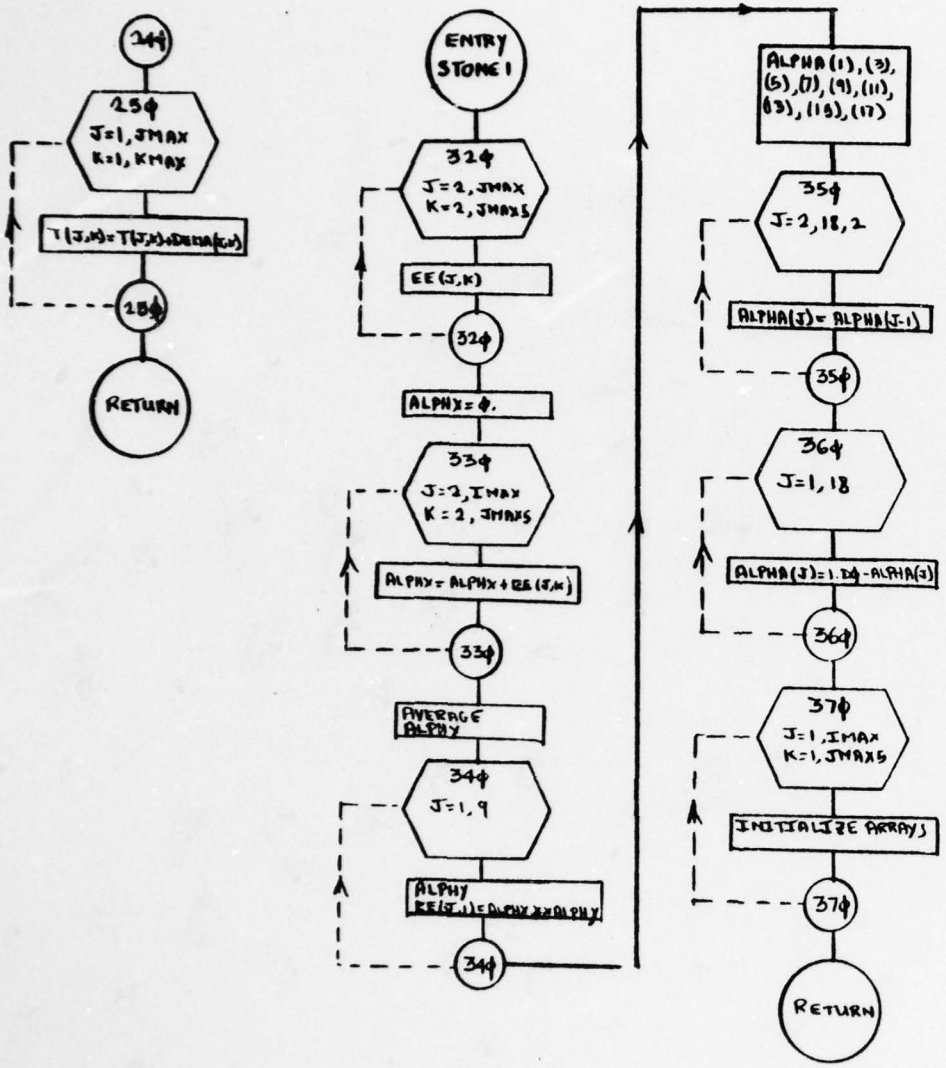


STONE SHEET 3 of 3

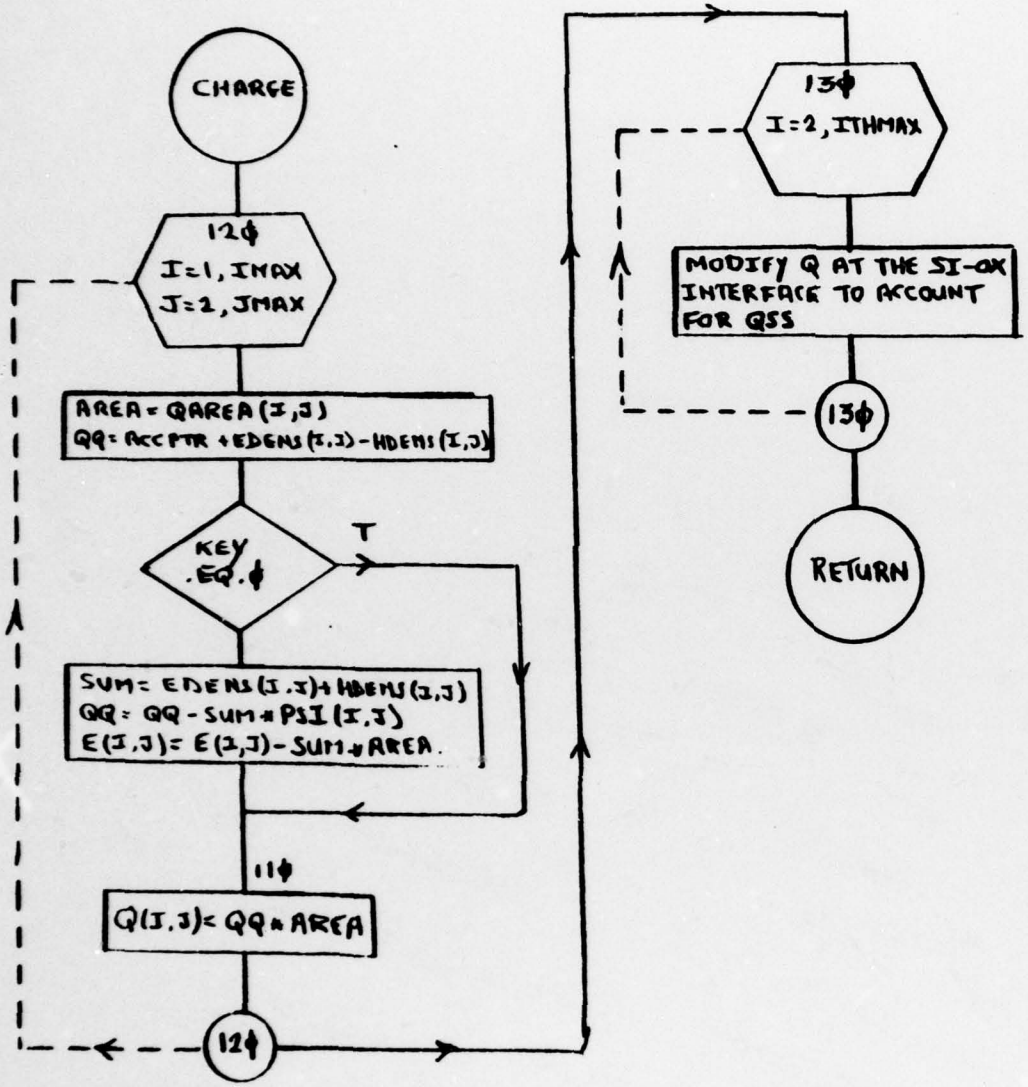
15.0 CHARGE

This subprogram calculates the charge term Q_p defined in eq. (23) of Chapter I, Section 3.01.2, using eq. (36) if the Picard iterative procedure is in use. As explained in Section 4.02 of Chapter I, it invokes appropriate modifications of both Q_p and E_p, in eq. (23), if Gummel's algorithm is in use. These procedures are almost trivial, and hardly call for a separate subroutine for their implementation. The fact that Subroutine CHARGE exists as such is a vestige of the fact that, at one time during the development, more elaborate procedures for calculating Q_p were tried; procedures which were ultimately found unnecessary.

```
2.1  ( JAN 75 )                          OS/360  FORTRAN H EXTENDED                    DA
TED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

S IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
             SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC NO

0002                 SUBROUTINE CHARGE(KEY)
                C
                C    CALCULATES THE CHARGE TERM APPEARING IN THE PICARD & GUMMEL
                C    FINITE-DIFFERENCE EQUATIONS FOR PSI.
                C
0003                 IMPLICIT REAL*8(A-H,O-Z)
                C
0004                 COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                    1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),Q(41,30),DELTA(41,30
                    2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
0005                 COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30)
                    1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25),
                    2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
0006                 COMMON/REAL/XCHNL,YSUB,YOXIDE,YJNCTN,DUNOR,ACCPTR,VGATE,VSUB,
                    1VDRAIN,DIELK,QSS,CURRNT,EDENS0,PSIEQ
0007                 COMMON/INTGR/IMAX,JMAX,JMAXS,JUNCTN,ITHMAX
0008                 COMMON/CNTRL/ OMEGA
                C
0009                 DO 120 I=1,IMAX
0010                 DO 120 J=2,JMAX
0011            100  AREA=GAREA(I,J)
0012                 QQ=ACCPTR+EDENS(I,J)-HDENS(I,J)
0013                 IF(KEY.EQ.0) GO TO 110
                C
                C    MODIFY E & Q FOR THE GUMMEL ALGORITHM
                C
0015                 SUM=EDENS(I,J)+HDENS(I,J)
0016                 QQ=QQ-SUM*PSI(I,J)
0017                 E(I,J)=E(I,J)-SUM*AREA
0018            110  Q(I,J)=QQ*AREA
0019            120  CONTINUE
                C
                C    MODIFY Q AT THE Si-Ox INTERFACE TO ACCOUNT FOR QSS.
                C
0020                 DO 130 I=2,ITHMAX
0021                 Q(I,JMAX)=Q(I,JMAX)-QSS*XTHDLT(I)/2.D0
0022            130  Q(I,JMAX+1)=-QSS*XTHDLT(I)/(2.D0*DIELK)
0023                 RETURN
0024                 END
```



CHARGE SHEET 1 of 1

16.0 OUTPUT

This subprogram performs the obvious function of providing for the printing of output data concerning the solution. It should be transparent to the user, and is the one subprogram which he is most likely to wish to modify to suit his own needs. About the only aspect of OUTPUT that needs clarification is the method used to calculate the inversion charge QI, and the depletion charge QD. In both instances, piece-wise exponential "fits" are utilized to approximate the electron and hole densities as a function of the vertical coordinate y between adjacent lattice points, and an analytic evaluation of the integrals required for those parameters is applied. No flowchart for OUTPUT is included herein, because it is deemed unnecessary.

```
EVEL 2.1  ( JAN 75 )                    OS/360  FORTRAN H EXTENDED                    DA
UESTED OPTIONS: NODECK,LOAD,OPT=2,SOURCE,MAP

IONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(74) SIZE(MAX) AUTODBL(NONE)
                SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT NOXREF ALC N

 ISN 0002              SUBROUTINE OUTPUT
                C
 ISN 0003              IMPLICIT REAL*8(A-H,O-Z)
                C
 ISN 0004              COMMON/ARRAYS/PSI(41,30),EDENS(41,25),HDENS(41,25),THETA(40,25),
                      1B(41,30),D(41,30),E(41,30),F(41,30),H(41,30),G(41,30),DELTA(41,3
                      2,XPOS(41),YPOS(25),XTHPOS(41),YTHPOS(25)
 ISN 0005              COMMON/COEFF/XDLT(42),YDLT(31),XTHDLT(41),YTHDLT(25),HNRTH(41,30
                      1HEAST(41,30),HSTH(41,30),HWEST(41,30),HCNTR(41,30),THNRTH(40,25)
                      2THEAST(40,25),THSTH(40,25),THWEST(40,25),GAREA(41,25)
 ISN 0006              COMMON/REAL/XCHNL,YSUB,VOXIDE,YJNCTN,DONOR,ACCPTR,VGATE,VSUB,
                      1VDRAIN,DIELK,GSS,CURRNT,EDENS0,PSIEQ
 ISN 0007              COMMON/INTGR/IMAX,JMAX,JMAX5,JUNCTN,ITHMAX
 ISN 0008              COMMON/NRMLZE/DNSNRM,PSINRM,DEBYE,GNORM,CRTNRM
                C
 ISN 0009              DIMENSION X(41),SISURF(41),PHIN(41),QI(41),QD(41),SCRTCH(41),
                      1YD(41),YI(41)
                C
                C       INTEGRATE JSUBX ALONG THE DRAIN TO OBTAIN THE DRAIN CURRENT.
                C
 ISN 0010              J=JMAX
 ISN 0011              YY=YSUB*DEBYE
 ISN 0012              VD=VDRAIN*PSINRM
 ISN 0013              VG=VGATE*PSINRM
 ISN 0014              VB=VSUB*PSINRM
 ISN 0015              CRNT=CURRNT*CRTNRM*DEBYE/1.D1
 ISN 0016              WRITE(6,1000) VG,VD,VB,CRNT
 ISN 0017              WRITE(6,1010) YY
 ISN 0018       1000 FORMAT(1H1,///,9X,'CURRENT VOLTAGE RESULT :',//,13X,'VG=',1PD10.
                     11X,'VOLTS',/,13X,'VD=',1PD10.2,1X,'VOLTS',/,13X,'VB=',1PD10.2,1X
                     2'VOLTS',/,13X,'ID=',1PD10.2,1X,'MILLIAMPS/CM OF CHANNEL WIDTH',/
 ISN 0019       1010 FORMAT(9X,'SUBSTRATE THICKNESS (AUTOMATICALLY DETERMINED) IS YS(
                     1',1PD11.3,' MICROMETERS',//)
                C
                C       CALCULATE THE INTERFACE SURFACE PARAMETERS.
                C
 ISN 0020              DYSURF=YPOS(JMAX)
 ISN 0021              I=1
 ISN 0022          100 I=I+1
 ISN 0023              IF(I.GT.ITHMAX) GO TO 190
 ISN 0025              X(I)=XPOS(I)*DEBYE
 ISN 0026              SISURF(I)=(PSI(I,JMAX)+PSI(I,JMAX+1))/2.D0+GSS*DYSURF/2.D0
 ISN 0027              PHIN(I)=PSI(I,JMAX)-DLOG(EDENS(I,JMAX))
 ISN 0028              DSI=SISURF(I)-PSI(I,JMAX)
 ISN 0029              QQI=EDENS(I,JMAX)*DYSURF*(DEXP(DSI)-1.D0)/DSI
 ISN 0030              QQD=ACCPTR*DYSURF
 ISN 0031              YD(I)=DYSURF*DEBYE
 ISN 0032              YI(I)=DYSURF*DEBYE
 ISN 0033              KFLAGI=0
 ISN 0034              KFLAGD=0
 ISN 0035              J=JMAX+1
                C
                C       INVERSION CHARGE
                C
 ISN 0036          110 J=J-1
 ISN 0037              IF(J.GT.1) GO TO 120
 ISN 0039              GO TO 100
 ISN 0040          120 IF(KFLAGI.GT.0) GO TO 150
 ISN 0042              IF(EDENS(I,J).LE.1.D0.OR.EDENS(I,J-1).LT.0) GO TO 140
 ISN 0044              YI(I)=YI(I)+YDLT(J)*DEBYE
 ISN 0045              RATIO=EDENS(I,J-1)/EDENS(I,J)
 ISN 0046              IF(RATIO.GT.0.95D0.AND.RATIO.LT.1.05D0) GO TO 130
 ISN 0048              QQI=QQI+(EDENS(I,J-1)-EDENS(I,J))*YDLT(J)/DLOG(RATIO)
 ISN 0049              GO TO 150
 ISN 0050          130 QQI=QQI+EDENS(I,J)*YDLT(J)/(1.D0-(EDENS(I,J-1)-EDENS(I,J))/(2.D
                     1EDENS(I,J)))
 ISN 0051              GO TO 150
```

```
2.1  ( JAN 75 )          OUTPUT          OS/360  FORTRAN H EXTENDED          DATE
0052        140 KFLAGI=1
            C
            C       DEPLETION CHARGE
            C
0053        150 IF(KFLAGD.GT.0) GO TO 180
0055            IF(HDENS(I,J).GT.0.9500*ACCPTR.OR.J.EQ.2) GO TO 170
0057            YD(I)=YD(I)+YDLT(J)*DEBYE
0058            RATIO=HDENS(I,J-1)/HDENS(I,J)
0059            IF(RATIO.GT.0.9500.AND.RATIO.LT.1.0500) GO TO 160
0061            QQD=QQD+(ACCPTR-(HDENS(I,J-1)-HDENS(I,J))/DLOG(RATIO))*YDLT(J)
0062            GO TO 110
0063        160 QQD=QQD+(ACCPTR-HDENS(I,J)/(1.D0-(HDENS(I,J-1)-HDENS(I,J))/
               12.D0*HDENS(I,J)))*YDLT(J)
0064            GO TO 110
0065        170 KFLAGD=1
0066        180 IF(KFLAGI.EQ.0) GO TO 110
0068            QI(I)=QQI*QNORM
0069            QD(I)=QQD*QNORM
0070            SISURF(I)=SISURF(I)*PSINRM
0071            PHIN(I)=PHIN(I)*PSINRM
0072            GO TO 100
            C
            C       WRITE THE INTERFACE PARAMETERS
            C
0073        190 J=JMAX
0074            QI(1)=0.
0075            QD(1)=0.
0076            QI(I)=0.
0077            QD(I)=0.
0078            SISURF(1)=PSI(1,J)*PSINRM
0079            SISURF(I)=PSI(I,J)*PSINRM
0080            PHIN(1)=(PSI(1,J)-DLOG(EDENS(1,J)))*PSINRM
0081            PHIN(I)=(PSI(I,J)-DLOG(EDENS(I,J)))*PSINRM
0082            X(1)=0.
0083            X(I)=XPOS(I)*DEBYE
0084            YD(1)=0.
0085            YI(1)=0.
0086            YD(I)=0.
0087            YI(I)=0.
0088            WRITE(6,1020)
0089     1020 FORMAT(1H0,9X,'SUMMARY OF CONDITIONS ALONG THE SILICON-OXIDE INTE
               1FACE :',//,15X,'X',9X,'PSI',8X,'PHIN',7X,'QI',10X,'QD',
               210X,'YI',10X,'YD',//)
0090            DO 200 I=1,IMAX
0091        200 WRITE(6,1030)X(I),SISURF(I),PHIN(I),QI(I),QD(I),YI(I),YD(I)
0092     1030 FORMAT(11X,1P7D11.3)
            C
            C       BULK PARAMETERS
            C
0093            WRITE(6,1040)
0094     1040 FORMAT(1H1,5X,'X(I),I=1,IMAX',//)
0095            WRITE(6,1050)(X(I),I=1,IMAX)
0096     1050 FORMAT(20X,1P10D10.2)
0097            WRITE(6,1060)
0098     1060 FORMAT(1H0,5X,'J',9X,'Y(J)',15X,'PSI(I,J),I=1,IMAX',//)
0099            YY=-YOXIDE*DEBYE
0100            DO 220 JJ=1,JMAX5
0101            J=JMAX5+1-JJ
0102            DO 210 I=1,IMAX
0103        210 SCRTCH(I)=PSI(I,J)*PSINRM
0104            WRITE(6,1070)J,YY,(SCRTCH(I),I=1,10)
0105     1070 FORMAT(15,5X,1PD10.2,1P10D10.2)
0106            WRITE(6,1080)(SCRTCH(I),I=11,IMAX)
0107     1080 FORMAT(20X,1P10D10.2)
0108        220 YY=YY+YDLT(J)*DEBYE
0109            WRITE(6,1040)
0110            WRITE(6,1050)(X(I),I=1,IMAX)
0111            WRITE(6,1090)
0112     1090 FORMAT(1H0,5X,' J',9X,'Y(J)',15X,'EDENS(I,J),I=1,IMAX',//)
0113            YY=DYSURF*DEBYE
0114            DO 240 JJ=1,JMAX
0115            J=JMAX+1-JJ
```

```
ISN 0116            DO 230 I=1,IMAX
ISN 0117        230 SCRTCH(I)=EDENS(I,J)*DNSNRM
ISN 0118            WRITE(6,1070)J,YY,(SCRTCH(I),I=1,10)
ISN 0119            WRITE(6,1080)(SCRTCH(I),I=11,IMAX)
ISN 0120        240 YY=YY+YDLT(J)*DEBYE
ISN 0121            WRITE(6,1040)
ISN 0122            WRITE(6,1050)(X(I),I=1,IMAX)
ISN 0123            WRITE(6,1100)
ISN 0124       1100 FORMAT(1H0,5X,'J',9X,'Y(J)',15X,'HDENS(I,J),I=1,IMAX',//)
ISN 0125            YY=DYSURF*DEBYE
ISN 0126            DO 260 JJ=1,JMAX
ISN 0127            J=JMAX+1-JJ
ISN 0128            DO 250 I=1,IMAX
ISN 0129        250 SCRTCH(I)=HDENS(I,J)*DNSNRM
ISN 0130            WRITE(6,1070)J,YY,(SCRTCH(I),I=1,10)
ISN 0131            WRITE(6,1080)(SCRTCH(I),I=11,IMAX)
ISN 0132        260 YY=YY+YDLT(J)*DEBYE
ISN 0133            DO 270 I=1,ITHMAX
ISN 0134        270 X(I)=XTHPOS(I)*DEBYE
ISN 0135            WRITE(6,1110)
ISN 0136       1110 FORMAT(1H1,5X,'XTH(I),I=1,ITHMAX',//)
ISN 0137            WRITE(6,1050)(X(I),I=1,ITHMAX)
ISN 0138            WRITE(6,1120)
ISN 0139       1120 FORMAT(1H0,5X,'J',9X,'YTH(J)',13X,'THETA(I,J),I=1,ITHMAX')
ISN 0140            YY=0.
ISN 0141            DO 290 JJ=1,JMAX
ISN 0142            J=JMAX+1-JJ
ISN 0143            DO 280 I=1,ITHMAX
ISN 0144        280 SCRTCH(I)=THETA(I,J)
ISN 0145            WRITE(6,1130)J,YY,(SCRTCH(I),I=1,10)
ISN 0146       1130 FORMAT(1H0,I4,5X,1PD10.2,1P10D10.2)
ISN 0147            WRITE(6,1080)(SCRTCH(I),I=11,ITHMAX)
ISN 0148        290 YY=YY+YTHDLT(J)*DEBYE
ISN 0149            WRITE(6,1110)
ISN 0150            WRITE(6,1050)(X(I),I=1,ITHMAX)
ISN 0151            WRITE(6,1140)
ISN 0152       1140 FORMAT(1H0,5X,'J',9X,'Y(J)',15X,'ESUBX(I,J),I=1,ITHMAX')
ISN 0153            YY=DYSURF*DEBYE
ISN 0154            DO 310 JJ=2,JMAX
ISN 0155            J=JMAX+2-JJ
ISN 0156            DO 300 I=1,ITHMAX
ISN 0157        300 SCRTCH(I)=(PSI(I,J)-PSI(I+1,J))*PSINRM*1.D4/(XDLT(I+1)*DEBYE)
ISN 0158            WRITE(6,1130)J,YY,(SCRTCH(I),I=1,10)
ISN 0159            WRITE(6,1080)(SCRTCH(I),I=11,ITHMAX)
ISN 0160        310 YY=YY+YDLT(J)*DEBYE
ISN 0161            DO 315 I=1,IMAX
ISN 0162        315 X(I)=XPOS(I)*DEBYE
ISN 0163            WRITE(6,1040)
ISN 0164            WRITE(6,1050)(X(I),I=1,IMAX)
ISN 0165            WRITE(6,1150)
ISN 0166       1150 FORMAT(1H0,5X,'J',9X,'YTH(J)',13X,'ESUBY(I,J),I=1,IMAX',//)
ISN 0167            YY=0.
ISN 0168            DO 320 I=2,ITHMAX
ISN 0169        320 SCRTCH(I)=(SISURF(I)-PSI(I,JMAX)*PSINRM)*1.D4/(DYSURF*DEBYE)
ISN 0170            SCRTCH(1)=0.
ISN 0171            SCRTCH(IMAX)=0.
ISN 0172            WRITE(6,1070)JMAX,YY,(SCRTCH(I),I=1,10)
ISN 0173            WRITE(6,1080)(SCRTCH(I),I=11,IMAX)
ISN 0174            YY=YY+YTHDLT(JMAX)*DEBYE
ISN 0175            JMAXX=JMAX-1
ISN 0176            DO 340 JJ=1,JMAXX
ISN 0177            J=JMAXX+1-JJ
ISN 0178            DO 330 I=1,IMAX
ISN 0179        330 SCRTCH(I)=(PSI(I,J+1)-PSI(I,J))*PSINRM*1.D4/(YDLT(J+1)*DEBYE)
ISN 0180            WRITE(6,1070)J,YY,(SCRTCH(I),I=1,10)
ISN 0181            WRITE(6,1080)(SCRTCH(I),I=11,IMAX)
ISN 0182        340 YY=YY+YTHDLT(J)*DEBYE
ISN 0183            DO 350 I=1,ITHMAX
ISN 0184        350 X(I)=XTHPOS(I)*DEBYE
ISN 0185            WRITE(6,1110)
ISN 0186            WRITE(6,1050)(X(I),I=1,ITHMAX)
ISN 0187            WRITE(6,1160)
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2.1  ( JAN 75 )          OUTPUT          OS/360  FORTRAN H EXTENDED          DATE
0188       1160 FORMAT(1H0,5X,'J',9X,'Y(J)',10X,'JSUBX(I,J),I=1,ITHMAX')
0189            YY=DYSURFACE*DEBYE
0190            DO 370 JJ=2,JMAX
0191            J=JMAX+2-JJ
0192            DO 360 I=1,ITHMAX
0193        360 SCRTCH(I)=CURRNT*(THETA(I,J-1)-THETA(I,J))*CRTNRM/YTHDLT(J)
0194            WRITE(6,1130)J,YY,(SCRTCH(I),I=1,10)
0195            WRITE(6,1080)(SCRTCH(I),I=11,ITHMAX)
0196        370 YY=YY+YDLT(J)*DEBYE
0197            DO 375 I=1,IMAX
0198        375 X(I)=XPOS(I)*DEBYE
0199            WRITE(6,1040)
0200            WRITE(6,1050)(X(I),I=1,IMAX)
0201            WRITE(6,1170)
0202       1170 FORMAT(1H0,5X,'J',9X,'YTH(J)',13X,'JSUBY(I,J),I=2,ITHMAX')
0203            YY=0.
0204            DO 390 JJ=1,JMAX
0205            J=JMAX+1-JJ
0206            SCRTCH(1)=0.
0207            DO 380 I=2,ITHMAX
0208        380 SCRTCH(I)=CURRNT*(THETA(I-1,J)-THETA(I,J))*CRTNRM/XTHDLT(I)
0209            WRITE(6,1130)J,YY,(SCRTCH(I),I=1,10)
0210            WRITE(6,1080)(SCRTCH(I),I=11,ITHMAX)
0211        390 YY=YY+YTHDLT(J)*DEBYE
0212            RETURN
0213            END
```

CHAPTER V
Conclusions

The two-dimensional model for an MOSFET described herein has proven to produce results in good agreement with those of the predecessor program developed by Mock. A single check with an experimental p-channel structure, whose parameters have been well characterized by means of "test sites" has also shown good agreement with experiment.

It has been found that Gummel's iterative algorithm, when overrelaxed, proves to be impressively efficient in terms of rate of convergence, when compared with Mock's, Picard and Gummel procedure. The author is unaware of instances in which such overrelaxation procedures have been previously used for solving the nonlinear equations involved with semiconductor devices, although they have been widely used for the equations encountered with vacuum tube devices. In view of the success of this approach, further investigations of the procedure, in the interest of optimization of convergence rates would appear desirable.

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